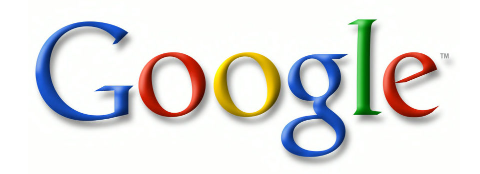


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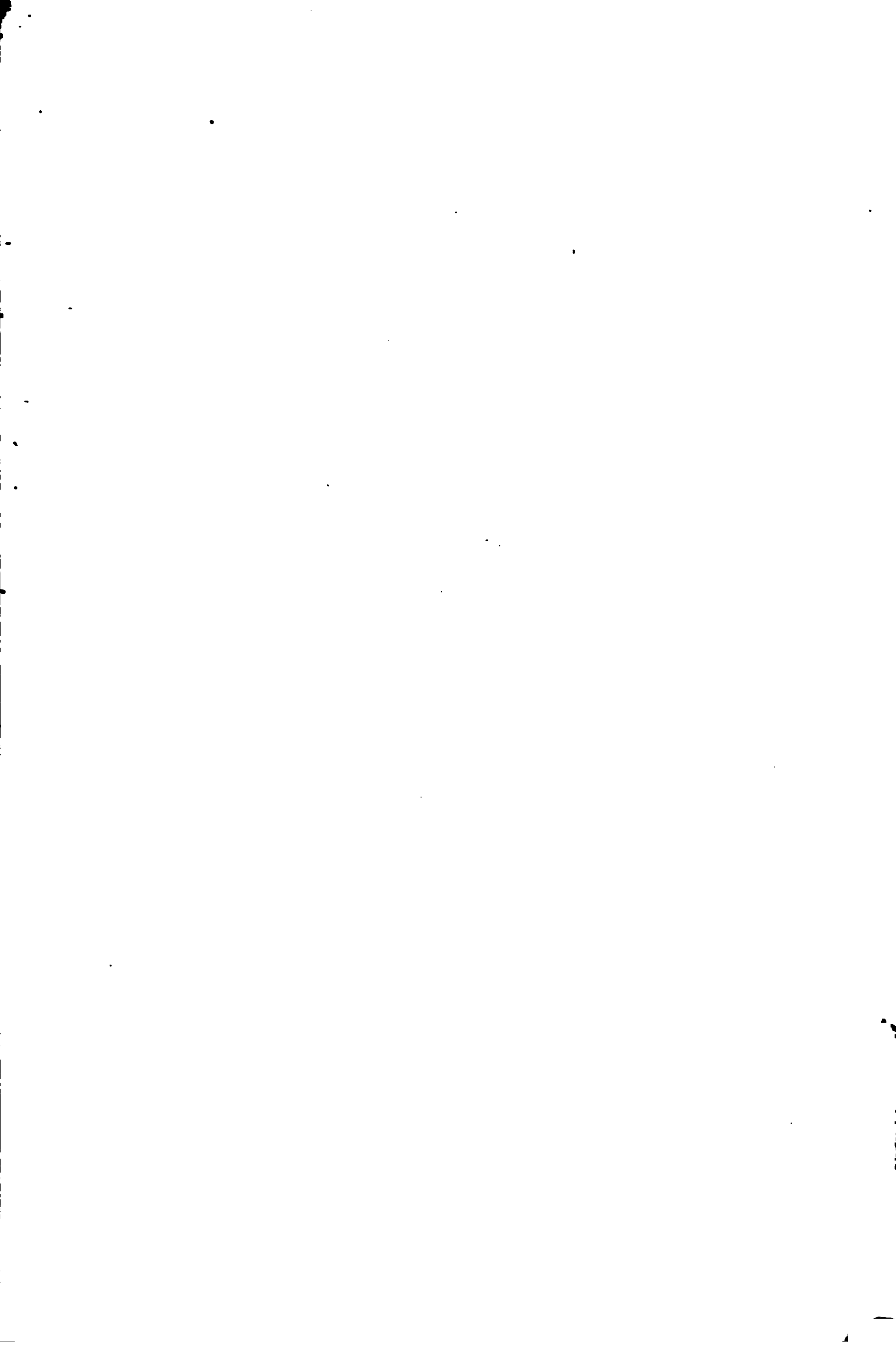


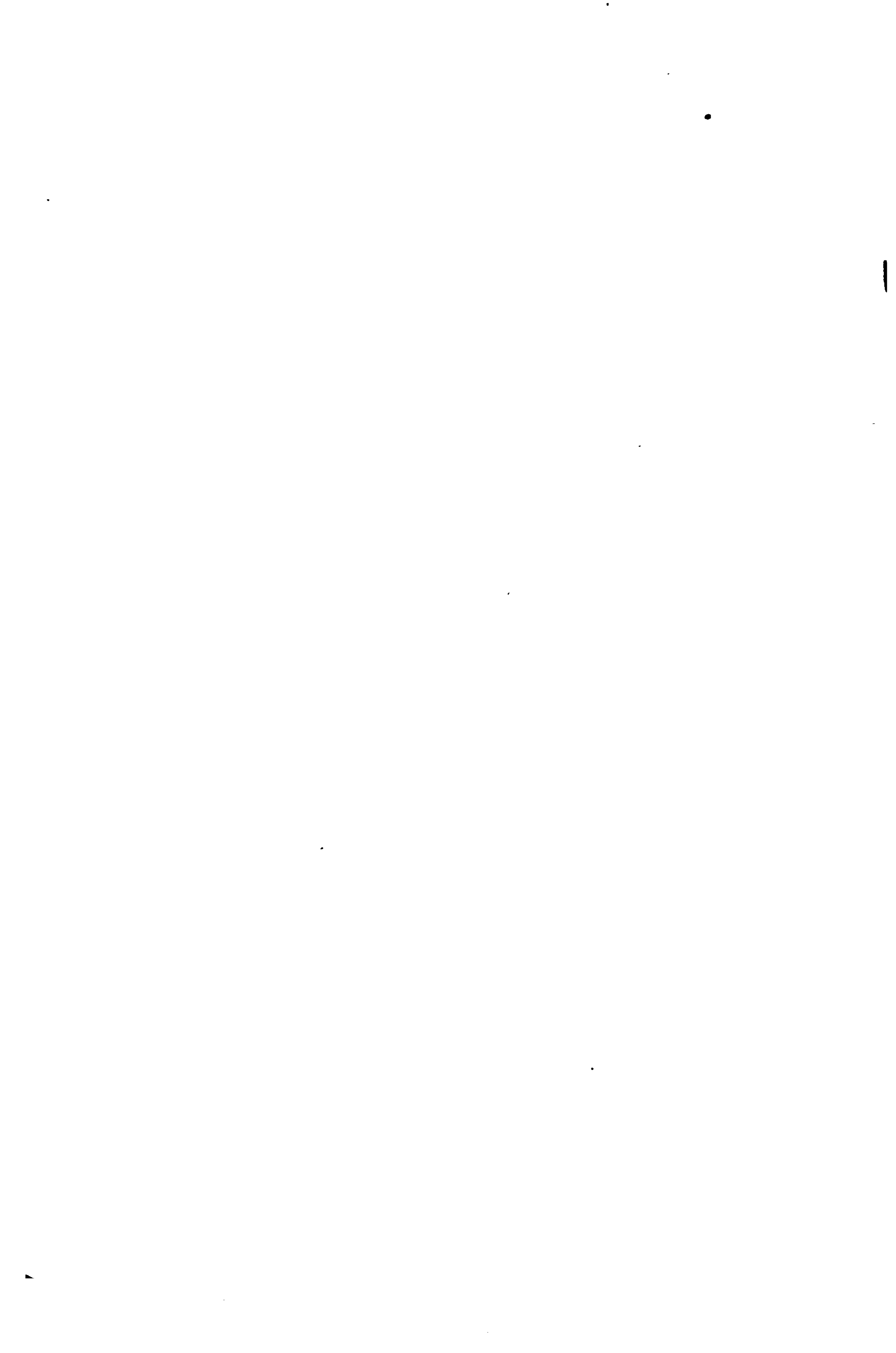
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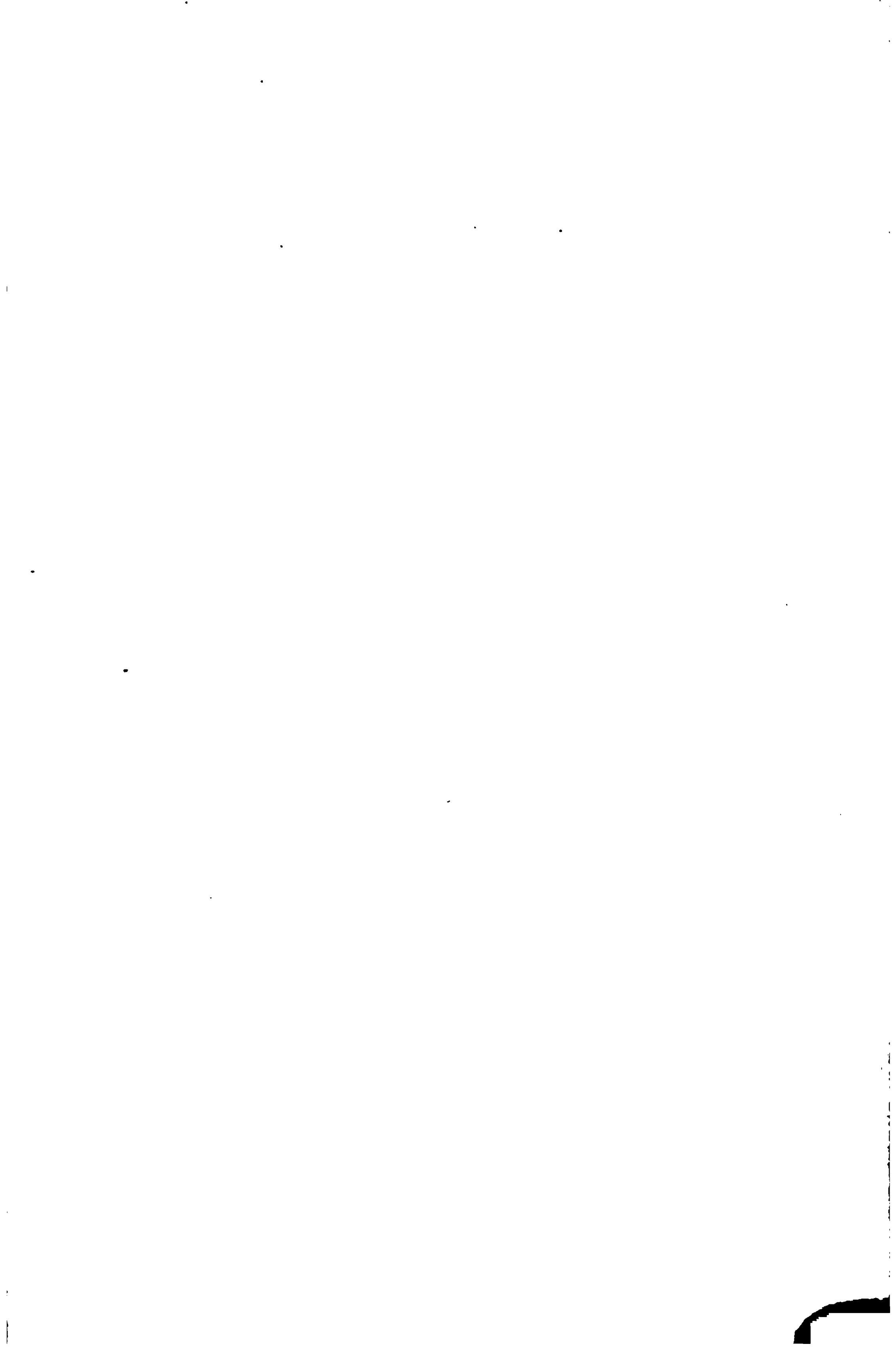
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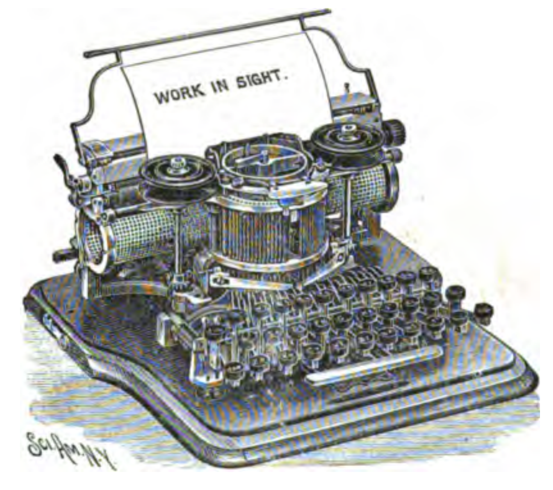
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KANSAS UNIVERSITY QUARTERLY.

VOL. IV. JULY, 1895. NO. 1.

Natural Gas and Coal Oil in Kansas.

BY E. H. S. BAILEY.

Petroleum or "Rock Oil" of some sort has been found in almost all countries and in nearly every state of the Union. Practically the production of Petroleum in this country began with the striking of oil by Col. Drake at Titusville in 1859. Most of the oil that is put on the market now comes from the states of Pennsylvania, New York, Ohio, West Virginia, California, Colorado, and Indiana, and considerable from Canada. As the production begins to diminish in the older fields, there are many prospectors hoping to find newer and better ones.

One of these newly-discovered fields for gas and oil is the state of Kansas. These products have been known in various localities in the state for a number of years, but it is only since December, 1892, when the first successful well was bored near Neodesha, that there has been any extensive development in this direction.

The gas and oil field in this state, from present developments, seems to extend from the vicinity of Paola, Miami county, in a south-westerly direction, to the southern border of the state, near Peru, a distance of about 120 miles in a straight line. The width of this zone has not been fully established, but from borings that have been made it does not appear to be over 25 miles at any point. These products have been found in commercial quantities in the counties of Miami, Allen, Neosho, Wilson, Labette, Montgomery and Chautauqua, with small quantities in Wyandotte, Linn and Bourbon counties. It is evident that the borders of this field are irregular, and many more holes must be bored to define it accurately. For convenience most of the borings have been made near the streams, and thus far the work has been done in the valleys of the Marais des Cygnes, the

Neosho, the Fall, the Caney and the Verdigris rivers. Successful borings have been made in the following towns:—Paola, Osawatomie, Louisburg, Iola, Humboldt, Thayer, Cherryvale, Mound Valley, Ft. Scott, Fulton, Coffeyville, Independence, Peru, Sedan, and Neodesha. Small developments may have been made in other places, for it is known that in some localities farmers have bored shallow wells which yield a sufficient supply of gas for domestic and heating purposes.

The method of boring the wells is that which has been used so extensively in the eastern oil fields. The outfit is as inexpensive as possible, and the material which is forced out of the boring by water is usually quite finely divided, yet coarse enough so that it can be readily identified. The casing of the well is often eight inches for the first 300 or 400 feet, then for the next 300 feet it is about six inches, while at the bottom of the well it is diminished to 4½ inches. After passing through the various layers of limestone and shale, frequently an "oil sand," as it is called, is struck some distance above the true oil-bearing layer. The oil-sand, which is really the porous rock alluded to below as the reservoir of the oil, is of various thickness but often from ten to twenty feet. If the conditions are favorable, yet the yield of the well in oil is not very large, and there is no strong flow of gas, it is ofen considered advisable to "shoot" the well. This is the term used to indicate the explosion of a charge of nitroglycerine in the oil-bearing rock, to produce a greater cavity near the bottom of the hole, so that the oil may flow more freely from the shattered rock For this purpose the well is carefully cleaned out and its depth noted, so as to get the charge in the right place. The man who makes it his business to "shoot" wells for a company, brings with him overland in a wagon from his factory, the requisite amount of nitroglycerine, from 10 to 60 quarts, in tin cans. When all is ready, he pours the explosive into a long cylindrical can which is suspended over the well. In the top of this can is fixed an exploder, and after the charge has been carefully lowered to the proper depth, some fortunate bystander is allowed to drop the "go-devil" onto the cap. This is simply a piece of two inch gas pipe, which sliding down several hundred feet, strikes a very heavy blow and explodes the charge. At the surface the force of the explosion is at first not very apparent, but soon a column of water, gas and oil shoots from the well, and gradually mounts higher till it is often many feet above the top of the derrick. This is often accompanied with a terrific noise and, in the case of a "gusher," the stream continues to flow for some time. Later, if the well is a producer of oil, the oil rises and begins to flow over the top of the well. Arrangements are then made to pump the well, or if there is no storage tank at hand, it is simply capped till such receptacles shallbe provided.

OIL AND GAS REGIONS OF KANSAS.

MIAMI COUNTY.

In the vicinity of Paola there have been at least seventy-five wells bored. There was great activity here in the gas industry five or six years ago, and a large amount of capital was invested in boring wells. About 12 wells are at present yielding gas, and three of these some oil. There are three of those not yielding gas that furnish some oil. The pressure of the combined wells as delivered at the regulator was 35 pounds per square inch in the summer. After the pressure had been reduced, the gas was delivered to customers at a pressure of about two inches, water pressure. The gas has considerable odor of sulphur compounds, so that there is little danger that it will escape into a room and not be noticed. Some difficulty has been experienced here in keeping sufficient pressure on the mains for the use of consumers in winter, when so large an amount is used for heating purposes.

In the vicinity of Osawatomie there is a stronger flow of gas than at Paola. There are at least eight wells here, most of them yielding an abundant supply of gas, and new wells have been bored recently. The pressure runs up above 200 pounds to the square inch, and the depth of the wells is from 324 to 686 feet. A large number of private families are supplied with the gas and it is used at the State Insane Asylum.

At Louisburgh, in the eastern part of the county, there is at present only one well yielding gas, and this only in a limited quantity.

ALLEN COUNTY.

The Iola Mineral Well was the first development in this line here. At a depth of 626 feet gas was obtained, and at one time the quantity was estimated to be over 5000 cubic feet daily; now, however, owing to the flooding and partial filling up of the well, there is practically no gas. The well still affords an abundant supply of mineral water. Previous to this year 5 wells were supplying the town with gas, but more recently a new well has been bored in the southwestern part of the town, that yields gas at a rock pressure of 350 pounds to the square inch. The gas issues from a 4⅞ inch pipe and comes from a depth of 835 feet, although the well has been bored to the depth of over 850 feet. A well bored some distance east of town proved to be unproductive. It is estimated that the yield of the new well is 7,000,000 cu. ft. per day. This is one of the strongest wells that has been bored in the state. The wells that supply the city discharge into a common regulator of the "Little Giant" type, where the

pressure is reduced to eight ounces, and gas is supplied to a large number of consumers at this pressure.

At Humboldt in the same county, there were two gas wells, about 600 feet deep, bored five years ago, which supplied a flouring mill and many of the residences of the city, but more recently the yield of gas has not been very satisfactory because the water was not properly cased off. A new company, however, has taken up the matter and four good wells have been bored. One of these, the Hamm well, is north of town, and the other three are south and west. The Hamm well yields considerable gas from the shale at a depth of 595 feet, and a more abundant supply of gas with considerable oil from a depth of about 900 feet. One of the other new wells is on Coal Creek, one on Owl Creek and the other directly on the banks of the Neosho river. All of these yield gas with some oil. Another well more recently reported, three miles from town, is an excellent gas producer. There have been no developments directly east of town yet.

NEOSHO COUNTY.

At Thayer a new field has been opened up during the past year. Here there are nine wells that are said to produce from 5 to 20 barrels of oil per day. They are from one to two and a half miles northwest of town and five miles southwest. Ordway No. 1 is about 800 feet deep, and is producing a good grade of oil. Ordway No. 2, which is 810 feet in depth, and contains 35 feet of sand rock, was "shot" in June last. Nye No. 1 yields a thick oil that may be of value as a lubricating oil. McFarland No. 1 is south of the Ordways, and is said to yield an excellent oil in abundance. There are several thousand barrels of oil already stored here in tanks. Several dry wells or "dusters" as they are called have been sunk in the vicinity of Thayer. At Chanute the borings so far have not been successful.

WILSON COUNTY.

It is in this county, especially in a district near the confluence of Verdigris and Fall rivers, that the greatest development has been made during the past two years. There have been at least eighty wells bored in this vicinity, and a large proportion of them are producers of oil or gas or both. As early as February, 1894, 25 wells had been bored. They are usually between 800 and 900 feet in depth. In order to test the strata at a greater depth, in one case the boring was continued to the depth of nearly 1200 feet, till a harder flint rock was struck, but no good oil sands were met in this increased depth.

Special mention may be made of a few of the representative wells in this district. About six miles north of Neodesha is the Hopkins well, 1034 feet deep. This is on the bank of Fall river, and from the quality of the oil is in a different "pool" from most of the wells in this vicinity. A very thick heavy oil is produced, which has already obtained quite a reputation in the vicinity as a lubricant, so that farmers come many miles to get it for use on their farm machinery. Salt water comes up with the oil, About two miles northwest of the town is the Haag well, one of the best gas wells in the state. From this and adjacent wells gas is piped to the town, and has been used for running engines in boring other wells. Still nearer the town, and on the west side quite near the bank of the Fall river, is a group of wells that promise to be excellent producers. Among these may be mentioned the Kimball wells, the De Moss wells, and the Pierce wells. These are nested about the De Moss No. 2, where a pumping station has been established. Some excellent wells have been bored south of town, and in the vicinity of the confluence of Fall and Verdigris rivers. In regard to the striking of a "gusher," as it is called, in the spring of 1894, a citizen of Neodesha writes: "Immediately after being 'shot,' most of the wells spout for a time at intervals until they have quieted down. Other wells, without being shot, but being drilled down into the oil-bearing rock, are often capped, and then whenever the cap is removed they will overflow. In the case of the Mann well, the drill had hardly entered the oil-bearing rock before the oil began to rise within the well to a height of 500 feet, the well soon began to spout a column of oil six inches in diameter, entirely free from brine, and then continued to flow at intervals for several hours till the workmen succeeded in capping it. Spouting and intermission are due to a contest between the oil and the gas each pouring into the well. For a time the gas may escape freely, but soon the oil, flowing in, overpowers the gas and is forced out by it." Another interesting well of the spouting variety is the Butcher well, four miles southeast, 840 feet deep, which at intervals of two or three minutes throws out a vast volume of brine into the air to a height of perhaps 20 feet. An analysis of the brine of this well shows that it contains quite large quantities of bromides and iodides. In this respect it is similar to the deep well at Independence that was bored some eight or nine years ago. There are other wells just as promising as these mentioned, all within a short distance of the town. It was not till November, 1894, that the houses of the town were piped for gas. The oil yield of these wells has not been completely estimated, but many are believed to be capable of yielding from forty to sixty barrels per day. The depth of the wells in this county is from 790 to 890 feet.

MONTGOMERY COUNTY.

There are three gas towns in this county: Independence, Cherryvale and Coffeyville. At Independence, during the past summer, there have been four wells supplying the gas for use in the city. The pressure of the gas is regulated at the tank by the "Fulton" Regulator, so that from 60 to 80 pounds of pressure is carried by the pipes leading to the town. The closed pressure on the wells runs from 300 to 400 pounds at different wells. The Regulator system that is used is for the purpose of preventing a higher pressure than that stated above from entering the lines of pipe, so as to avoid any accident that might occur from the bursting of pipes. For domestic use in the city the pressure is reduced by an automatic regulator from 75 pounds to 5 ounces, and the regulator is so arranged that a higher pressure will cause the oil to blow out of the apparatus so that the excess of gas will escape. For supplying factories and mills a special pipe line is laid and a pressure of 20 pounds is maintained on this. The new ice factory on the banks of the Verdigris is thus supplied. During the past season the gas company has connected four other wells with the system for supplying the city. A new well that was bored in December a mile west of the city is said to pass through gas sand 40 feet in thickness, and to be the best well yet, yielding an abundant flow of gas, and giving a rock pressure of 350 pounds per square inch. The oil produced here has not yet been utilized.

At Cherryvale during the season there have been eleven wells from which gas could be drawn. The amount of gas that these can yield is not known. There is oil in smaller quantities than at the Neodesha pool, but no attempt has been made to utilize it yet. The wells here have not been "shot." Several companies are in the field and it is expected that soon more prospect holes will be bored. There are three regulators in use, of a new and superior pattern. It is noticeable that the wells here are not as deep as most of those that have been previously mentioned, being only about 600 feet deep. The "log" of well No. 2 has been kindly furnished us by the superintendent of the Gas Company there. It is as follows:

	Feet.		Feet.
Soil	4	Dark shale	10
Clay and lime boulders	10	Drab sand (gas)	8
Blue stone grit	30	Soapstone	14
Dark clay like fire clay	10	Light sand (gas)	7
Gray lime, hard	18	Shale and soapstone	85
Soapstone	10	Drab lime	26

Blue shale	37
Gray lime	14
Dark shales	15
Black muck and salt water	2
Dark shale, soft	30
Lime, shell	1
Light shale	5
Dark shales	13
Sand shale (gas)	1
Light shale	14
Magnesian lime	22
Light shales	19
Hard white lime	10
Soapstone	2
Light shale	5
White lime	6
Light shale	5
White lime	4
Dark shale	18
Black sand (gas)	6
Brown sand (salt water)	7
Blue shale	5
Gray sand (water)	17
Dark shale	1
Gray lime	31
Dark shale	7
Gray lime	6
Dark shale	2
Gray lime	20
Dark shale	5
Black sand, hard	6
Dark shale	19
Gas sand	2
Total	590

The rock pressure is 260 to 300 pounds per square inch. It is delivered to customers at a pressure of one and one-half ounces. In Cherryvale the gas has been used more or less for over four years, but recently its use has been much extended. As the wells are in different parts of the city and connected in the same system there is little loss from friction from conveying the gas long distances and no trouble has been experienced by consumers since the pipes have been properly protected in the winter. It is said to never have a less pressure than 115 pounds even in the coldest weather.

At Coffeyville on the southern border of the county there are six wells in use for supplying the gas to the city, but they are not all in constant use. The rock pressure is about 300 pounds, and the gas is supplied to factories at a pressure of 50 pounds and to consumers at six ounces, water pressure. The wells are approximately 800 feet in depth. Other companies are at work here, in addition to the one that has been supplying the city for some time.

There was a very curious explosion at Coffeyville in July last. It occurred near gas well No. 2 of the Coffeyville Gas and Mining Company. A local reporter says: "The earth was torn up for a distance of several hundred yards. Where the main part of the upheaval occurred is a hole probably twenty yards wide by ten or twelve feet deep. Huge masses of rock are lying around which came from the very bowels of the earth. The gas company say that the stratum of rock lies at least thirty feet below the surface.

Several pieces of rock weighing fully fifty pounds were found a block and a half away, where they had been carried by the force of the explosion." All around this crater are cracks as if the earth had been lifted up and had fallen back nearly in the original place, and in many places it looks as if a veritable earthquake had passed that way. There is one very important industry that is fostered by the natural gas in this place. A large establishment, with a capacity of 25,000 per day, is making an excellent grade of vitrified brick from the bluff shale south of the city. In addition to this, there are numerous factories and mills that depend on this gas for fuel.

CHAUTAUQUA COUNTY.

At Peru a well has been bored which yields both gas and oil. It is 920 feet to gas and 940 to oil, though it was bored to a depth of 1300 feet. The "oil-bearing sand" is said to be much thicker than that at Neodesha. A flow of water made it necessary to case the well for 800 feet down, and when hot salt water was struck at a little over 1300 feet, the well was plugged and further exploration stopped. The closed pressure of the well is said to be 400 pounds. The oil belongs to the black heavy lubricating class.

At Buckle's well, two and one-half miles west of Peru, between that town and Sedan, another well has been bored which is estimated to flow 20 gallons without pumping. At Niotaze, east of Peru, a well has also been bored. The well at Sedan, 1100 to 1200 feet deep, proved a dry hole. Another at the county poor farm is under contract.

LABETTE COUNTY.

At Mound Valley a well was sunk ten years ago to a depth of about 750 feet and a fair supply of gas obtained, which has been utilized ever since at a "Health Home" in the vicinity. Other prospectors are now in the field and numerous leases have been made.

BOURBON COUNTY.

At Fort Scott, several years ago, wells were bored in the bottom near the Marmaton river and the gas obtained was piped into the city and used at houses near the wells.

Of this field Robert Hay, in his report to The Kansas State Board of Agriculture, 1885-6, says: "One company has drilled four wells, three of which are yielding an abundant supply of gas. Mains have been laid and gas is now in use in Fort Scott hotels, private houses, car-barns, &c." In the report to the same Board in 1891-2 he says: "The wells have gradually diminished their yield and the

city is no longer using natural gas." One well 400 feet deep was quite successful and the artesian well (see Trans. Kas. Acad. Science, Vol. IX), which was bored 621 feet, yielded a little gas. At Fulton, in the same county, natural gas is used locally on a small scale. There are about twenty-five wells within a radius of twelve miles, and each of these furnishes gas enough for the use of a family. The "sand" is struck at a depth of 270 to 300 feet. The cost of drilling is not over 50 cents per foot.

WYANDOTTE COUNTY.

At Kansas City, Kansas, some wells were bored several years ago, and enough gas was produced so that it was used locally for fuel. Oil was also produced in small quantities (see Quarterly Report Kas. State Bd. Agric., Dec., 1885).

There are many other counties where oil and gas have been produced in small quantities, but the yield is at present not large enough to consider that they are of commercial importance.

NATURAL GAS: METHODS OF COLLECTING AND ANALYZING.

There are numerous theories as to the accumulation and storage of natural gas, but most writers agree that the necessary conditions are: (1) a reservoir of porous sandstone or limestone; (2) a cover of impervious rock so that the gas cannot escape; and (3) such an arrangement of strata as shall allow the gas to collect in the upper part of a natural arch of rock.

The writer has visited all the important gas fields of the state and collected samples of the gas and oil. The method of collecting the gas was as follows: Flasks holding 300 cubic centimeters were drawn out at about the middle of the neck to a small bore, and a glass tube attached by a rubber hose to the gas burner, was inserted in the flask and was pushed nearly to the bottom. The gas was allowed to run at about full pressure for some time, the flask being inverted till the air was thoroughly washed out by displacement. The flask was then held in the same position, the glass tube was withdrawn and the cork was inserted in the neck. After warming the flask and removing the cork for an instant, by means of a small alcohol lamp, the glass tube at the narrow part of the neck was drawn out and the neck thus sealed. The gas was transfered to a measuring apparatus at the laboratory, by breaking off the end of the neck, beneath the surface of the water under a bell glass. For the analysis of the gas, Hempel's Apparatus for Gas Analysis was employed. In this apparatus the carbonic anhydride was first absorbed by caustic potash, then the heavy hydrocarbons were absorbed by

passing the gas over fuming sulfuric acid; the oxygen was removed by means of stick phosphorus, and the carbon monoxide was removed by cuprous chlorid. A part of the remaining gases; nitrogen, marsh gas and hydrogen, was exploded over mercury, and from this the amount of each of the two latter was calculated, and the nitrogen was thus absorbed, and the other gases were determined as before. This gas contains over 90 per cent. of marsh gas, a gas producing but little light, but valuable for heating purposes. It contains small quantities of carbon monoxides, carbonic acid and oxygen with traces of nitrogen and olefiant gas. Like most natural gases it is, as far as examined almost entirely free from hydrogen. It is on account of the small quantity of olefiant gas or other illuminating materials, that the gas is not burned satisfactorily in an ordinary "bat wing" burner, but a special burner, must be used. The results of some of the analyses were as follows:

	Paola.	Osawatomie.	Iola.	Cherryvale.	Coffeyville.	Independence.	Neodesha.
Carbon dioxide	.33	.22	.90	.22	.00	.44	1.00
Olefiant gas, &c.	.11	.22	.00	.00	.35	.67	.22
Oxygen	.45	trace	.45	.22	.12	trace	.65
Carbon monoxide	1.57	1.33	1.23	1.16	.91	.33	.50
Marsh gas	95.20	97.63	89.56	92.46	96 41	95.28	90 56
Nitrogen	2.34	.60	7.76	5.94	2.21	3.28	7.07
Hydrogen	0.00	0 00	0.00	0.00	0.00	0.00	0.00
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00

DISTRIBUTION AND USE OF GAS.

As the gas is produced at the wells and held in the mains at tremendous pressure it is of the utmost importance that before it is supplied to customers it be reduced in pressure to nearly the same pressure as that on the ordinary street mains, where illuminating gas is used. The regulators that are used for this purpose are of various patterns, but in the better class the gas is passed through a governor, in which the pressure is reduced as desired, by weighting the end of the lever, and then through the valve to a gas holder. By means of an automatic valve the pressure on the mains is kept constant.

The gas is sometimes used in the ordinary burner, provided with a special lava tip, but as it contains so little illuminating material, and as in order to get sufficient light it is allowed to "blow," the light is not satisfactory, except perhaps for a street lamp. There are two special burners that are very largely used with natural gas and with poor illuminating gas. One of these is the Welsbach Incandescent, in which a "mantle" of oxides of some rare earths is heated to a

very high temperature. The other burner for natural gas is the Lungren Regenerative Lamp, in which the air used in the combustion is heated to a high temperature.

For fuel purposes the gas is used in the ordinary gas stove.

Some investigations were made as to the cost of natural gas to consumers in the different towns where it is supplied. Heating stoves cost the consumer $1.50 per month at Cherryvale; the same at Independence, and $9.00 per year at Coffeyville. Mills are supplied at $2.00 per day. Cooking stoves are supplied for $2.50 at Paola, $1.00 at Cherryvale, $2.00 at Independence, $2.50 at Iola and the same at Osawatomie. Gas jets cost 39 cents at Paola, and 25 cents at other places. The gas for the Lungren burner costs from $1.00 to $1.50 per month, and sometimes a special charge of 50 cents is made for the Welsbach burners. Complete details are not at hand as to the number of consumers. In Cherryvale there are 550 stoves and 500 lights, and in Independence there are 400 stoves in winter and 400 burners. At Iola about 200 stoves and houses will be heated by the gas this winter. The price of gas by the thousand feet is 20 cents at Coffeyville, and the maximum price at Neodesha will be 30 cents.

COMPOSITION OF THE PETROLEUM.

In making an analysis of the oils, that was distilled first which came over at a low temperature, and then that which came over at a high. These were kept separate; then beginning again a fractional distillation was made, at the temperature stated below. The distillations were continued till the glass of the flasks melted. In the determination of the flash points, a wide glass tube open at both ends was used. In the bottom of this a cork was inserted through which passed a glass tube drawn to a small jet. Oil was put in the large tube and it was placed in a water bath. A thermometer was hung in the oil and a current of air blown through.

The results on some of the Neodesha wells are as follows:

MANN WELL.

	Temp. C.°	Flash point.	Specific Grav.	Amount.
1.	40 to 110	Below 26.°	.7058	61 cc.
2.	110 to 150	Below 26.	.7314	82 "
3.	150 to 200	Below 26.	.7778	107 "
4.	200 to 250	Below 76.	.8112	77 "
5.	250 to 300	91.	.8377	101 "
6.	300	Above 100.	.8691	215 "
Original oil		Below 30°	.8900 37.75 Baumé.	

The relative amount that came off at the different temperatures is graphically represented below.

Amount.	Temperature.
▬	Below 150°
▬▬▬▬▬	150 to 200
▬▬▬▬▬▬▬▬	200 to 250
▬▬▬▬▬▬▬▬▬	250 to 300
▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬	Above 300

About 14 per cent. of this oil remained in the retort as coke and material that could not be distilled over in a glass flask.

DE MOSS WELL.

	Temp. C.°.	Flash Point.	Specific Grav.	Amount.
1.	Below 150	Below 30°	....	25 cc.
2.	150 to 200	Below 30	.7814	130 "
3.	200 to 250	51	.8089	208 "
4.	250 to 300	88	.8366	230 "
5.	Above 300	Above 100	.8774	525 "
Original oil		68°	.8700 31.25 Baumé.	

A little over 10 per cent. remained in the retort.

KIMBALL NO. 2.

	Temp. C.	Flash Point.	Specific Grav.	Amount.
1.	70 to 110	Below 30	.7002	118 cc.
2.	110 to 150	Below 30	.7417	178 "
3.	150 to 200	Below 30	.7793	157 "
4.	200 to 250	57	.8099	153 "
5.	250 to 300	Above 95	.8343	210 "
6.	300+		.8739	500 "
Original oil		Below 30	.835 38.25 Baumé	

About 12 per cent. remaind in the retort.

HOPKINS WELL.

This oil was of about the same specific gravity as water. It was very thick and difficult to distill. One distillation was obtained having a specific gravity of .8477, and another .9012. The flash point of these distillates was of course high. Over half of the oil distilled over above 300 degrees Centigrade.

ORDWAY WELL.

	Temp. C.°	Flash Point.	Specific Grav.	Amount.
1.	70 to 110	Below 27	.7124	45 cc.
2.	110 to 150	Below 27	.7374	82 "
3.	150 to 200	Below 27	.7742	70 "
4.	200 to 250	69.5	.8046	115 "
5.	250 to 300	Above 95	.8341	121 "
6.	300+	Above 95	.8663	250 "
Original oil		Below 30	.984 37 Baumé.	

There remained a residue amounting to 12 per cent. that was not distilled over.

A graphic representation of the different distillates from this well and their relative amounts is shown below:

Amount. — Temperature.

70 to 110°

110 to 150

150 to 200

200 to 250

250 to 300

Above 300

It may be remarked in general in reference to the products of distillation that those that come off at the lower temperatures are good oils for burning and those that distill at higher temperatures are valuable as lubricating agents. The manufacture of lubricating oils was begun on a small scale in Paola several years ago, but was discontinued on account of competition. The claim for that oil at the time was "No other natural oil combines the high fire test, 300 F., and low cold test, 5 F., high viscosity, 220, and heavy gravity, 24 B."

In the distillation of the oil as made above, the results arrived at are not the same as those that would be obtained, on a commercial scale, for the high temperature there obtained would no doubt "crack" the oil more thoroughly, and thus change the quantity of illuminating oil.

In the oil fields of the state at present the great necessity seems to be some economical method of storing the oil and of disposing of it. In addition to the numerous wooden tanks of 200 to 300 barrels capacity that have been built at the wells at Neodesha, a tank of boiler iron about eighty feet in diameter was constructed last summer and oil has been pumped into it from the various nests. Its capacity is about 36,000 barrels. There is talk of a refinery here.

This oil is a valuable fuel without refining. It is utilized at many places by being sprayed into the fire-box with a jet of steam, and in this way produces intense heat. It is also used in the manufacture of gas and in the enriching of the "water" gas.

Altogether, the outlook is good for gas and oil in Kansas. Their production must add very largely to the wealth of the state. In the eastern part of the state, it is true, fuel is not such an item of expense as farther west, but the use of natural gas and oil here will tend to increase the manufacturing industries, and in that way benefit the people very materially. It is conceded that last year there was only one state where the yield of natural gas had not decreased. If the same thing should prove to be true in this part of the country, and a maximum production should be reached in a few years, still the effort will not be in vain, for not only will the gas be of use while it does last, but it will pave the way for the general introduction of some cheap "fuel gas," which can be made from the petroleum of the state, or from other cheap combustible material.

Experiments in the Solution of the Labor Problem.

BY F. W. BLACKMAR.

Experiments in the manipulation of social forces are much more difficult than those in other branches of science where the material to be dealt with may be brought immediately into subordination to the will of the experimenter. In the former case the experimenter has to contend with the vagaries of human nature and the potency of individual will force. He must first take society as it is, investigate the conditions under which it operates, examine the forces that create and propel it forward, and determine the moment and direction of each. The question is rendered even more difficult, on account of the fact that the instant he considers any given force, a score of resultant and deflective forces interfere with analysis and application. But the real problem in social dynamics is made more apparent as the experimenter becomes a reformer and attempts to direct the social forces toward a certain end. It is the herculean task of the reformer to determine the nature and direction of social forces and impel them in the line of actual progress. As one surveys the evolution of society, it is marvelous to observe how little has been accomplished in the attempt to force social life into especial channels and how much has been determined by the spontaneous activities of human nature. But it yields wholesome lessons to the person who by his own will says he will change the trend of society or build upon the old foundation a new structure which will permanently change the relations of men. He will at least be convinced of the persistency of the social forces that have been developing through the progressive stages of man's existence.

The social problem of the day is the creation of the largest possible amount of general well-being of the entire number of individuals which compose the social organism. This involves many questions. It includes not only the utility of existing forces but it includes the utility and the happiness of each individual. As the progress of social life is measured only through a long period of time, the "greatest good to the greatest number" may include those of future generations. Consequently it is difficult for the

social scientist to measure with exactness the permanent results of his experiment. It requires, therefore, a careful estimate of the ends sought for, a clear understanding of the nature of the forces and material dealt with, a wise management of the experiment and great wisdom to determine whether the results obtained after all are anything more than changed conditions rather than permanent advance.

The solution of the labor problem has had more formal attempts by actual experiment than any other form of the modern social problem. As a part of the social problem it seeks to adjust economic relations between employer and employee. It involves something more than the proper and just apportionment of the net products of industry. It seeks to insure contentment, happiness and progress. If the distribution of a certain amount of goods in proportion to service rendered in production was the only object sought, the question would be readily solved. But when it involves the idea of happiness and contentment, the harmony of social relations must be considered, and then it is that the problem becomes more difficult, for law has failed to force out of man a spirit of discontent. There must be a readjustment of conditions which time alone can perfect. If the idea of progress is insisted upon, then the individual and the community must move forward with power, increase in knowledge, in skill and intellectual growth which involves also a progressive happiness to accompany this, in short it means a constant harmonious progressive activity, which is indeed difficult to obtain.

A few experiments in the attempt to improve the condition of the laboring classes will be given, the data of which have been obtained through original research of the writer. In these it will be seen that the foregoing principles have been partially observed. The first one presented will be that of Mr. Pullman, the founder of the famous town of Pullman, and the owner of the Pullman patents. The object of the experiment is best set forth in the founder's own words:

"The object of building Pullman was the establishment of a great manufacturing business on the most substantial basis possible, recognizing as we did and do now that the working people are the most important element which enters into the successful operation of any manufacturing enterprise.

"We decided to build in close proximity to the shops homes for the working men of such character and surroundings as would prove so attractive as to cause the best class of mechanics to seek that place for employment in preference to others. We also de-

sired to establish the place on such a basis as would exclude all baneful influences, believing that such a policy would result in the greatest measure of success from a commercial point of view, and also, what was equally important, or perhaps of greater importance, in a tendency toward the continued elevation and improvement of the condition not only of the working people themselves but of their children growing up about them."

To one who reads closely this statement of the founder of Pullman, it will be observed that the prime motive was to build a manufacturing interest which would yield a sure and steady profit to the investors without the usual accidents that arise from uncertain and unskilled labor. The laborer being a necessary adjunct to this business enterprise should be cared for in order that the money-making plan could be successfully carried out. Manufacturing was to be placed on a higher plane. Attracted by the surroundings and by good wages, it was hoped and expected that skilled workmen would seek Pullman as a place of residence "*in preference to others.*" Thus the Pullman company would have the advantage over other companies in the production of goods. Thus far the idea is purely commercial and represents the extreme spirit of modern competition. But the plan was accompanied by altruistic motives, for it was thought that the workmen would be interested in the Pullman plan and thus improve their condition and furnish a rational plan for the peaceful solution of the labor problem. So the plan, having been symmetrically conceived, was formally carried out. A model town was built adjacent to the shops composed of long rows of brick houses joining eachother and facing on streets well macadamized and drained. Systems of sewerage, water works, and gas supply were established for the convenience of the residents. A central market was erected provided with rooms and stalls which were rented to parties who would conduct the business for the convenience of the people. A large arcade or department store was so arranged that an individual after once entering could do all of his shopping under the cover of one roof. A play-ground, a church, a theatre, and a school house were provided by the town authorities. And thus it happened that a city of twelve thousand people sprang up, nearly all of whom were employees of the Pullman company and who had all municipal advantages thrust upon them with no anxiety except as to wages received and rents paid. They rented the houses and the company did everything else. It did the thinking, the planning, the working, the doing. The result of it was that a symmetrical town, well arranged for convenience and economy, sprang up, com-

posed of a group of citizens who were virtually denied the right of self-government as much as if they were an old-time feudal village governed by a lord of the manor.

For many years the company carried out their plan of paying better wages than in similar shops in the United States, and thus proceeded upon their first supposition that a superior class of laborers should receive superior wages. The rents charged in the beginning were based upon an income of six per cent. on an actual investment. In many instances the rents were higher than the wages would warrant, although possibly no higher than the business interests of the investors had good reasons to demand. In 1893, the company felt obliged to reduce the wages of the employees in order to keep up a regular dividend and surplus. The laborers thereat became discontented and struck for higher wages, although the company assured them that they were manufacturing goods at less than cost. Prior to the reduction the rate of wages was about $2.10, which included men, girls, and youths, and all skilled and unskilled labor. The maximum rate was $4.50 per day. After the reduction the average was $1.67, while the maximum was $3.50. The entire reduction, which occurred at three different times, amounted to between 20 and 25 per cent. A careful estimate shows that at the same time a sudden depression of 33⅓ per cent. in wages had occured throughout the United States, and also that the sudden panic had thrown a large amount of manufactured goods upon the market upon a decreasing demand, and this reduced their price from 20 to 25 per cent., so that had the laborers only known how to accomplish it they could have purchased as many goods in the market with their wages after the reduction as they could have done before when times were better.

Not so, however, with rents. The following table exhibits a complete statement of rentals:

No. Rentals.	Amt. per Mo.	No. Rentals.	Amt. per Mo.	No. Rentals.	Amt. per Mo.
2	$4.00	4	$11.50	8	$26.00
6	4.50	173	12.00	1	27.00
23	5.00	25	12.50	13	28.00
4	5.50	4	13.00	9	30.00
24	6.00	263	14.00	4	32.50
41	6.50	70	15.00	20	35.00
5	6.75	61	16.00	3	37.00
55	6.00	222	17.00	4	40.00
9	7.50	58	18.00	10	45.00
83	8.00	1	19.00	1	46.00
5	8.25	26	20.00	1	50.00

No. Rentals.	Amt. per Mo.	No. Rentals.	Amt. per Mo.	No. Rentals.	Amt. per Mo.
106	8.50	3	22.00	1	55.00
226	9.00	2	22.50	1	60.00
68	10.00	6	23.50	1	70.00
4	9.50	4	23.00	2	65.00
38	10.50	1	24.00	2	75.00
53	11.00	45	25.00	1	77.25

The average monthly rental of 1799 tenants was $13.50; of 1200 the average rental was $10.00, and of 600 it was $8.00. Not all of the Pullman employees were renters, though fully 35 per cent. of them were direct renters, 30 per cent. sub-renters, and the remaining 35 per cent. had lodgings outside of the town of Pullman. The rents ranged from $4.00 a month to $77.25 a month, although a large proportion, as the above statement will show, ranged from $8.00 to $18.00 per month. A flat of five rooms on the first floor with interior water closets, and a basement capable of use for washing and cooking, on one of the best streets, rented for $14.00 per month, with street water rents of 71 cents per month. It would seem that this was not an extravagant rent. Yet the tenants sublet one room for $9.00, and another for $6.00, retaining the other three for themselves, thus living practically rent free for three rooms in the house. However, it will be seen by a careful examination that the rents conformed fairly well to the income of laborers before the reduction, but after the reduction they were too high. The average normal rent expenditure of a laboring man in the state of Illinois, according to economic laws, would be about 17 per cent. of his income; but we find after the reduction of wages that the relation of rent to income had changed so that the rent amounted to from 20 to 25 per cent. of the income.

So the company which agreed to furnish superior conditions to the laboring man and free him from the burdens of unjust pressure and to raise his standard of life, had finally succeeded in placing these same laborers in a condition which would not support the standard of life which had been practiced. The result was discontent, distrust and inharmony, the very things which the Pullman company sought by its plan to avoid. It is not sufficient for the Pullman company to say that laborers were unreasonable and therefore the plan did not work. It does not free the plan from the stigma of failure, simply because the originators of it miscalculated the conditions of human labor.

For the improvement of the social conditions of the laborer, there was established a well equipped library, with fine furnishings, containing standard works which the laborer had the privilege of

reading on payment of the small sum of twenty-five cents per month or three dollars per year. Quite a large number of the more intelligent classes found it convenient to patronize the library. With it were connected several courses of lectures for the benefit of the employees. But the great majority of the employees remained outside of the direct influence of the library. There was, however, from year to year a slow increase in the number of persons using the library, and many young men and women were materially helped by it. The influence on those who were constant patrons and readers at the library was very noticeable. They were among the better class of employees, were not found with the malcontents and strikers, and were reasonable in the consideration of the relations of employer to employee. A church was built by the company where regular services were had, but this, too, became as all churches do when supplied by individual power, a monotonous affair. The theatre was more of a success, for here convenient to their labor the people could find the means of an evening's entertainment and observe the performance of some of the best talent of the country. The base-ball grounds and picnic grounds were also of some service to the people. In order to make a moral town, great care was taken to eliminate all baneful influences, such as saloons, gambling houses, and brothels, although it was allowed later to peddle beer on the streets or deliver it in kegs at the houses. The social life of the people, however, was mostly based upon the gossipy communications of families more intimately açquainted which associated in groups. With all of the symmetry of the town, there was not a homogeneous society. And the first universal idea which prevailed in the majority of the minds of the people, and upon which they could all unite, was that the Pullman company had finally grown to be a great overbearing corporation without sympathy for the laboring man, and this idea, once thoroughly established, could not easily be eradicated.

An inquiry into the causes of the failure of the plan will show first of all that laborers do not care to have too much done for them, but prefer to be placed in a condition to do for themselves. There are laborers who would prefer to have a poor house surrounded by a small yard into which they could allow the pigs, ducks, and the chickens to come and go at will, with poor water and poorer sewerage, for the sake of bare independence, of having the absolute control of the premises for the time being, than to live in a row of brick houses, with lawns in front, well macadamized streets which were kept by the company of whom they rented, and in the management of which they have no voice. Such people

cannot be impressed with the advantages of the aesthetic, the uniform and the symmetrical, nor, indeed, to a great extent, with the conditions of health and comfort.

The attempt of the company to make a better-living animal possessed of full physical capacity and skill with a better mind and a better life, and a better civilization, that the same creature might yield a larger return in the manufacturing interests, was indeed a noble conception. The fact that the system of social improvement of the laborers was to rest upon an economic basis, was in accordance with sound economic doctrine; for no attempted improvement of any class or group of people can be considered of any value except as it rests upon this basis. And the principle that each individual should pay for everything received, and that nothing should be granted gratis, is also to be considered fundamental in the doctrine of social improvement. Yet when the desire for these things has been excited and the means of attaining them fails, the foundation of the plan is destroyed.

Another potent cause of the discontent arose from the fact that the magnitude of this business seemed to remove the company far away from the laboring population; sympathy was soon lost between laborers and employers, and discontent gradually developed. This the employers failed to recognize until the discontent was fanned into open revolt. If the friendly feeling of the managers continued to exist, there was at least the lack of the expression of sympathy between employer and employee. It may be said that the laborer is narrow and bigoted in his conception of affairs and therefore unreasonable. To a certain extent this is true, not only with the laborers at Pullman but with the laboring class at large. While there has been an increase in skill and in general intelligence of the laboring population, there has remained with them a sort of illogical conception of the relation of things. The excessive division of labor has brought about great skill, but in teaching each individual to do one thing well and that alone, it has narrowed his life to a minute operation requiring a single conception. The result of the excessive division of labor is to make men illogical and unreasonable concerning the exact condition of affairs. But those who would solve the labor problem must take this into consideration. If anyone doubts the truth of the assertion, let him mingle with the laborers and he will see how the man who stands at the machine cutting bolts all the day long, day after day, year after year, becomes monotonous in life and provincial in thought, so that when new conditions arise he is unable to grasp thoroughly the situation. He is then open to conviction by the man who

can gain his confidence and move him through passion and prejudice.

So far as the library is concerned, a large proportion of the laborers would have been better contented and received as much benefit by having a reading room adjoining a billiard room, card room, and smoking room, than in the well equipped library with its fine furniture and stately volumes. In this respect the ideal plan overreached the condition of the laborer, and consequently had its effect upon but few. And finally it may be said, that had the company taken the laborers into partnership not only in the plan of improvement but in the actual building of the town, asked them to assist in its control and management, and given them an opportunity to obtain stock in the business and to become sharers in the great enterprise, doubtless the plan might have realized the hopes of the founders. Not that things would have been done any more nearly right, or in better form than when accomplished by the company itself, but they would have been done to some purpose. But as it is, the model plan of Pullman for the harmonizing of the interests of labor and capital, of employer and employe, and the elevation of the laboring population to a higher standard of life, cannot be said to have been productive of much permanent good. Whether, in the future, plans will be so modified as to overcome the defects of the system remains to be determined.

A more successful plan of harmonizing the interests of labor and capital has been instituted by the firm of Proctor & Gamble whose manufactory is located at Ivorydale, a small town in the suburbs of Cincinnati. The company employs about 500 laborers at the factory besides another hundred in the Cincinnati office and on the road. The average wages of men is $10 per week; of women, $4.75; of boys, $3.50 to $7. The wages are considered only fair, yet the method the company has adopted of dealing with its employees has been such as to prevent any discontent, strike, or revolution. The firm was established in 1837, but the plan of profit sharing was adopted in 1887. It provided for the distribution of the profits among the employees after allowing a reasonable salary of $4000 to each member of the firm who was actually engaged in conducting the business. The laborers were to receive the same proportion of the profits as the total wages paid bore to the total cost of manufacturing and marketing the product. For example, if the total amount of business done was $100,000, the amount of wages paid $20,000, the amount of profit made $10,000;

then the total cost of making and marketing the goods was $100,000 less the the profit of $10,000, or $90,000. The amount of wages paid was $20,000. The amount of profits given to employees would be then in the ratio of 20,000 to 90,000 or two-ninths, and the proportion to the firm would be as 70,000 to 90,000 or seven-ninths of the profits. The laborers' proportion of the profits was distributed among them in accordance with the amount of wages earned by each. This plan was in force for three years, during which the dividend or share of the profits averaged 12⅛ per cent. of the wages.

In 1890 the firm of Proctor & Gamble was reorganized on the basis of the payment of 12 per cent. on the common stock, if this amount should be earned. This being practically the same rate earned by the employees under the old plan, it was an easy and advantageous arrangement to adopt a plan of paying to employees as their share of profits the same rate of dividend upon their wages as was paid upon the common stock of the company. This method was adopted and under it profit sharing is now carried on. The dividends are paid semi-annually. To illustrate this, suppose a man earns $500 a year in wages; he receives in addition a dividend of 12 per cent. on this amount or $60. The man that has $500 worth of stock in the company also receives 12 per cent. or $60. Thus the laborers and the stockholders are upon an equitable basis. All employees are entitled to begin to share in the dividends after being in the employ of the company for three months; but if one quits work or is discharged before three months' labor in the service of the company he receives no dividend. At first the laborers were divided into full participants and half participants in profits. This was not found to be desirable and all employees were placed on the same basis. Now fully 98 per cent. of the laborers participate in the profits. The company reserves the right to deny the dividend to the employee for cause, but the amount of this unpaid dividend must be paid to other laborers and does not go to the stockholders of the company.

The company not only allows sharing in the profits but also encourages employees to acquire a part of the capital stock. Any employee may obtain a share of the common stock upon the following terms: $10 at the time of application, the balance in installments of not less than $5 each. Upon this balance he must pay interest at the rate of 4 per cent. per annum. In the meantime all dividends declared upon the stock accrue to the purchaser. But the certificate of stock is held by the secretary of the company as trustee for the subscriber until the final payment is made. There

have been up to date about seventy or eighty shares taken by the employees, nearly all of which were purchased at prices varying from $100 to $128. The company has under consideration a plan to go one step further and guarantee the employees who hold stock against loss upon their investment. They find a good many difficulties in the way of the practical working of such a guarantee, but hope to make it a permanent part of their system.

Another important feature of the Ivorydale system is the pension fund inaugurated for the benefit of the employees. This fund is created by setting aside the sum of $500 semi-annually, half of which amount is taken from each profit sharing dividend and one-half is paid by the Proctor & Gamble Company. The management of the fund is in charge of a board of trustees composed of employees and members of the company. A pension is granted to any employee who has been in the continuous employment of the company for not less than seven years when partial or total disability to work has been caused by accident, sickness or old age, and it is the company's intention so far as possible to provide those who are entitled to pensions with such work as they can readily perform at such wages as the work is worth. The introduction of the pension fund is of recent date, but on January 1, 1895, there was $2000 in the fund with one pensioner upon the rolls.

One other economic condition is found in the building and loan associations which have enabled a few to build their own homes, and this is encouraged by the company.

The attempt to improve the social life of the employees has met with less success. Although library, reading room and card room have been provided free, they have not met the success anticipated when inaugurated. This is doubtless owing to the many mutual aid clubs which furnish greater attractions than the reading room and the library. In seeking enjoyment laborers have a tendency to scatter into other groups rather than to associate among themselves in a single group; also the widely separated position of the homes render compact grouping almost impossible, as about one-half live near Ivorydale and the rest live in the city of Cincinnati.

When an employee is injured or sick, the physician employed by the company cares for him. The company also continues the wages of the injured employee through the period of his disability, and seeks to emphasize the fact that employer and employee are associated for a common interest. Many methods are taken by the managers to show their interest in the employees. Thus, on Christmas day, 1893, three hundred turkeys were distributed among the heads of families. And after each semi-annual pay-day in

January and July, a day is set apart for a general celebration in which employers and employees engage. The day is taken up with games, sports and general jollifications.

The entire profit sharing enterprise is established on a business basis. Although altruistic motives may have been at the foundation of this scheme, it was originated for the improvement of the business with the belief that the benefit of the employee was in the end to be to the benefit of the employers. Most of the laborers being unskilled at this time and below the average intelligence of skilled workmen, it was difficult to persuade them that it was not a scheme to get more work out of them for a corresponding equivalent. Also they were disposed to take the dividend as a matter of course and spend it freely and sometimes foolishly. But time and experience have dispelled this idea. The success of profit sharing there as elsewhere is a matter of education, and many efforts of profit sharing have failed elsewhere simply because the employers failed to remember that the employees must be educated up to it. Patience as well as justice is required for success. During the first two years the profit sharing was not a success as a money making investment, but as the men became more and more convinced that they were treated with justice they became more careful and more intelligent in the work, until it is plainly demonstrated and freely admitted that the saving under profit sharing is much in excess of the sums paid to wage earners as profits.

The success of the plan has exceeded the expectations of the company. The gain is in the saving of time, in the diminishing of material, in making a better quality of wares, in keeping men of experience, and finally a saving in oversight. These are the principles which have been maintained by the advocates of profit-sharing and it is gratifying to find that they agree with the experience of those who have carried it out. There have been no strikes or labor troubles of any kind at these works since this plan has been in force. Employees remain longer in the service of the company, and it is very seldom that a man is discharged on account of lack of work. It demonstrates that the interests of employer and employee are the same and any warefare between the two classes is an unnatural warfare and works against the interests of both parties engaged in it.

Another very important example of profit sharing is furnished by the N. O. Nelson Manufacturing Co., a corporation for the manufacture and sale of plumbing goods, steam goods, and machinery.

The firm was first established in 1877, incorporated in 1883, and began profit sharing in 1886. Its factories are in St. Louis, Mo., Mound City, Ill., and Leclare, Ill. The number of employees varies from 400 to 500, and the wages range from $1.25 to $1.50 per day for common labor, and from $2.00 to $2.50 per day for skilled mechanics. The company runs full time with the possible reduction to three-fourth's time for perhaps a month in mid-winter. About one-half of the company's works are located at Leclare. Here the factories are well built, heated with steam and lighted with electricity. The company owns 125 acres of land, 15 of which are reserved for factory uses and 110 for residence purposes. There is no attempt to build a model town, as there are no models for houses or modes of action for people. It was held that in every respect life should be as free from restraint as on a farm. But the streets are well laid out. They are paved with cinders and sprinkled in dry weather. Plank sidewalks prevail where needed, and shade trees have been planted on all the streets. Water and street lights extend wherever there are dwellings.

The company has made it possible for employees to purchase land at a very low rate and build their own homes, paying for them in monthly payments. Should a person desire to move from the town the company takes the property from him without his loss.

The plan of profit sharing adopted varies somewhat from that of the example just given. It sets aside one-tenth of the profits for a reserve fund, one-tenth for a provident fund, and one-twentieth for an educational fund; then the balance is divided equally between employers and employees. The reserve fund was set apart to meet the loss of a bad year and to equalize dividends when profits were small. The provident fund was created to take care of the sick and disabled and the families of deceased laborers. This latter fund is in the hands of a committee of five of the employees elected by the employees themselves. Special rules are made for the control of the expenditure of this fund so as to meet all the requirements of the capitalist. There are no conditions attached to employment and profit sharing except a man's capacity for work. There are no agreements respecting unions, time of service, nor the manner of quitting. Finally the manner of division was modified so as to yield 2 per cent. on wages to every one per cent. on capital, and the earlier practice of setting aside 10 per cent. for provident fund, and 5 per cent. for educational fund was displaced by paying out whatever was necessary for these funds and charging the same to the gross profits.

The results of the first year's business after profit sharing was

adopted gave a dividend of 5 per cent. on wages; second year 10 per cent.; third year, 10 per cent.; fourth year 8 per cent.; fifth year, 10 per cent.; sixth year, 8 per cent.; seventh year, 4 per cent.; and in the eighth year, which was 1893, no dividend was declared. The total dividends paid to wage earners as their share of divided profits has been $65,000, or an average of 9 per cent. on wages paid. In the beginning dividends were payable in cash or the stock of the company; but in recent years in order to make a solidarity between capital and labor, the company insists that all dividends shall be paid in the stock of the company. Perhaps there is no other feature that binds the employees and employers so closely together as a participation in dividends and sharing in the profits of the concern.

One other economic condition here is worthy of especial attention: this is the co-operative store which was opened in May, 1892, and incorporated under the laws of Illinois for the purpose of furnishing consumers with goods at moderate rates. The laws under which it is incorporated provide, among other things, that no one shall hold more than one share or cast more than one vote, and that all profits above interest shall be divided among members. The capital stock is two thousand shares of $50 each. Its business consists in buying and selling any class of goods required by members, and manufacturing. It is under control of a board of directors elected annually by the members. All business is conducted on a cash basis. All goods are retailed to members or non-members at the ruling prices as indicated by the actual prices prevailing in stores in the vicinity. At the end of each quarter the books are balanced and dividends declared according to the recommendation of the directors and the approval of the stockholders, in proportion to the amount of the purchases of each individual during the quarter. Only half-rate dividends are given to non-members. The dividends have varied from 10 per cent. to 20 per cent., except in one quarter when there was no profit at all. The business is carried on entirely upon an economic basis.

Some attempts have been made to improve the social condition of the laborers. There are free billiard rooms and bowling alleys, a small grove where swings and benches are provided, and a base ball ground. The company maintains a landscape gardener and keeps a green house from which residents are supplied free of charge with as many flowers as they wish to set out and care for. A well organized literary society is in existence where occasional lectures by distinguished men are given. There is a well trained band which gives open-air concerts during the week on the grounds

adjoining the clubhouse. A library containing about 600 carefully selected books is free to all. A large number of the readers reside outside of Leclare. There are provided also a kindergarten and public schools. In the school, students are admitted on part time and allowed to work in the shops or the farm during a portion of the day, for which they receive compensation.

As to the results of the entire system as practiced by the N. C. Nelson Company it is the opinion of the managers that the waste of time and material has been greatly reduced, that there has been a better attention to business, and that there has been established a solidarity of employer and employee in a common business in which they are mutually interested from which they draw mutual profits.

The brief statistical presentation of these examples of attempts to solve the labor problem reveal to us the fundamental proposition in the process of its solution, namely, that as interests of capital and labor, of employer and employee are common and all warfare between them is unnatural, any system which will tend to establish this fact will have within itself the basis of success, and any system which fails to establish this certainly will not succeed. There must be established a solidarity of interests of employer and employee upon an economic basis. There must be established a feeling that their interests are common. Having established this, and acting upon it on the basis of absolute justice, any rational plan has the probability of success. If this be continued further in the social life, so that the employer and employees mingle together on a common basis, the barriers now existing between the classes will be broken down and there will be a common sympathy and trust between them. From the foregoing examples we may infer that a successful solution of the problem rests upon the observance of the following principles:

1. The laborer must have an economic interest in the product of his own industry to insure care of tools, saving of time, saving of material and the creation of a better quality of goods.

2. He should be received into total or partial partnership in the management of the business through stock ownership or some similar means.

3. Both employees and employers should co-operate in furnishing means of social improvement.

4. While working together the utmost sympathy should prevail between the employer and employee and at the same time due respect should be given to the respective position and rights of each class.

5. In order to bring about the above conditions the employers must cease to combine against the interests of laborers and the latter must cease to combine against the former.

6. To gain the confidence of the public in all efforts to their own improvement laborers must cease to militate against members of their own class and recognize the rights of all men to work for wages according to their own choice.

7. And finally it may be said, to accomplish the above there must be a constant education of both employers and employees concerning the rights, duties and limitations of each class and the mutual interests of each as if no class distinction existed either on an economic or social basis. And this leads to consideration of individual character as causation in social improvement.

The Servant Girl Problem.

BY MARTHA BOUTELLE SNOW.

The heated discussion which this question has lately aroused seems, undoubtedly, quite needless to many, who regard it as a very simple matter. Simple it may seem to the uninitiated, but, as a recent writer says, "it is as momentous as that of capital and labor and as complicated as that of individualism and socialism."

Everyone knows something of the depraved mental and moral—to say nothing of the physical—condition which results from a diet of bad bread and burnt steak, and most families have experienced the inconvenience and discomfort of a household after "Maggie" or "Susie" has "given warning." The worry of domestic life affects in no small degree all the other relations and departments of life.

At no other time and in no other country has this problem become so serious as in the United States and at present. Within the last fifty or sixty years, foreign immigration has to a great extent furnished material for this department of industry. During this time there has been a very marked falling off in the number of American servants, and not because of any crowding out, as the supply seems never to equal the demand. "The plain truth of the matter is," says one writer, "that the whole native population of the United States has almost suddenly and with one accord, refused to perform for hire any of the services usually called menial or indoor. No country has ever before refused to do its own chores."

The foreigners who have stepped into these positions have received little or no training for such work. A great majority of the young women who come to this country, expecting to be taken into our homes to do work which requires skilled labor, are untaught and inexperienced in every branch of household work. They come because they are unable to obtain positions at home. Many even of the better trained are quite ignorant of our ways and tastes, and our household arrangements are to them mysterious and bewildering.

Of the American girls who have gone into service, we find that a very large number are from the poorest and most ignorant classes,

while the better classes of young women prefer hard all-day work with small wages in almost any other occupation. For instance, one girl, in preference to housework with a weekly salary of $3.00 at least, with board and lodging, chose employment in a shop where she received only $2.00 a week, and was compelled to walk three miles a day.

It is indeed hard to understand why girls hold in greater favor positions in laundries, where they are required to do heavy work, and of a kind which gives no chance for rest at any time in the day, or in sweat-shops, the slow torture of which is too well known to need description; or in factories where the work is positively injurious to health. Take for example the soap factories, where, in one department, girls stand all day wrapping the bars of soap in papers. It is found, after a little time, that the fingers and nails actually become eaten away by the caustic soda in the soap. On this account one of these positions can be kept for only a comparatively short period, but there are always girls ready to take the vacant places.

One cause of suffering among working women in our large cities, about which, especially in these hard times, we have heard so much, is the fact that women crowd every industry except domestic service, and seem willing to starve in the city rather than take places in private families in the country. Why is this true, and what has brought about this state of things?

It is the object of this paper to discover if possible the reason of this prevailing prejudice against domestic service, and the remedies, if any exist. Do young women dislike to be servants, because these positions are unprofitable financially when compared with others? Take, for example, the position of dry-goods clerk, and let us compare it with domestic service. The former is generally conceded by the public to be the higher of the two positions in the social scale. The wages are apparently much greater, and it seems, at first thought, to be a preferable employment in all respects.

It must, however, be remembered that a servant incurs no expense for board and room. Her expenses for clothing are not necessarily great, for a dress appropriate to the kitchen must not be of costly material, while a dry-goods clerk is obliged to spend a large portion of her salary on dress. Moreover, a certain rivalry in extravagance in dress is fostered among clerks, which in some instances, has been so marked that proprietors have insisted upon a uniform costume.

In order to show the relative incomes and expenses of servant

girls and dry-goods clerks as illustrated in a town in Kansas I have secured statistics from representatives of both occupations. The following results were obtained from answers relating to investigations concerning wages, expenditures for clothing, incidentals and family support, and savings.

I.

SERVANT GIRLS.

No.	Yearly Wages.	Clothes.	Incidentals.	Family.	Savings.	Nationality.
1	$ 78.00	$ 40.00	$ 10.00	$ 28.00		American.
2	100.00	35.00	15.00		$ 50.00	Germam.
3	104.00	13.00	19.00	72.00		Colored.
4	120.00	60.00	30.00	30.00		Swiss.
5	130.00	115.00	15.00			American.
6	130.00	96.00	34.00			German.
7	133.00	60.00	48.00		25.00	French.
8	140.00	50.00	90.00			American.
9	156.00	50.00		106.00		German.
10	156.00	75.00	6.00		75.00	Canadian.
11	156.00	58.50	19.50	19.50	58.50	American.
12	182.00	62.00	30.00	30.00	60.00	Swede.
13	208.00	10.00	5.00	193.00		Colored.
14	240.00	100.00	90.00	50.00		German.
Av'ge	145.21	54.61	29.39	37.75	19.18	

2.

DRY-GOODS CLERKS.

No.	Yearly Wages.	Board and Room.	Clothes.	Incidentals.	Savings.
1	$208.00	$104.00	$ 78.00	$ 26.00	0.00
2	360.00	168.00	198.00		0.00
3	416.00	Home.	300.00	16.00	$100.00
4	420.00	Home.	240.00	60.00	120.00
5	480.00	180.00	120.00		*180.00
6	480.00	180.00	300.00		0.00
7	520.00	240.00	250.00	30.00	0.00
8	780.00	260.00	*400.00		120.00
Average	$458.00	$188.67	197.50		

The wages of the servant girls vary from $1.50 a week to $20.00 a month, the yearly average being $145.21. The salaries of the dry-goods clerks range from $16.00 to $60.00 a month with a yearly average of $458.00.

The average yearly expense for clothing on the part of servants

*Including incidentals.

is only $54.61 while that of dry-goods clerk is $197.50. The latter in some cases spends as much as $300.00 yearly for clothing, while $115.00 is the greatest amount recorded by the former.

The two clerks who live at home save respectively $100.00 and $120.00, spending $16.00 and $60.00 for incidentals, but it will be readily seen that these amounts, $116.00 and $180.00, are neither of them equal to the $188.67, the average expense for board and room.

Taking each table separately, if we add together the amounts under the headings "Incidentals," "Family," and "Savings," the following result is secured. From the sum obtained from Table No. 1, we get an average amount of $86.33. This represents the money not consumed by clothing, board and room, while in the case of the clerks, the corresponding average is $57.00 or just $29.33 less.

It will be necessary to explain that in making out the average just mentioned, the savings and incidentals of the two girls who live at home were omitted because the money in their case is not saved, and $100.00 was added to the list because one clerk has included incidentals in the $400.00 for clothing.

From the study of all the figures obtained from these reports the fact is strikingly evident that the servant girls save more money than the dry-goods clerks, and have more to use for incidentals and for the assistance of friends.

Employees in these stores, moreover, have the advantage of reduction in the price of the dry-goods they purchase. At one of the largest establishments visited, clerks have a discount of ten per cent., at another they have goods at cost.

Two or three cases of thrift among the servants inteviewed are perhaps worth mentioning. One American girl receiving $3.00 a week, laid up enough money to invest in cattle. She was married, and, adding her own to her husband's savings, they were able to buy a quarter section farm.

The colored people are noted for their shiftless habits, but there are occasionally striking exceptions among servant girls. One colored woman who has for years received $3.00 a week has recently bought a place for $150.00, and has a neat sum invested in the building association. Another has from $3.50 to $4.00 with which she supports herself and three children, and pays $5.00 a month for a small house.

The fact is, the wages of servants are not affected by financial embarrassments. They were raised in a marked degree during the war and have never been lowered. There is one important point which should be mentioned in favor of the position of dry-goods

clerk, though this is noticeably true only in the great stores of the large cities. In the establishment of a well-known Kansas City firm, one of the largest dry-goods stores in the West, young women are given the same advantages as men, with salaries ranging from $15.00 a month, or $3.75 a week, to $3,500.00 a year.

In another house the range of wages is from $3.00 a week to $1,500.00 a year. On securing a position in this store, young women are frequently told, "You can come on trial at such a price; it will depend on you how rapidly your wages are advanced." This has, however, upon many no effect, and the higher positions certainly require an unusually shrewd business faculty. It is for the great class of girls who do not possess such ability that we desire to make domestic service attractive as well as profitable.

The life of a dry-goods clerk is by no means an easy one. Of the three clerks who expressed an opinion on the subject, all preferred clerking on account of social standing, one considered the work of both employments equally arduous, and two greatly preferred the position of servant, leaving out of consideration the question of social prejudice. One young woman acknowledged that in the latter occupation she could make more money, have easier work, and more time to rest and improve her mind.

In one of the stores visited, clerks are provided with stools and may sit down if they have time, but an opportunity seldom comes. At another store they are never allowed to sit down.

Clerks are required to work in the evening during the winter holidays, for two or three nights a week during several weeks before Christmas, and every night through three weeks of invoicing; also on Saturday evenings throughout the year, and at any other time when new goods are received or when the pressure of work is especially great. It is true that they have all of Sunday, while servants have only half a day, but this is more than made up for by "the afternoon out" through the week—and of such privileges town servants receive many.

Again, let us compare the servant's position with that of workers in factories and shops of all kinds in the cities.

From statistics taken from the report of the Commissioner of Labor for 1888, it may be seen that the average yearly expense for board and lodging for 277 girls in twenty-five different trades is $164.80. Adding this to the average income of servants, according to table No. 1 above, we have a total income of $310.01.

According to other statistics in the above mentioned report, gathered from twenty-two of our largest cities, the average yearly wages of 13,822 women representing 337 trades is $272.45. This

is $37.56 less than $310.01 which represents our average yearly income of servant girls.

The feeling of social degredation seems, then, to be the chief barrier against a servant's position for girls.

If we endeavor to discover the reasons for this feeling, we must remember that times have changed, and these changes are felt in all departments of life. A spirit of freedom and independence has ruled this country from the very beginning. This spirit has grown and has brought with it a hatred of superiority and a desire for equality which affect all classes of people.

Some one says, "The declaration of independence, universal suffrage, and unrestricted immigration are the causes of our minor domestic woes." However this may be, it is certain that we cannot now expect to find the old fashioned family servant who has devoted her whole life solely to the interests of her employers, and who seems almost part of the family possessions.

Another cause for this prejudice is to be found in the fact that, though slavery has been abolished in this country, there are feelings and relations closely connected with that institution which have become so deeply rooted in the minds of the people that it will take more than the civil war and the emancipation proclamation to destroy them entirely.

Moreover, Americans are proverbially a restless people. Wage-earners, as well as other classes, enjoy the change and variety which city life affords, and house work in the country is therefore to many distasteful.

Comic papers and the stage have done not a little with their jokes at the expense of servants, and their slurs against the position to bring domestic service into disfavor.

Then, too, girls feel, though without cause, that their position in the social scale has been lowered when they cease to be called "Miss" and are known only as "Bridget" or "Maggie." Small disagreeable kitchens and other apartments for servants, poorly regulated work with no definite and well defined hours, are responsible in no small degree for the prejudice of society against the position of servant.

Again, we must consider the not unnatural effect produced on the minds of the people by the poor class of untrained servants which has to be tolerated for lack of better. The comparatively few faithful, well-trained and intelligent servants are compelled to suffer for the short-comings of their associates.

Now, cannot something be done to make this profitable occupation more desirable for young women, and to remove from it the

idea of disgrace wrongfully associated with it? Ruskin says that the commercial relation between employer and servant has proved a failure, and that all that remains is slavery or adoption. However, adoption has been tried, especially in the west, to the advantage in some cases perhaps of the servants but to the great discomfort of the family. The commercial basis just as in other occupations is the only true basis. One writer even goes so far as to advocate giving up all personal interest in one's servants. She thinks there is no more reason for inquiring into private affairs of servants than into those of the seamstress or the occasional carpenter employed about the house. But one's relations with a person who spends most of her time in one's house would of necessity be more intimate than with an occasional employee.

There are some who suggest a special training for servants in one particular line of work. They are to go about from one house to another performing the same duty over and over again through the day. This would result in a disastrous lack of responsibility on the part of the half dozen or more servants employed at different times in one household.

The idea has also been advanced that girls should never live under the same roof with their employers. This, however, is not practical as it involves unnecessary expense, and it is far better on both sides that the mistress shall know something of the life of her house-maids outside of working hours. It is true, they need the intercourse with companions of their own class, from which they are otherwise somewhat cut off, but visitors should be allowed in the kitchen, and it is now becoming customary to provide a servant's sitting-room. House-maids must have pleasant rooms to live in, and when more women become architects more thought will be given to that part of the house.

There are few, however, who would think it necessary as one writer suggests, to have the kitchen in the front of the house, because servants as well as mistresses like to be entertained by looking out of the window while engaged in their work.

There is no industry in which skilled labor is more necessary than this, but there is also no industry in which there is so little done to procure it. Our standards of requirement are high, but what right have we to expect that servants will meet them when their training has been such that this is impossible?

We have schools for all other trades and professions. Manual training schools are becoming everywhere common, and a marked advance was made when it was thought necessary to have trained nurses. To be sure, cooking is taught to some extent in our

agricultural colleges and manual training schools, but the requirements and conditions of these schools make them out of the reach of most servants.

This subject is not altogether a new one and many are the objections which have been made against servant's training schools. It is said that servants will not see the advantage to be gained from them and therefore will not attend them; also, that housekeepers will not recognize the superior ability of these trained young women, and they will therefore be in no greater demand than their untrained associates.

But the experiment has been tried, and these objections can be met and answered with knowledge based on facts. Among New York society women who have interested themselves of late in social subjects is Mrs. Spencer Trask. She is the wife of a wealthy banker on Wall street. Two years ago Mr. and Mrs. Trask erected in memory of two children a large and beautiful building which is called "St. Christina's Home." It is for the education of servant girls. This is one of the finest buildings on Ballston Avenue, Saratoga. Here forty girls are trained in the common branches and in all departments of household work.

Girls from any part of the country are received, and the number is always full. If a girl is able to pay for instruction, she is permitted to do so, but otherwise, free instruction is given. The age for entrance is from ten to fourteen years, and the course from three to five years. The school provides a pleasant home during this time, and the general air of refinement which pervades it furnishes no small part of the education of these young girls. Graduates from this school are in great demand, and they find no difficulty in obtaining high wages.

Mrs. Trask has tried in her own home the experiment of giving to her servants a pleasant sitting-room with pictures, and literary and musical advantages. This departure at first called forth only ridicule from her friends but they now have cause to envy the domestic comfort of Mrs. Trask's household.

Mrs. John Sherwood is another New York woman who has practical ideas on this subject. To quote from a newspaper account of her theories, "She believes that girls from cities would be induced to go to such schools because they would have good homes for the year or two they would be under instruction.

"The home and training would be free. But to give the girls a feeling of independence they would work after graduation for a year or six months in some family, where they would be located by the school, and thus assist in its maintenance.

"A diploma will be given on graduation from the school. If a girl remains a year in the place she goes to on leaving the home school, she will receive a second diploma. This would be a guarantee to anybody desiring her services that she was thoroughly proficient in the branches of household work which she had studied. With the supplementary diploma she would be given a sum of prize money and could without doubt command the best of pay afterward."

It is to be hoped that the time will soon come when others who have money and wish to do good, will see the great work to be done for servant girls and will follow the example of Mr. and Mrs. Trask. Why should not the establishment of such schools be considered the duty of the state?

Many of our domestic troubles arise from the ignorance, not always of the servants themselves, but very often of the mistresses. They expect results which are absolutely impossible on the part of any human being, and only because they know nothing of the work they superintend.. If every young woman should receive a thorough training in cooking and in household management, she would find in the future that she would be able to instruct and direct her servants with good results. Every house where servants are employed, should include a training school on a small scale. If a girl is unfortunate enough to take her first position in a slack and disorderly household she acquires habits which will always hinder her advancement.

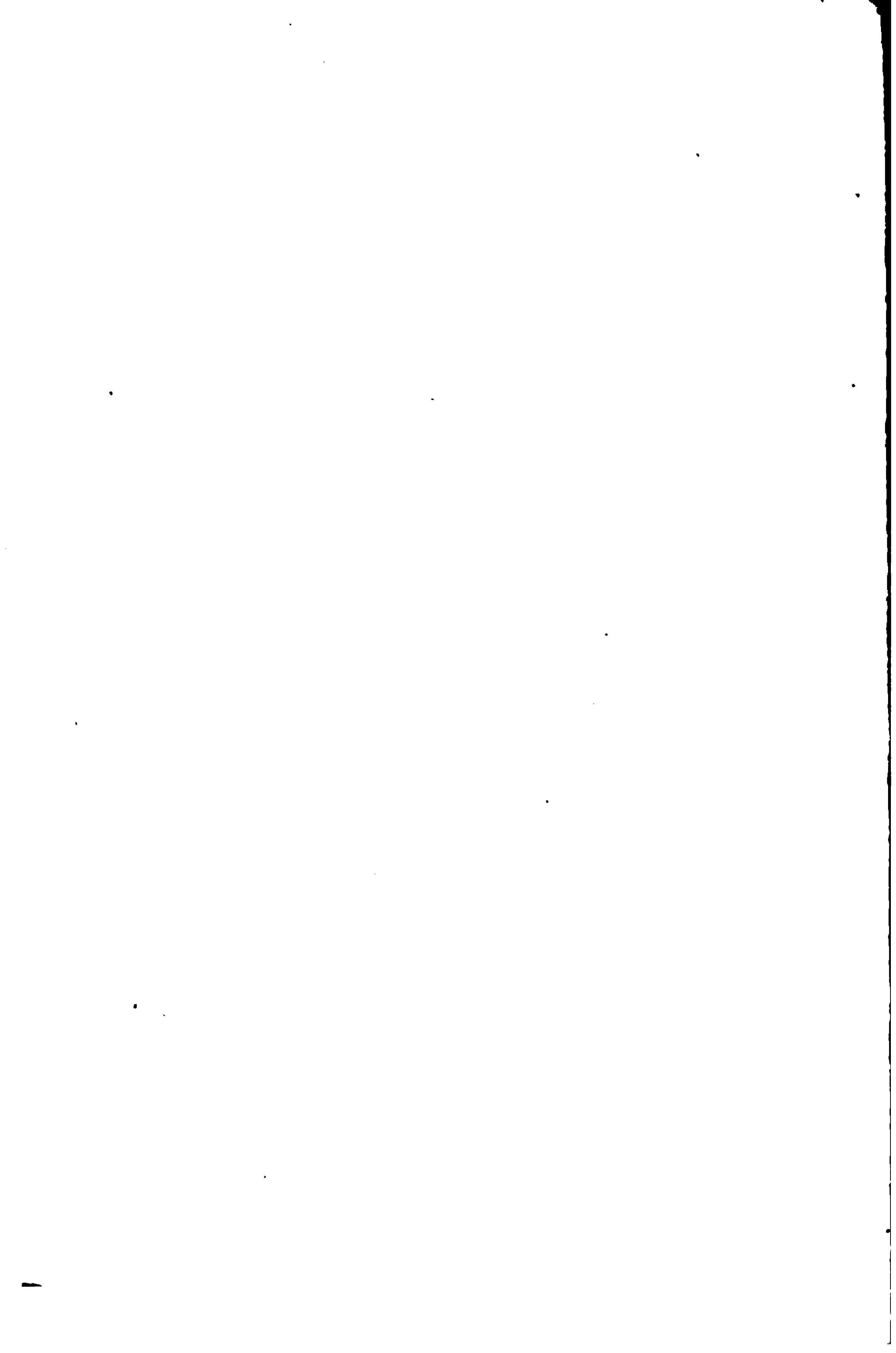
The petty fault-finding, which is habitual with many housekeepers, should give way to kind reproof and careful correcting of shortcomings when necessary, and to praise when it is merited.

A servant must know her place, but kindly dignity on the part of the mistress who is not afraid of being imposed upon, is better than lofty superiority. So long has the latter method been in vogue that it may be considered one of the causes of the low esteem in which domestic service is held among the better class of girls.

More system with regard to work and well-defined working and leisure hours should be insisted upon, as also a greater regularity in payment of wages.

Above all is it important that the instruction given in the public schools should overthrow the foolish notions of disgrace which are so common, regarding labor with the hands.

The solution of this problem is really to be found in the words "harmony and sympathy on the part of both mistress and maid," and all efforts expended upon it should aim at the establishment of such a relation.



The Condition of Packing House Employees.

BY A. E. MOODY.

[Study in the Armour Brothers' Plant, January 1st, 1895.]

The junction of the "L" Road and the Kansas State Line is a locality teeming with gambling houses, policy shops, saloons, restaurants, stores and shops of every description. An air of cheapness and poverty pervades the neighborhood, the stores are dirty, the streets are dirty, the children are dirty. A fringe of two story buildings along the street hides, in some measure, the squalid dwellings of the people who live in the rear. We are in the heart of the "West Bottoms" of Kansas City—an area lying between the bluffs on the Kansas and those on the Missouri sides of the line, or we may define it as the area lying between the railroad tracks and the Kaw river. This area contains over 10,000 people and is the center of the packing-house industries of the two cities.

General Superintendent Tourtellot of the Armour Packing Company gave permission to interview the foremen of departments and ask all the questions desired concerning the men under them, and tried in every way to make the visit a success.

For the week ending December 29th there were 2700 names on the pay roll, but this number varies by a thousand according to the demands of business. This thousand constitute the class known as floaters, to which the majority of the common laborers belong.

Work is now done on the basis of an 8 hour day which went into effect December 29th, 1894, but this does not necessarily imply that all the employees work 8 hours daily, on the contrary, some are now working only 30 or 40 hours per week. A large number of the employees are paid by the piece; others, working on the basis of an 8 hour day, earn from 10 to 35 cents per hour—the length of time a man works each day depending upon the amount of work on hand; the foremen and some of the most skilled workmen are paid by the month. This paper will not touch at any length upon the condition of the several hundred office employees but will deal almost entirely with the common laboring people. The following is a summary of the condition of the different departments:

1. CANNING ROOM.—Full force is 700 men and girls, all of whom are over 17 years old—540 of this force work in the Label Room; 96 girls earn from $1.50 to $1.60 per day on piece work here, working 8 hours daily and lunching in the room where they work (a light, airy, clean and sweet-smelling room). Tinners (young men 18 to 20 years old) earn from $2.50 to $3 per day. Girls, by day work, make from $4.50 to $6 per week; by piece work, up to $12 per week. Men, by day work, make from $6 to $12 per week. The majority of the girls live at home; a few are widows earning their own living. The Label Room is under the supervision of a forewoman who is much valued by the Superintendent because of her close attention to business. Employees in this department save very little; some of the girls wait on table (at dinner time) in the company's restaurant, for which they receive overpay and their dinners. This restaurant is run as a matter of accommodation and is for the use of the office employees and the foremen only. It is the general impression, among certain people, that the ordinary working girl is of loose or immoral character, but I was repeatedly assured by the various persons with whom I talked that the girls and women employed in the plant *are not so*.

2. TIN SHOP OR CANNERS' SHOP.—Running 80 hands at present writing, of whom five or six are women and girls. Men and boys range from twelve years up. All piece work in this shop—no makers of finished products, hence none of the men belong to trades' unions—average class as regards intelligence and morality—wages range from $1.50 to $2.75 per day, by piece work.

3. LITHOGRAPHY.—Full force, eight men, fourteen women, all unmarried—men work by the day at from $1.50 to $5 per day; women by piece at from $ .75 to $2.50 per day. The latter figure is seldom reached, $1.25 being the average limit. The foreman said that he found a good education a good thing in any line of work. The workmen in this room have all received their training in the plant and are exceptionally bright: one of them will soon remove to a western state where he has secured a position at $25 per week. The foreman is a member of the Young Men's Christian Association and is one of their best gymnasts.

4. BONING ROOM.—Full force, two hundred men, most of whom have families. Swedes predominate here. Six are time men, others are piece workers. This gang is continually changing.

5. POULTRY ROOM.—Full force, 150 men and 35 girls. This room corresponds in its essential features to No. 4.

6. SAUSAGE ROOM.—100 men and boys, 25 women; of these twenty-five, four are widows. There are thirty boys aged thirteen

and upward. One seventeen year old boy who has worked in the packing house for three years can neither read nor write. Only three or four of the men are forty years old. Some are buying homes on the installment plan. Average wages are made here on both piece and day work. Workmen are of the common type—work is dirty but healthy. The foreman has been fifteen years a foreman.

7. TANK ROOM.—Three men at $1.75. Four men in press room—married—steady workers—have been here for years. Twenty-six men and ten girls in pigs' feet and tripe department—steady and industrious. Eight of these men are single. Girls work by the piece. No labor organizations have members in this department. An old negro from Mississippi in A. P. H. since 1879, has five children, owns his own home, is fifty-six years old, has saved money, doesn't attend church because he spends his money in fitting his children to go. Though he can scarcely read or write, he is giving his children a good education. Another man has been in the tank room for twenty-one years. He has six children and owns his home; sixty-four years of age. He says that wages are better now than when he first came.

8. HOG-KILLING.—Foreman has worked fourteen years here. From one hundred ten to seventy-three killing hands. Eight boys from ten to fourteen years of age, fifteen cleaners; men are twenty-one to fifty years of age. They are steady industrious men. The work is fairly healthy. They all work by the day except the head gang. A mixed crowd—negroes, foreigners. Head men receive $2 a piece per day.

9. CARPENTERING AND BRICK MASONRY.—Fifty men—a few union men—all have families and are steady workmen—ages range from twenty-two to thirty. These men do all the repairing and building for the plant.

10. HOG-CUTTING.—Fifty-four boys, forty-six men—all day workers. Nothing unusual in this department.

11. SUPPLY DEPARTMENT.—Eighty men and boys—some boys under eighteen—sober and industrious men. Foreman says that he must have good men in his department. They are all day workers above the ordinary intelligence. Some few blacks. Freezer boys work at 50 cents per day. The freezers are kept at a temperature of 14 degrees above zero. It is not an especially healthy job for a man to work in the freezers.

12. SAUSAGE TRIMMING.—Four men, two boys, forty-one girls. Foreign and American. Boys, thirteen to twenty years of age. Girls are piece workers—girls average five hours per day during

the month of January. Men and boys are day workers. This department is very clean and healthy.

13. CURING, SMOKING, PICKLING.—Two hundred to two hundred and forty-five, nearly all men. Ten to twelve boys, twelve to eighteen years old. Very few men in labor organizations, therefore we term them nearly all common laborers. Sixty per cent. of the men in cellars and cool damp work are foreigners and negroes. Swedes and Germans are far stronger to do this work.

14. LAUNDRY.—Four girls and one man. All the house linen is washed here free of charge. The girls are waitresses in the company's restaurant and also make all the aprons, etc., for the house. They work by the day for eighty cents and their dinner—eight hours. They are girls of integrity.

15. HOG CASING.—Twenty-four men and two boys—thirteen men married. Unhealthy work—slop and steam tend to make it so. All work here is day work at $1.40 for eight hours.

16. ICE PLANT.—Six men on the day force, five night—work twelve hours for $1.80. At the engine, eight on the day force and same number at night, at from $2 to $3.50. All married but two. Boiler room, fourteen men by day, same by night, at from $2 to $2.75. Some colored men. Few in trades' unions, but nearly all in benefit orders of some kind. Majority are renters.

17. BUTTERINE.—Twenty-six men, six girls, three boys. All piece workers. Clean, airy work. Average wages $1.

18. TIN SHOP.—Seven men, all tinners—$2.25 per day. Five of them married. Intelligent men—renters, for the reason that taxes are too high in localities near their work. The foreman has a fair house of six rooms at $11 per month, with a railroad passing less than eleven feet from his door.

19. FERTILIZER.—Thirty-two men from twenty-eight to fifty years of age. Six boys sixteen to seventeen years. All day work at 15 cents per hour. This work is healthy, one's nose to the contrary notwithstanding. The foreman was a consumptive when he entered this building, but now he is as brawny as a prize-fighter. The American is too aristocratic for this work, and it has been delegated to the Bohemians, largely, who live in the "Patch."* Many fires occur in this department.

20. HIDES.—Twenty-one married men. 15 cents to 20 cents per hour for an eight hour day. The work is healthy. The men are mostly Austrians, illiterate and uncouth—live in the "Patch."

21. NEUTRAL.—Eleven men, one boy fourteen years old. Three of the men are married—all work by the day—average wage is

*An irregular tract lying west of Armour's packing-house. See infra p. 56.

$1.50, although some get as high as $2. No skilled labor in this department.

22. STORE ROOM.—Two clerks and a foreman, of necessity intelligent men. Here all requisitions for supplies are made. The clerks are young men.

23. PACKING AND LARD.—Sixty men, married, of mixed races. Average wage for eight hours, $1.60. Fairly intelligent.

24. CAR SHOPS.—For repairs only. Forty men, all skilled laborers except two or three. Scarcely any of them are unions. Half the force is on piece work at an average wage of $2 per day. Nearly all are married. About one-third own their houses. Many lodge and benefit society members.

25. MACHINE SHOPS.—Machinists, blacksmiths, steam-fitters, boiler-makers—seventy-one all told. Twenty to thirty-five years of age. Good workmen. There are seventy-three day and night engineers and firemen in the four plants of the company, in engine and boiler rooms. No finished products come from these shops—just repairs made.

26. CARPENTER SHOPS.—Six men. A cabinet-maker, wagon-maker, pattern-maker, saw-filer. Average wage, $2 per day. All learned their trade abroad, and are non-union men. All are married. One owns his house. Good steady men, working eight hours per day.

27. PAINT SHOP.—Eight men, four of whom are painters; the others attend to the scales, elevator ropes and belts. All are married. Wages range from $2 up. Two own their homes. These men are steady, industrious men.

28. GLUE.—Sixteen men, twenty-six to twenty-eight years old, work by the hour at 15 cents to 22½ cents per hour, from an eight to a ten hour day. Some few lodge members. The work is clean and healthy. One man has been in the glue room for eighteen years, and owns two houses on leased land in the bottom. Has a thirteen year old boy whom he is sending to night school.

29. FIRE DEPARTMENT.—Plant can be emptied in five minutes. There is no general alarm. The outfit consists of two hose reels, four chemical wheelers, two chemical stationary, Armour water towers (at a nearby station), ninety-five watchman's boxes, automatic extinguishers in box factory, lard house, and fertilizer buildings. Private water tank in three buildings. Seventeen night watchmen, five day watchmen, six hundred feet of hose. All alarms center in General Superintendent's office. Marshal and his assistants are on salary, and the eight firemen get 25 cents per night. These firemen work in the packing house during the day.

30. COOPERAGE.—Forty-two men. Thirty-six are skilled workmen at $3 per day of ten hours. Others earn from $1.50 to $2 for ten hour day. Mostly married men—such are preferred. Union and lodge members found here. The men make finished products, but repairing is their chief work. They make special sizes of casks which may be needed, but others are bought. Ages range from twenty-two to forty-five years.

31. LARD.—Foreman has been thirty-six years in this business. Sixty-eight men here and sixteen in the oil house. More than half are boys from twelve to twenty years of age. The rest are married men under thirty. Work dirty but healthy. Foreigners and Americans found here. About one-third are house owners. All but four are day workers earning from $1.50 to $2 per day—some few get $1.20 for an eight hour day.

32. BOX HOUSE.—The biggest one in the world. Sixty men and boys are employed, fifteen of whom are skilled workmen. Wages range from 50 cents per day to $1200 per year. Two nail sorters get 50 cents per day. More than one-half the force are married. Only one man unable to read and write English, though he is an intelligent Swede. No members of labor organizations, as they are too young and do not make finished products. There are no box makers' or nailers' unions in Kansas City. The men are sober and industrious. Most of them carry accident insurance. Soap and towels are furnished to squads who wash in the overflow tanks, each squad taking care of its own soap and towels. Each man seems to be satisfied. The foreman says, "We get what we ask for." The company spares no expense to prevent the occurrence of accident. In many cases the savings are large. One man has saved $6,000, made here in the plant. Another man in the saw-mill is worth $2,000 which he has saved while working here. Another man is worth $3,000. One man has one or two very swift trotters in his stable and takes delight in showing them to his friends.

33. SHEEP.—Twenty-eight men, eight boys, from thirteen years up. One-half are skilled workmen. The others are roustabouts, at $1 to $1.75 per day. The skilled men are piece workers. Only six men are married. This is an unsteady gang. Men are illiterate. Mostly renters. The work room is low, has no outside windows. It has ventilators and air fans for use in summer time, but notwithstanding this, it seems a most unhealthy place.

34. MARKET PIGS.—Twenty-five men, one-half of them negroes. Average class. Work healthy. Wages range from 10 cents to 30 cents per hour, averaging $1.65 per day. About one-half of

them are married. Eight are skilled workmen, good butchers and dressers.

35. BEEF KILLING.—Large, airy top room. Largest beef slaughter house in the world. Employs one hundred and fifty men and twelve boys; fifteen to forty-five years of age. One-half are married. Fifteen or twenty own their houses. Wages on day work from 15 cents to 40 cents per hour. Some boys get $2 for a ten hour day. The work is healthy. Most of the men are fairly intelligent. The sanitary condition throughout the plant is good, but there is room for improvement.

36. BEEF CASING.—Force, one hundred men and ten boys—wages range from $1 to $4 by day or piece work per day. The boys earn $1 to $1.25. Work here is healthy. Workmen are of average intelligence, some few are lodge members. Most of them are not house owners.

37. NEW TANK ROOM.—This room has an intelligent negro as foreman—the only one who holds that position in the house. Ten men work here, receiving from 15 cents to 22½ cents per hour, most all of them married. Three or four of them can neither read nor write. This gang is mixed as regards nationalities. In another part of this room are eight piece-workers (men), averaging $2, eight hour day. This work is hard and only the very strongest can stand it. Of this gang all are married, save one. The foreman is a widower with two children—he is boarding with a relative of his who cares for the children, but August 19th, 1894, he began to make payments on a forty acre farm near Oskaloosa, Kansas, which he expects to make into a home for himself and little ones. He has ten years time in which to pay for it.

38. OLEO OIL.—Fifty men and boys. Wages range from $1.50 to $2.50 per day. Work is by day and piece. All are intelligent. The gang is composed of young married men, mostly. The majority are renters, who save nothing. They are as sober as the average run of men. The work is healthy. They are not members of any labor organizations, but some few are lodge men. Some foreigners. One man has been in this department for eleven years.

39. ELECTRIC LIGHT PLANT.—Six men and one boy, all of whom have been brought up in the house. Four of them are married, and earn from $1.50 to $3 per day. Four work twelve hours, the balance work eight. Five are on the day force, two on the night. This plant has 8000 incandescent and 240 arc lights, and is one of the most complete plants in the country. None of the men are in labor organizations; some in benefit orders. All are of the higher class of sober, intelligent workingmen.

40. No. 3 Engine House.—A sample plant, four day firemen at $2. Nine men in boiler room. Four men in engine room. Three skilled men, others laborers. None in labor organizations. Average wage for twelve hours, $2.25. Most all married. Some few are house owners.

41. Canvass Room.—This foreman has been in the house twenty years and has been in this room since '85. He has accumulated quite a snug sum, having at the present time about $5,000 out on interest, all of which he has made here. He is unmarried and has the reputation of being the best dressed man in the bottoms. His parents are members of the Lutheran church. Five girls work here, earning 85 cents per eight hour day by day work, averaging 90 cents per day on piece work (sewing hams at 22½ cents per hundred). There is one widow here; the others are unmaried girls. All live at home. They are good, steady American girls, some of whom have been here for nine years, three Protestant and two Catholic church members. They have the nicest work room in the plant—they sew canvass and coarse bagging and are not, therefore, necessarily good seamstresses—ham sewing is done at a long table which has little tables projecting from it, behind each one of which is a little stool on which a girl sits. These stools give no support to the back of the one sewing.

42. Local Shipping.—Four laborers, three teamsters, all outside work, wages, 17½ cents per hour; two are married, one is a house owner, men of average intelligence, some are lodge members.

43. Packing Room.—For all sugar-cured meats. Forty men with wages from $1.50 to $2.50 per day, they are all married, are fairly intelligent, some few are house owners. Four clerks and weigh masters, earning from $2 to $2.50 per day. These are good intelligent men. A skilled tester who sorts the meats into different brands—"Gold Brand" ham is the best.

44. Livery Stable.—Thirty different nationalities—no negroes. Eight drive dump carts; others delivery and freight wagons. Men earn from $1.50 to $2 per day (ten hours). Some few own homes. A few are in lodges. Each man cares for his own team; fifty-four horses in the stable. Foreman has been here sixteen years.

45. Roustabouts.—Thirty-seven men, six of whom are negroes, no foreigners in this gang. These men have no fixed work, but go to any department which needs more labor temporarily, consequently the gang is constantly changing. For ordinary work they receive 15 cents per hour; for boiler room work, 17½ cents per hour; for firing, 20 cents per hour. These men are mostly renters

in the bottom and work when there is anything for them to do. About two-thirds of them are married. The cart gang numbers nine men, eight of whom are married. They receive 17½ cents per hour. No colored men in this gang; the entire gang is composed of "floaters."

46. UNLOADING.—Thirty-five men, majority of whom are married—sixteen are negroes, the rest are foreigners. Twenty-nine can read and write English. Average wage is $2 per eight hour day. These men board in the "Patch" or are renters in the bottoms. All their work is piece work—i. e. so much for a car-load. The following are some of the prices paid by the car-load: Cooperage, $5; coal, $1.40; cooperage in large packages, $2.50; salt in bulk or sack, $2.50; sawdust or lumber, $5. This gang is also composed of "floaters." Few of these men or of the roustabouts can save any money.

47. BEEF LOADING.—Sixty men, mostly negroes; all are married. Mostly illiterate, live in Wyandotte. Average wage 15 cents per hour; piece workers by the thousand pounds average $10 per week. Checkers and weigh masfers receive $12 per week. Not all of this gang belong to the "floaters," as some have been in the gang from six to twelve years.

From the foregoing a brief summary may be given:

1. The majority of the employees are unmarried and live at home or board; while those who are married for the most part live in rented houses.

2. Comparatively few belong to labor organizations of any kind; a large number are in benefit orders, lodges or life insurance companies, membership in which is encouraged by the Armour Co.

3. The average workman is fairly industrious and moral, but there is a large proportion of the employees (men, women and children) who live in homes of degredation and vice, outside of working hours. The home life of this latter class is fearful. They live in the immediate vicinity of the packing houses, crowded together in dirty little shanties, whose door-yards are the depositories of refuse and, in some cases, human filth.

4. The majority of the employees live in Kansas although some few live on the Missouri side.

5. Throughout the plant ample protection is afforded against fire.

6. Buildings are kept neat and clean, and are, in most cases, well lighted and ventilated.

7. The employees, while at work, are civil and well-behaved.

8. Nearly all nationalities are represented: German, Austrian,

Bohemian, French, Danes, Swedes, Dutch, Irish, Scotch, Poles and Negroes. These foreigners constitute more than one-half of the working force of the plant.

9. Wages range from 5 cents to 40 cents per hour; only a few save any part of their earnings.

10. Workmen of no nationality excel in any line of work, although men who have learned their trade abroad are generally considered more thorough workmen.

The Armour interests in Kansas City and Chicago form the largest packing establishment in the world. The new Beef House at Kansas City is the largest one in the world, being seven stories high with a cold storage capacity of 15,000 dressed cattle, and covering an area of 300 by 500 feet.

The general statistics of the Kansas City plant are as follows:

Ground acreage covered by buildings and used for other purposes .... 30 acres.
Floor acreage in buildings........................ 90 "
Cold air rooms.. 30 "
Storage capacity................................ 200,000,000 lbs.
14 Ice Machines producing a refrigeration equal to the melting of 1350 tons of ice every 24 hours.
Electric Light capacity equal to that used in a well lighted city of 20,000 inhabitants.
Daily killing capacity: Hogs, 12,000; Cattle, 4,000; Sheep, 5,000.

The term building, as used in the plant, means any building or portion of a building which is separated from another building or portion of a building, by a wall.

There are 4 main offices in the plant, viz: Executive, Fresh Meat, New Beef House, General Superintendent's. In these offices about 225 people are employed. Scattered throughout the plant are 20 smaller offices with a force of 50 clerks. There are 3 time-keepers, 2 for the "hands" and 1 for the office employees—time-keepers work from 6 A. M. to 6 P. M. The General Superintendent's office is the working centre of the plant; all work done is reported here; all repairs needed, all requisitions for men and for materials, first come to this office for an "O. K." The General Superintendent receives a salary of $25,000 per year; formen average $2.75 per day. The Traveling Superintendent, who goes out on short trips to near by points in case several towns should together wish to order a car-load of goods, receives $300 per month for over-seeing the correct unloading of these goods.

We have obtained some information concerning the plant itself, let us now consider the condition of the working people—how they

live, and what is being done for them. They have clean quarters to work in. Under the 8 hour system work is begun at 8 A. M. and stops at 4:30 P. M. (foremen at 7 A. M. and quit at 5:30 P. M.). Although the buildings are from three to seven stories high, there are but two or three passenger elevators in the whole plant.

Profanity, vulgarity and the use of tobacco in the buildings, are not tolerated, although a great profusion of lewd pictures everywhere prevails.

The firm has taken no stand either for or against labor organizations, but encourages membership in benefit orders of all kinds. A physician is nearby in case of accident. The General Superintendent said: "We insist on sober men in responsible positions; we want clean quarters, overflow tanks furnish abundant opportunity for employees to clean up; we furnish them soap and towels in squads; we use as much water as the city of Topeka; we have good girls, a better class than those in the mills." Mr. Tourtellot is one of the firm's standbys and virtually lives for the Armour Packing Co. alone. His hours are from 7 A. M. to 6 P. M. No Sunday work is done except when repairs or a rush of work demands (?) it.

One of the "characters" of the plant is an old darky 60 years of age. He has been in the house 22 years, 14 years of which time he was employed as hog-scraper, earning $14 to $15 per week. Although he is a perfect giant, physically, the work became so wearing upon him that he has been compelled to give it up. He is now receiving $1.50 per day for doing odd jobs—picking up waste paper, rubbish, etc. In the Cook Room is an old negress who earns $4 per week. Her husband works in the house also, earning $7 per week. Their united earnings support a family of five children, the oldest of whom is 10 years old. Both she and her husband are church members. A very common game, so I am told, among the children of the bottoms is what might be called "Family Scrap;" in other words, the children do just what they have seen their parents do, and pound and punch each other very vigorously. This game to them is probably what keeping house and tea parties are to the children of the better classes.

Saturday and Monday are pay days. Paying off begins at 5:30 A. M. and continues till after dinner. Saloons used to charge ten per cent. discount on meat checks, but now the house has ruled that only $2 per week be given in meat checks to any one person, and these must be cashed in Armour's meat market by the employee in person. Thus they have thwarted the saloon in its illegitimate gain. Today no pay checks are discounted, but go on face value.

No employee is allowed to loan or borrow money while in the plant, under penalty of discharge. This is simply a measure of protection for the employees. The paying is done by each man calling out his number and receiving his check. It sometimes happens, though very rarely, that a man comes into the office and says that someone has called his number and received his money. He is therefore minus his week's wages.

The company's restaurant has two cooks, a dish washer and head waiter, and here are served 15 cent and 20 cent meals to foremen and office employees. The common working people either sit down in their work-room and munch a cold lunch or go to a restaurant for a 10 cent or a 15 cent dinner or to a beer saloon where they can get a free lunch with a 5 cent glass of beer. In the winter of 1893-94 a long, gloomy room was fitted up on the dock. Its gloom was partially dispelled by electric lights and it was well heated and provided with pigeon holes in which dinner pails might be put. For some unknown reason this room is not run this year.

On Thursday, January 3rd, the "K. C." restaurant sold two hundred and fifty dinners while the three saloons in the immediate vicinity fed an equal number from their free lunch counters. This lunch in one saloon consisted of hot stew, onions, cheese and rye bread. At O'Brien's across the way they had sardines, bologna sausage and rye bread. This saloon draws the color line very sharply, having a sign above the counter which reads, "No blacks allowed to help themselves."

A large number of policy shops were in full blast a little farther up the street. Their drawings are held at 12:00 and 5:00. It is said that a person has one chance in one hundred to make any money on his drawing.

The packing house provides scantily for anything in the cloakroom line for the working people. There are no pleasant lunch rooms or rest rooms for the girls, some two hundred of whom work in the house. Half of this number are under sixteen. In the summer a large number of girls are employed whose ages range from eight to twelve. The girls are released from work ten minutes before the men, and they pass out and have their time taken by the time-keeper at the gateway, and are well on their way home before the men come out. Employees buy their working linen from the house. This is washed free of charge in the company's laundry. Armour makes no gifts to his employees. The sanitary arrangements throughout the plant are fairly good—more serviceable than elegant. The company has no system of profit-

sharing with its employees. Wages in this packing house are somewhat lower than in Fowler's. Armour's was the first packing house to employ a professional chemist.

Mr. Armour has done very little, practically nothing, to better the condition of his employees in their home life, and in this he is somewhat behind Mr. Fowler. It is the testimony of men who have been in the packing house for years that it is a desperately hard place to reform, and men who have been Christian men for years are still the butt of jokes and sneers from their fellow workmen.

I shall now ask you to visit with me some of the houses in the west bottoms. No. 1. In a little house we found an old colored woman and her brother. For three rooms she pays $6 per month and washes in part payment. She is a widow, is religious but not moral. The brother is a roustabout in the packing house. The house is very dirty, very crowded with furniture, such as it was, and had a rare odor. I was accompanied in these visits by one of the slum workers of the Salvation Army, to whose kind offices I owe much, for without her I would doubtless have been summarily ejected from some of the houses visited.

No. 2. Boone's. A four room house at $7 per month. A white man with a colored wife, a colored step-son and a little baby. The husband is a consumptive and unable to do any work. Harris, the step-son, is a cook earning $5 per week, but now out of work. They were paying $2.50 interest per month on $30 worth of furniture. The wife keeps one or two boarders. The front, or living room, is heated by a tiny stove that holds one and one-half small shovelfuls of coal. This house is owned by Mrs. Hickey. The husband yielded to the entreaties of a Salvation Army lassie and accepted Jesus Christ. The family as a whole, I believe, while poor are fairly moral.

No. 3. In a two room house we found Mrs. Hickey, an Irish widow. This place takes first rank for filth. Mrs. Hickey is a peculiar character. Some eight or ten years ago she owned a large number of cottages but these have been destroyed by fire. Borrowing money she has built several little shanties on leased land which she rents. From the Boone's mentioned above she has had no rent since December, 1893, but her heart is too kind to turn them out. Her ground lease costs her $37 per month. She borrowed $1000 at 10 per cent. and was damnably tricked into borrowing another thousand at 60 per cent. She has paid out $13 for a hydrant, $25 for a water meter and her monthly water rent, and supplies city water to her renters free of charge. She

has been defrauded out of her rightful pension, which has been recalled since her husband's death. Her shanties range in price from $7 for four rooms, to $1 for one room, monthly. Mrs. Hickey personally is kind and religious. Only a few days before our visit she had bought some dress goods of warm, serviceable cloth, and given them to a poor woman for dresses for herself and children. Her front room was her living room. In it were tables, sofa, chairs and boxes. The back room was her bed-room, kitchen and general workshop. As an illustration of the filthy condition I may only say that the sheets were so encrusted with dirt that they fairly cracked in one's fingers.

No. 4. In a one room shanty we found two old bachelors, both employed by the packing house. This place cost them $3 per month, notwithstanding the fact that the roof lets in the sunshine and the rain. The man whom we found at home evidently had seen better days, and his whole manner seemed to apologize for the condition in which we found him. Under the same roof at the other end, in a single room, we found an old negress with a little child, living in open immorality.

No. 5. In this house we found a grandmother, her daughter and sons—sons who work in the packing house. This outfit consisted of two rooms and a lean-to, and was sadly given over to dogs, dirt and devil.

No. 6. We failed to gain admission here, but through a partly open window we saw four negresses and a white man, smoking and carousing.

No. 7. An Irish woman with three or four small children—her husband absent at his work in the packing house. For their three rooms they pay $6, and as their front door is six feet from the railroad track, the landlord has kindly put a small fence around this and the adjoining houses in order to keep the children from the wheels. The poor woman had to give almost her entire time to her children, and her housework was sadly neglected. But had she time for it, doubtless her ignorance of all methods of keeping the house clean would have rendered her work futile. The children were dirty, the house was dirty, and above all arose a fearful stench. In this and neighboring yards the vaults and cisterns are side by side. How people live and thrive under such conditions is perhaps one of the wonders of the century.

No. 8. A two room ex-stable. Two washerwomen and two boys—the boys work in the packing house when they can get work; the women, who are young, do washing and ironing, though how they manage to dry their clothes and have them clean in the midst

of smoke, cinders and soot, is a mystery. One needs a chart and compass to get about this tiny house, for unless he is very wary he will stumble over some half-hidden obstacle and fall full length.

No. 9. A mother and daughter—husband out on a jewelry peddling trip. They live in a two room house which is furnished in that shabbily genteel manner so often found among poor people. With a sang-froid that was laughable she plied us with questions about our visit to the poor people. She said that she had heard that there were some in that neighborhood, but her time being so taken up with shopping, she had been unable to visit them.

No. 10. A two room house with a shed, in which we found a man by the name of Kishner, his wife and little child. The baby was sick with inflamed eyes. Kishner is one of the roustabouts and has averaged for the past few weeks thirty hours' work per week, which, at 15 cents per hour, has not furnished them with enough to live upon. He and his wife are both Christian people and express themselves very thankful to God for the warm weather that had so marked the winter. They pay $6 a month rent for this shack. We directed Kishner to the dispensary at the Bethel Mission where he procured medical aid for the baby and also had his own eyes treated. His wife was terribly discouraged, but seemed cheered up by the words of the Salvation lassie.

No. 11. A neat colored family living in a three room house. The husband works in the packing house at $1.50 per day, steady work. Nineteen-year-old girl, stunted and simple minded, works in the canning room for $4 per week. She can neither read nor write.

Saturday night I made a tour of the gambling houses. Maltby's was the largest one of these, and was running full blast when we entered. The room was comparatively quiet and the men orderly, to a certain extent. We cannot but be impressed with the large number of poor working men in these gambling houses. An ex-gambler told me that fully 60 per cent. of the frequenters of these places, and a much larger per cent. in the policy shops, is composed of poor working people. On good authority it is stated that this class of people pay the running expenses of these establishments. Thanks to a strictly enforced law, these gambling hells have since been closed.

On Sunday evening, in company with a friend, we visited the Bethel Mission church on James street, in the very heart of the West Bottoms. The membership of this church is largely composed of packing house people, and in the congregation we saw many faces, both of foremen and of laborers, that we had seen in the

packing house. The work done by this church in dispensary and employment bureau and other phases of so-called institutional church work is sadly crippled for lack of means.

On Sunday morning the "Patch" was visited. This is an irregular tract lying west of Armour's packing house and bordering on the river. About eleven hundred people live in this place. One two story house with a multitude of shanties and small houses of from two to four rooms, comprise the buildings. The land is under litigation and accordingly is leased to tenants. Fifty cents per month will secure enough land upon which to build a small house. The roads are narrow and winding, very filthy, even in winter time, with refuse and debris of all sorts. Vaults, cisterns and houses adjoin each other in close proximity. This settlement is almost wholly given over to foreigners, and here one finds club life existing. In these clubs, numbering from five to thirty-five in membership, board is obtained at from $1.40 to $1.50 per week. Each man pays the cook $2.25 per month, in return for which she cooks, takes care of the entire house, and does each man's washing and mending. Most of the clubs own their houses. Five hundred eighty men boarders live thus in these areas. These facts are reliable, as they were given me by a young man named Mott, a grocery clerk who lives in the "Patch." On Sunday afternoon furniture is packed out into the street and the house is cleared for a carousal. Beer, tobacco and jokes are the order of the day, and woe be to any unlucky missionary who goes among the Poles and Austrians on that day, for if he refuses to drink with them his life is not secure. On the bank of the river live some humble washerwomen, and so great—and justly, too—is their fear of these men that they dare not leave their homes after dark. It is needless to say that morality is very low.

Just before we left the "Patch" we visited a boarding house of four rooms in which lived, besides the man and his wife, ten boarders, all Bohemians. The man receives $9 per week in the packing house. His wife works like a slave, and is on terms of intimacy with all the men. We are wondering if there is any light in this darkness, and we find here and there signs that there is light.

A little farther on we visited Mrs. Lewis, whom we found sick in bed. She earns about $2 per week washing in a room in which it is impossible to stand upright. On account of an injury to her hand some years ago, from which she still suffers occasionally, she is unfit for work in the packing house. Her daughter, some fifteen years of age, lives at home in their house, which is offered for sale at $15. About three years ago she professed conversion,

and later joined the Bethel church. Some time after, an acquaintance of the mother took the girl to a town in Missouri, ostensibly for a visit, but really to make her the prey of an old man of fifty-five years of age, a prominent physician of the place. The child was easily led into this by promises of money and plenty of finery, and later the female who had designed this wickedness turned it to her own account through blackmail, and parties interested were paid liberally to close their mouths, even the mother accepting $50. The girl is now home again and seems to be desirous of leading a better life. This incident speaks for itself and is only an average illustration.

On Splitlog Avenue Mr. Fowler has built a Methodist church in which we found a revival in progress, whose total number of conversions to date had been seventy. A little farther on we find a Pleasant Green Colored Baptist church. In the bottom are also found, on the Missouri side, Liberty M. E. church, a Jewish synagogue, a Catholic church, a colored Baptist church, and the Railroad Y. M. C. A. On the Kansas side, Bethel church, State Line Mission (since closed). The Salvation Army was forced to leave the Bottom on account of lack of funds, but the people are rejoicing in the fact that they will soon reopen. It is a noticeable fact that while these people cannot always tell you about the churches, they are quite well informed as to the whereabouts of the Salvation Army—a fact that goes to prove that there is needed here more of the "good Samaritan" and less pulpit preaching.

On Hog Avenue Mr. Fowler has erected ten two story tenements of twelve rooms each. Three rooms up stairs rent for $5.00 per month; three rooms down stairs for $6.00. There is one entrance for each two suites of rooms, thus securing comparative privacy. The buildings are well made. Each suite has its individual wood and coal shed, and city water is furnished from several large hydrants in the street in front of the houses. This water is turned on twice a day by an employee of Fowler's. These houses face the street and do not line the alleys as the shanties in the "Patch" and other places do. Coal is peddled to these people by hucksters at 15 cents per bushel. These, it seems to me, are far preferable to shanties at the same price, and Mr. Armour might well follow Mr. Fowler's example.

On James street board ranges from $3.00 to $3.50 per week. On this street the best stores in the vicinity are found—in fact, this is *the* thoroughfare of this little community. In the Bottoms are four schools, viz., Kansas City public, colored and Catholic schools. The Unitarians maintain a night school on North James street with a loan library in connection and sixty pupils in attendance.

Anyone desiring to know more concerning real life in the Bottoms may read a few incidents in a little pamphlet published by the Bethel Evangelization society, of Kansas City, under whose care Bethel church has grown to its present usefulness.

The slums in the winter time are comparatively clean and free from odors for the simple reason that filth and decaying matter are frozen, but in the summer time the odor from the filth, in connection with that from the packing house, tends to make this locality one frightful stench.

We shall now briefly state what seems to us most needed by the people of the West Bottoms. In the greater number of cases the many children whose homes are here have no playgrounds save the railroad tracks and public streets. There are vacant lots here which are so situated that at small cost they could be utilized as public play grounds, and managed in the same way as the Hull House playgrounds in Chicago. There exist in the Bottoms no halls for concerts or entertainments of any better class than that afforded by the dime museum or the phrenological fake. I am sure these people would appreciate some good music—some of the old songs that perhaps they used to know in the days gone by, and the little children would have a bright spot in their own lives because of this, where they too might hear the songs. Of course Wagner and the highest classical music would not be appreciated, but there is a great amount of music which would be.

There are no brothels in the Bottoms—there is no need of any—morality is too low. In some sections of the Bottoms night travel is unsafe. One saloon in particular is notorious from the fact that it is the headquarters of a gang of thugs who lie in wait for the working men Saturday nights and relieve them of their hard-earned money.

One thing that might be of great benefit would be a lodging house for men, in which they might have wholesome food and a place to wash themselves. A dispensary with one or two physicians in charge is another needed improvement. Mr. Fowler has made a financial success of his ten tenements, and it is probable that other tenements could be speedily filled should they be erected. In fact, we may sum up the needs of the Bottoms in the one statement that more of institutional church work or applied Christianity is needed.

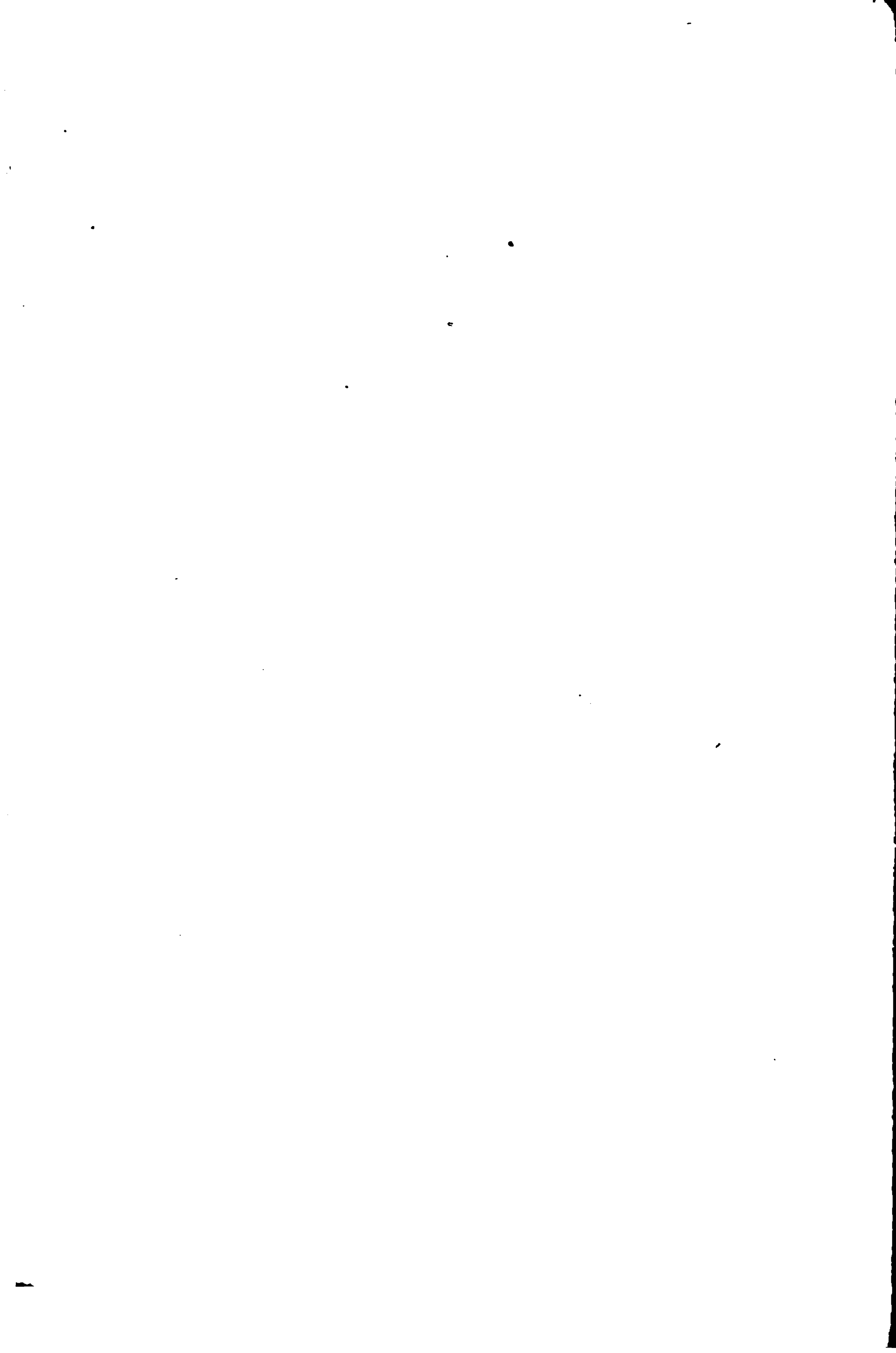
There are men in Bethel church who are ashamed to come to Sunday School because they cannot read the Sunday School lesson, and if there could be started in the Bottoms a college settlement with its full equipment of classes, clubs and other lines of work,

although it would be hard work for some years, it would eventually mean a great improvement in the condition of the people, so many of whom are forced to live here.

Kansas City compared with New York—especially this district—is not so congested. There are no tenement houses here, only one or two story houses or shanties. But there exists in Kansas City more coarse crime than in all that part of our country east of the Hudson river line.

The work in the packing houses is comparatively free from accident, although some jobs bring a man in contact with machinery and increase liability to accident. Generally speaking the work is healthy, although men who have to work in the steam, such as hog-scrapers and others, find the work hard on their lungs, being compelled to stoop over and come very closely in contact with the steam.

For the young life here in the Bottoms the future is anything but hopeful. Brought up in an atmosphere of poverty, of squalor, of sin, in a district where dirt leads to disease, disease to death, and where the devil himself seems to reign, what can be in store for them? We have it on very good authority that in one section of these Bottoms, by careful estimate, ninety per cent. of the girls between the ages of twelve and fourteen are unchaste.

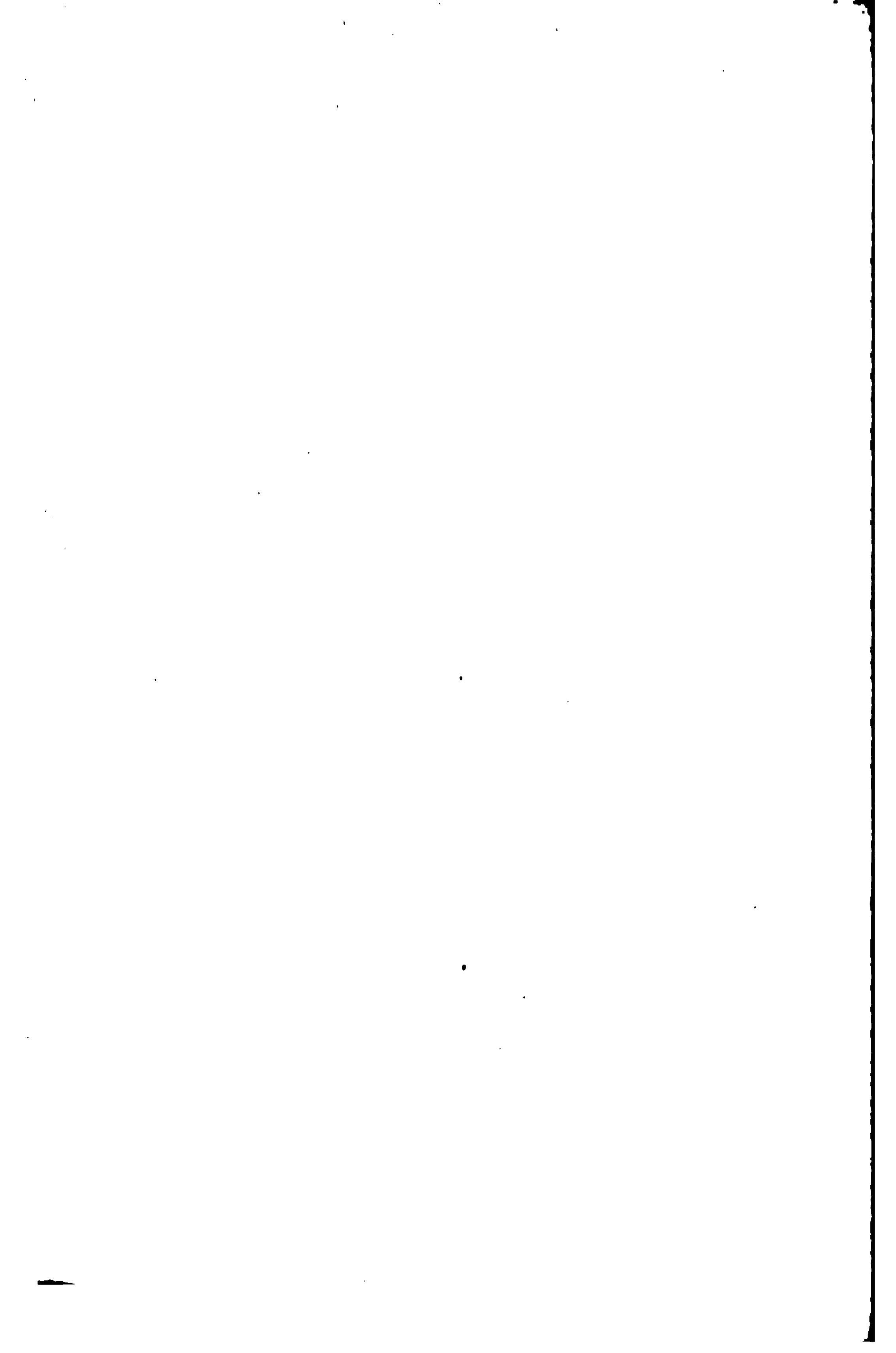


Note on the Mandible of Ornithostoma.

BY S. W. WILLISTON.

(With Plate I).

Almost invariably the skulls of specimens of *Ornithostoma* from the Kansas Cretaceous have been found crushed together laterally, obliterating the palatal surface of the maxillæ and the buccal surface of the mandible. For this reason, the figures hitherto given of the side view of the mandible have been too broad. A specimen recently discovered by Mr. H. T. Martin and now in the Universiy Museum, is fortunately flattened downward and not greatly distorted. In the accompanying plate a partially restored figure is given, showing the upper view of the specimen. The only points in which the figure may be erroneous are the obliquity of the posterior part of the rami and their convexity of outline. From comparison with other specimens in the museum, it is evident that only a little more than half of the jaws are preserved, the symphysis being fully twice the length of the rami. The specimen is of interest because of its thin, gently concave or flat, horizontal floor of the mouth, which ends in a free, concave and slightly thickened border about one inch in front of the posterior extremity of the symphysis, enclosing below it a long, free cavity, opening backward. At each lateral margin there is a free, thin, sharp parapet of bone, about six millimeters in height, standing vertically and having a perfectly smooth border. The floor was probably gently concave, though now flat, In Fig. *b* is given a cross-section of the jaws at the place marked *a*, showing the shape of the cavity and the crushing the bone has received. At *c* is given a restoration of the section at *a*, showing the outline of jaws very nearly as they must have been during life. It is apparent that the symphysis was rounded and not sharp below. How far the cavity extends forward, I cannot say, as the walls are in close contact and pressed flat, towards the distal end of the specimen. The palate seems to have been flat like the floor of the mouth, and it is very evident that if the tongue extended far forward it must have been extremely thin.



Notes on Air Resistance and Pressure.

BY E. C. MURPHY.

(With Plate II).

The following experiments were made by the writer in the machine shop of this University for the purpose of determining the relative resistance and pressure of air on surfaces of different degrees of curvature. The object in view was to determine the advantage gained in the use of curved instead of plane fans or slats in windmills, and more especially that for the jumbo mill which is now being used to some extent in Western Kansas. Many experiments have been made to determine the relation between pressure and velocity in plane surfaces. The results do not agree, though the results of recent experiments agree more nearly than those of earlier date. Very few experiments have been made to determine the relation between velocity and pressure for curved surfaces, and the results differ widely. These differences are probably due largely to the method of measuring the velocity of the wind. The velocity as found by the cup anemometer may be in error 20 per cent. or over for velocities of 30 miles or more per hour; and small pressure plate anemometers may be in error 40 per cent. or more. (See *Eng. News*, Vol. 33, p. 183.)

Space will not permit us to speak even briefly of the results of other experiments. Captain W. H. Bixby, in his report on "Maximum Spans Practicable for Suspension Bridges," reprinted in *Eng. News*, Vol. 23, gives a brief review of the history of past investigations in this subject. The work of 65 experimenters and writers is reviewed. A series of experiments not mentioned in Captain Bixby's report, made by J. Irminger, C. E., are worthy of special mention, in that they measure not only the pressure on the windward side of surface, but also the section on the leeward side.

Figures 1 and 2 show the apparatus for judging the resistance. Fig. 1 shows a part of a lathe, the belt A passes over the pulleys A, B, C and D, thus giving to the pully K, which drives the fans four velocities. The belt K passes over the pulley L in Fig. 2. The power is transmitted from the face plate E to the pulley K through the spring balance I, thus the number of pounds pull required to run the fans at any speed is seen.

Three forms of fans were used: one plane, x Fig. 2, and two curved, y and z, Fig. 2.

The pull P was found for each speed for four fans; then one fan was taken off and P found for the two highest speeds; then a second fan was taken off and P found for the four speeds; a third fan was then taken off and P found for the two highest speeds; finally all the fans were taken off and P found for friction.

After thus finding the pull required to drive the fans with concave surface forward, that required to run them with the convex surface forward was found in the same way.

Column one, table I, gives the speed in revolutions per minute of the fans; column two the velocities of center of fans in feet per second. In the other columns are given the net pull in pounds, after slight adjustments. The subscripts x, y, z refer to the curvature as shown in Fig. 2; the subscript f measures fan moving with concave surface forward and b convex surface forward.

TABLE I.

SPEED.		4 FANS.					2 FANS.					3 FANS.					1 FAN.				
Revolutions per min.	Velocity ft. per sec.	P_x	P_{yf}	P_{yb}	P_{zf}	P_{zb}	P_x	P_{yf}	P_{yb}	P_{zf}	P_{zb}	P_x	P_{yf}	P_{yb}	P_{zf}	P_{zb}	P_x	P_{yf}	P_{yb}	P_{zf}	P_{zb}
109	13.56	32	26	25	24	21	19	16	14 5	15	13	26	21	19	18	16	11	8	8	8	8
72	8.64	12	10	9	9	8	7	7	7	7	7	9.5	9	9	9	8	5	5	5	5	5
46	5.52	5	4	4	4	4	5	3	3	3	3										
29	3.84	2	2	2	2	2	2	2	2	2	2										

Fig. 3 shows diagrammatically the relation between the velocity and P_x for four fans. The points 1, 2, 3 and 4 are very nearly on a parabola whose equation is $V^2=6.1P_x$.

The following facts are easily deduced from the table:

(1) The resistance is very small for low speeds and nearly independent of the curvature of fans.

(2) The resistance to plane surfaces is greater than that to curved ones, the difference increasing with the speed.

(3) The difference decreases as the radius of curvature decreases.

(4) The resistance to a convex surface is somewhat less than that to a concave one.

(5) The resistance to the y form of fans, for a fan center velocity of 13.5 feet per second, is 22 per cent. less than that to x form, and the resistance to the z form 34 per cent less.

(6) The relation between P_x and the number of fans for a given

velocity is nearly of the form $P_x = a + bn$, in which a and b are constants and n the number of fans.

Fig. 4 shows in principle the apparatus used for finding the relative pressure on surfaces of different degree of curvature.

T is a 2⅞ inch tube which brings compressed air from a centrifugal blower to the fan, O a U-shaped glass tube for measuring the pressure, T a counterpoise for balancing the weight of the fan, and G a weight for balancing the air pressure.

Two sizes of fans were used, namely, 12 x 24 inches, those used in the experiments described above, and a 3 x 4 inch.

The blast had a temperature of 78° F. and a velocity, as measured by an air meter, of 60 feet per second. It was directed normally against the fan, the end of tube being 2½ inches from the fan.

The following results were gotten:

(1) The pressure on the concave side of y was the same as on the concave side of z.

(2) The pressure on the convex side of y was the same as on the convex side of z.

For the large fans:

(3) The pressure on the concave surface of y was 14 per cent. greater than that on x.

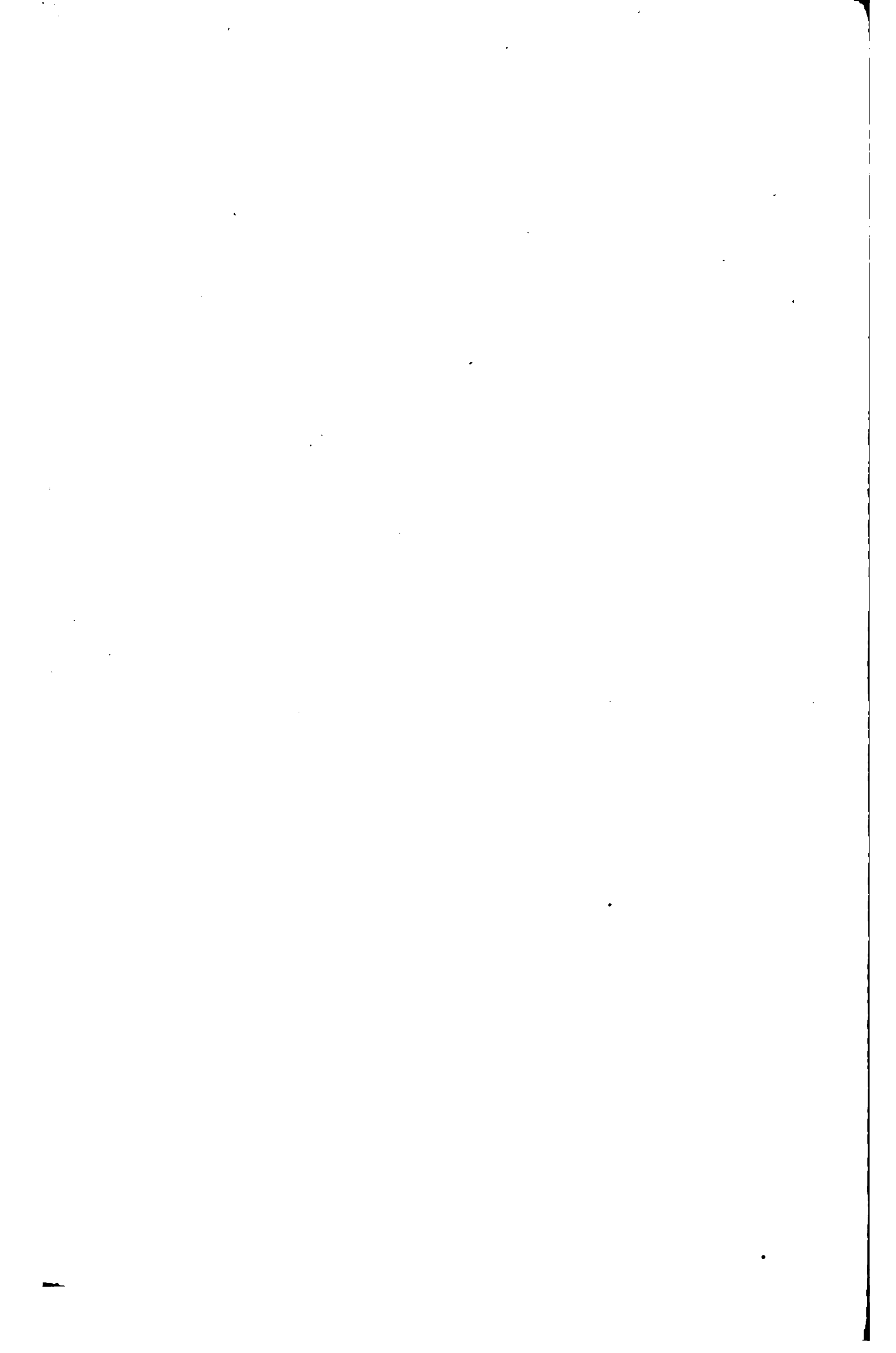
(4) The pressure on the convex surface of y was 28 per cent. less than on x.

For the small fans:

(5) The pressure on the concave surface of y was 17 per cent. greater than that on x.

(6) The pressure on the convex surface of y was 25 per cent. less than that on x.

The reduction of pressure or suction on the back side of the fans was measured by the water column. It was found to be too small to measure for the plane and concave fans, and $\frac{1}{16}$ inch or 8 per cent. of the pressure on the windward side for the convex fan.



The Coffeyville Explosion.

BY ERASMUS HAWORTH.

(With Plate I, Fig. 2.)

In the early morning of July 26, 1894, a terrific explosion occurred in the northern suburbs of Coffeyville, Montgomery county, Kansas, which was of sufficient force to throw from two hundred to three hundred tons of rock and earth to the surface. The noise produced was not very great in comparison with the force manifested as is shown from the fact that perhaps less than half the citizens of the town were awakened. The usual sequence of sensational press dispatches with startling head lines followed, as might have been expected, and the University was flooded with inquiries, personal and by letter, asking for an explanation of such an unusual occurrence. Nearly all of the various theories advanced by those making inquiry connected the explosion in one way or another with natural gas, which was known to exist in that vicinity in great quantities under a pressure of more than three hundred pounds to the square inch. The gas operators and the well drillers, on the contrary, throughout southeastern Kansas, stoutly affirmed that no such incident ever had occurred in the gas fields of the east, and that it was a physical impossibility for gas to explode so far beneath the surface.

Upon a careful examination of the grounds some months after the explosion the following conditions were observed: To the southwest of the gas well a short distance many fissures were seen in the earth's surface trending southwest and northeast which approximated parallelism. They were from six inches to eighteen inches in width and at the time observed the walls had caved to such an extent that the widest appeared to be less than three feet deep, although immediately after the explosion rumor has it that some of them were known to be from six to ten feet deep. Some of them bifurcated, while others did not as far as could be seen. Fig. 2, Plate I, drawn one hundred fifty feet to the inch, shows the location of the gas well, G. W., the fissures, the piles of earth thrown up the break in the gas main at b, and the fissure through the cistern C., near the house H. The larger pile of earth is located principally in a shallow ravine which has been

eroded to about six feet below the general level, although the two ends pass up onto the bank. In this pile of debris one mass of arenaceous shale measures sixteen feet long, five feet wide and three feet thick for the greatest diameters, and will weigh about fifteen tons, while more than half a dozen other pieces will average nearly half as large. In some parts of the pile the debris is fully six feet deep. In the smaller piles to the southwest the fragments were not so large, but many of them showed plainly that they were brought up from the regularly stratified shale which is reached at a short distance below the surface. Smaller fragments of the debris were thrown to the northeast for at least two hundred yards. One piece was driven through the wall of the upper story of the house, H., and lesser fragments aggregating tons in weight were strewn over the surface in that vicinity. A cistern, C., just north of the house was split from top to bottom by a fissure, and a gas main was pulled in two at the point b, opposite the cistern.

The gas well had been drilled months before and according to the record kindly furnished by Mr. Brown, secretary of the company, passed through the following strata:

RECORD OF GAS WELL AT COFFEYVILLE, NEAR POINT OF EXPLOSION.

THICKNESS OF STRATA.		TOTAL DEPTH.
12 feet	Soil, clay and gravel	12 feet.
40	Shale	52
15	Blue limestone	67
2	Shale	69
10	Grey sandstone	79
8	Shale	87
4	Limestone with salt water	91
20	Black shale	111
8	Shale	119
5	Limestone	124
35	Sandy shale	159
5	Limestone	164
2	Shale	166
25	Limestone	191
5	Black shale	196
17	Brown and grey sandstone	213
75	Shale	288
37	Limestone	325
2	Black shale	327
2	Limestone	329
2	Black shale	331
32	Limestone	363
4	Shale, salt water and gas	367
16	Limestone	383
15	Black shale	398

THICKNESS OF STRATA.		TOTAL DEPTH.
28 feet	....Blue sandstone	426 feet.
6	....Black shale	432
19	....Light shale	451
10	....Blue sandy shale	461
20	....Shale	481
10	....Brown shale	491
32	....Shale	523
15	....Grey sandstone, gas-bearing	538
42	....Shale	580
20	....Light colored sandstone, gas-bearing	600

It is quite evident that an ordinary explosion due to the oxidation of the gas could not have occurred on account of the impossibility of getting the requisite amount of oxygen mixed with the gas while it was below the surface. Neither is it probable that the phenomenon was seismic in character, for it is confined to too narrow limits. On the other hand at least one similar occurrence is known in the gas field of Indiana* near Kokomo in which a fissure was formed in the solid limestone from which natural gas escaped with explosive violence and caught fire from a burning log heap near by.

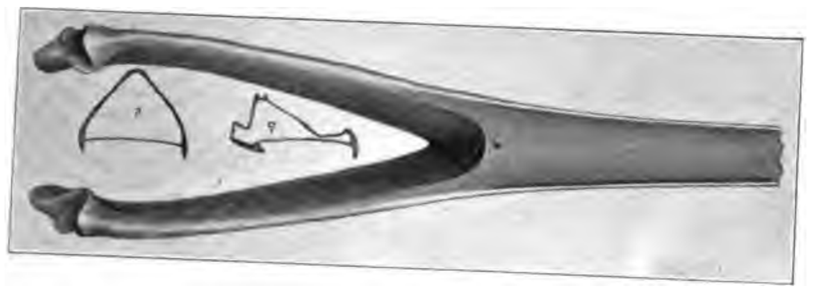
A short consideration of the possibilities of the case may not be wholly without interest. We will assume that the gas pressure was 300 pounds to the square inch. We know that in some of the Coffeyville wells it was more. The specific gravity of the materials near the surface is about 2.75. Now let us suppose that in some way a leak from the depth below occurred, possibly by the drilling of the well, but quite as likely from natural causes. Suppose this gas was conducted upwards to near the surface where it was permitted to spread laterally in a thin seam or opening between the layers of shale. It should be remembered that a layer of gas one-fourth or one-half inch thick would exert a pressure as great as though it were much thicker. The leak may have been very slow so that months, or even years, were occupied in the accumulation. If the gas sheet covered an area 300 feet square the upward pressure would be equal to nearly four billion pounds, or nearly two million tons. The character of the eruption indicates that the material came from near the surface, probably from less than twenty feet. If we consider the gas pressure and the specific gravity as above mentioned we would have the pressure per square foot equal to 43,200 pounds, and the weight of a cubic foot of earth equal to 172 pounds. It would therefore require a depth of a little over 250 feet

*The writer is indebted to Mr. Wm. Moore, of Kokomo, for his information on this subject. It is understood that an account of the explosion has been published by Prof. Jordan, State Gas Inspector, but exact reference is not available just now.

for the weight to equal the upward gas pressure. If the gas reservoir was only 20 feet below the surface the pressure of 300 pounds would give an excess pressure of 43,200—(172 × 20)=39,760 pounds for each square foot of surface, or 3,578,400,000 pounds or 1,789-200 tons for the area above considered. With the weight of the strata overcome by a portion of the gas pressure, the excess pressure would tend to bend the strata upwards. With the points of resistance 300 feet apart it is hardly probable that the enormous pressure of more than three and a half billion pounds could be withstood. From the conditions observed on the ground it is probable that the sheet of gas was more than 300 feet across, rather than less, for the total length of the fissures is more than that distance. This would increase the probability of an explosion.

It is therefore concluded that the most probable explanation of the Coffeyville explosion is the gradual accumulation of gas in the form of a broad, thin sheet between the rock strata near the surface where it was held until the upward pressure became so great that it was sufficient to overcome the weight of the super-imposed material and the rigidity of the strata, at which time the explosion occurred.

Fig. 1.



Mandible of Ornithostoma.

Fig. 2.

300′

E.H.

250′

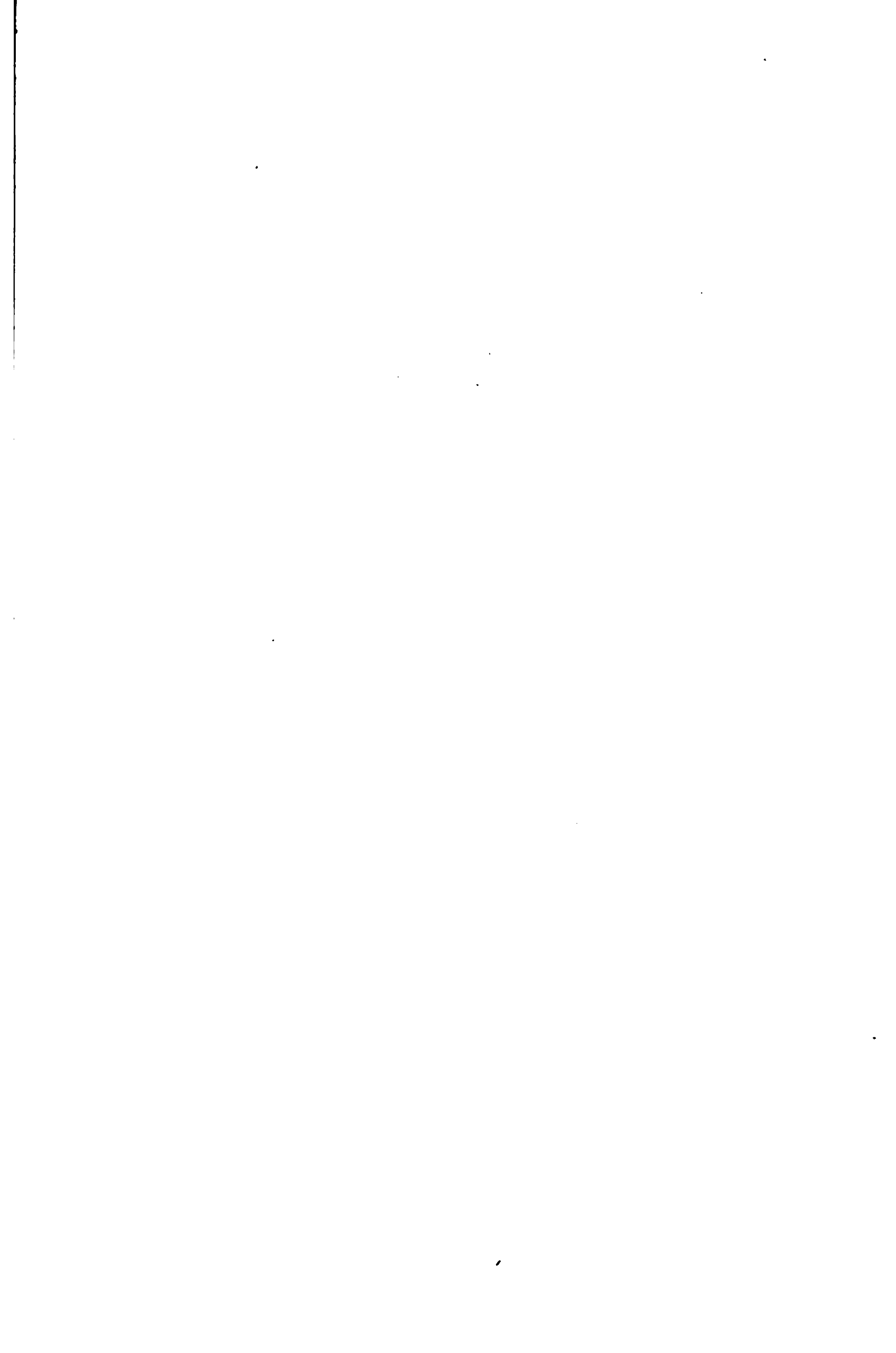
G.W.

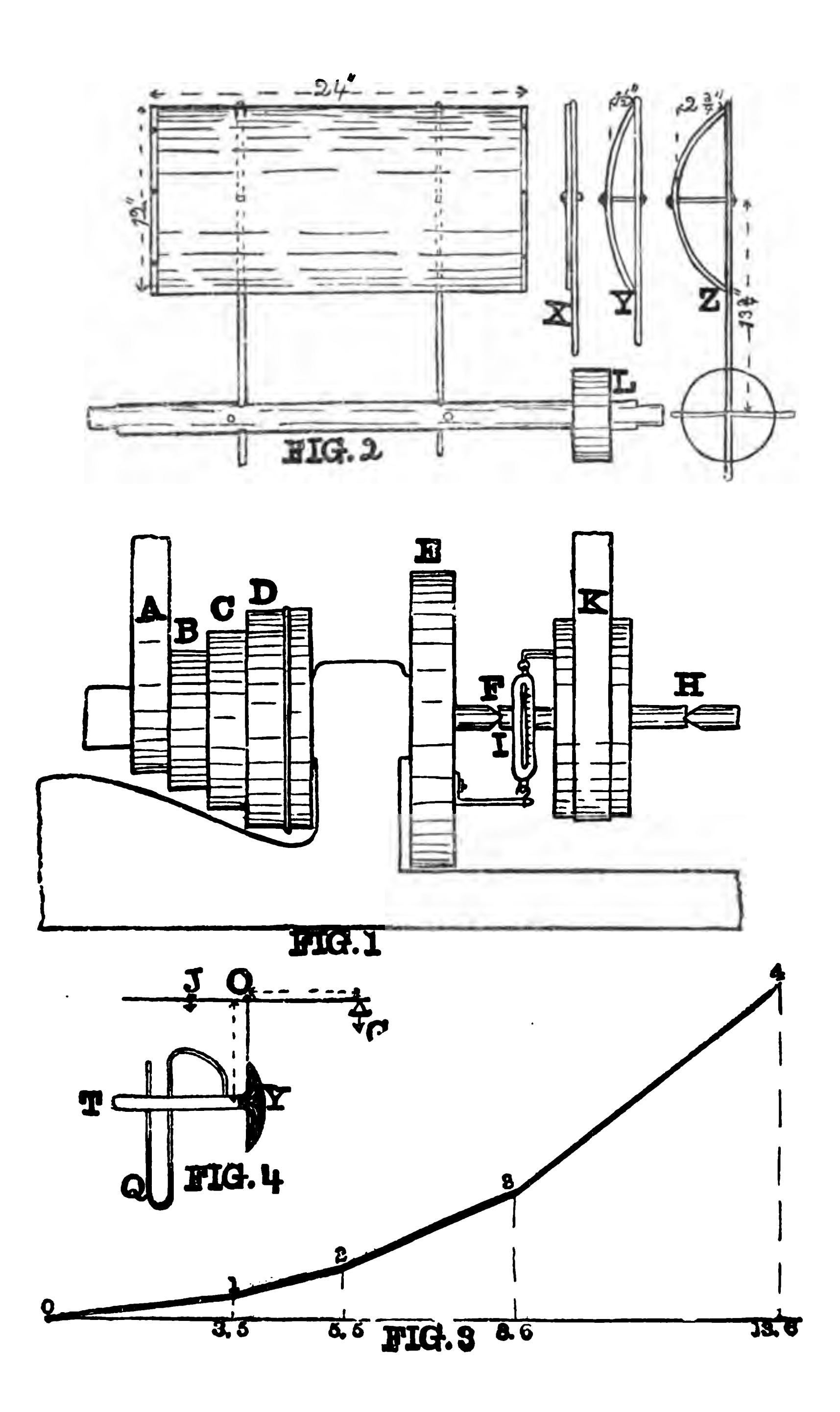
Gas Main

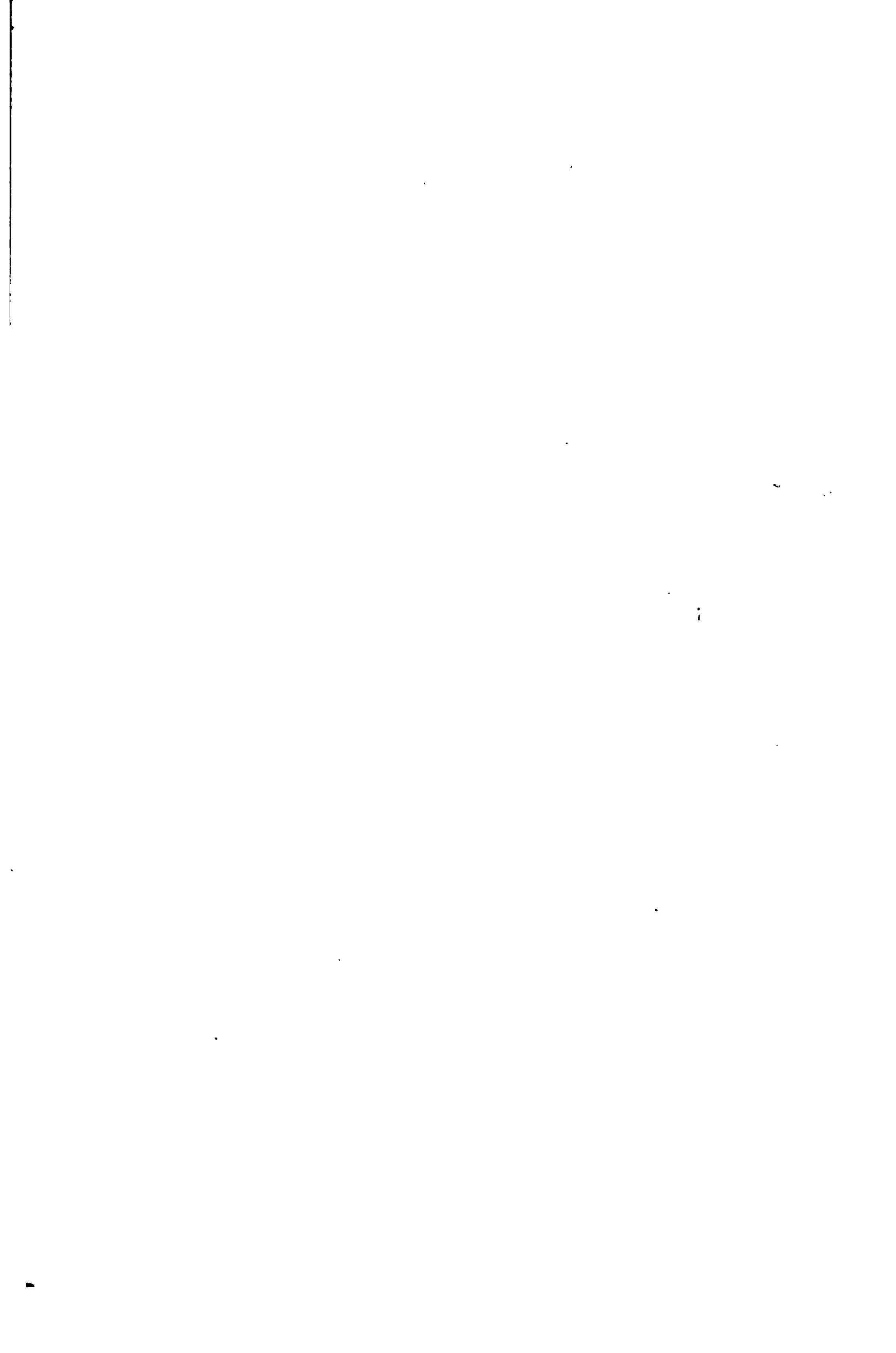
Street

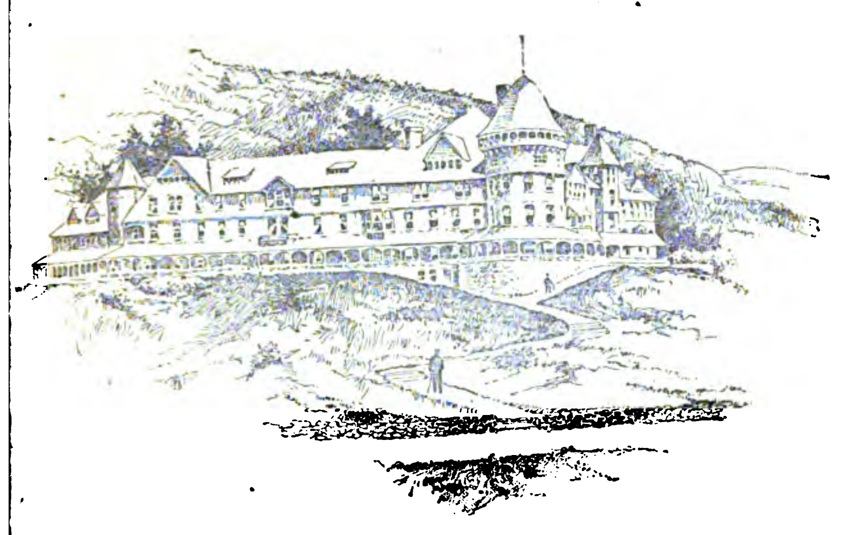
H.

C




24"
12"
X
Y
Z
13¾"
L
FIG. 2
A
B
C
D
E
K
F
I
H
FIG. 1
J
O
G
T
Y
Q
FIG. 4
0
1
2
3
4
3.5
5.5
8.6
13.6
FIG. 3





ANNOUNCEMENT.

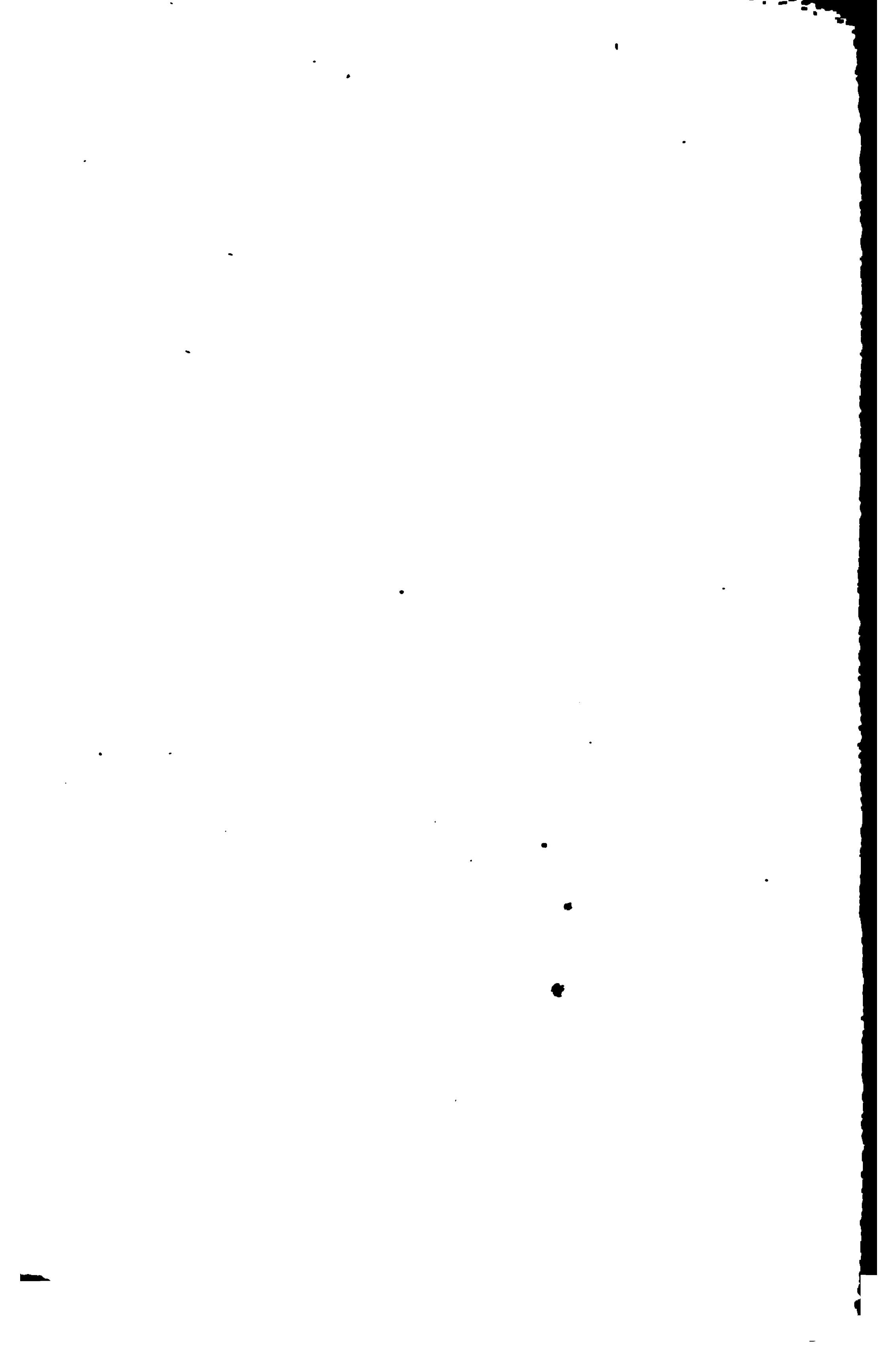
. . . **THE MONTEZUMA HOTEL,** at Las Vegas Hot Springs, New Mexico, has been **reopened** under the management of Mr. Jno. O. Plank. This will be a welcome announcement to a very large number of health and pleasure seekers who have known the Montezuma as one of the most perfectly appointed hotels in the Southwest, and have proven by experience the extraordinary virtues of the mineral waters of the Hot Springs. The Montezuma **has been refitted,** and various improvements have been made in the grounds, the bath house, etc. The capacity of the main hotel will be supplemented by the Mountain House (closely adjacent), by comfortable rooms in the second story of the bath house, and by attractive cottages, the arrangements being such that **several hundred patrons** can be amply accommodated at the same time.

. . . **The rates** at the Montezuma will be $3.00 per day and upward, and $17.50 per week and upward; at the Mountain House, and bath house and cottage annexes, $2.00 per day, and from $12.00 to $14.00 per week.

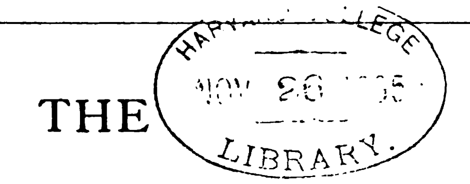
. . . **Round-trip tickets** to Las Vegas Hot Springs are on sale from the principal points. The through trains of the Santa Fe route pass Las Vegas daily, with sleeping car service from Chicago, St. Louis, Kansas City, Denver, and intermediate points.

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CONTENTS

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Continuous Groups of Projective Transformations Treated Synthetically.

BY H. B. NEWSON.

Prof. Sophus Lie, in his *Vorlesungen ueber Continuierliche Gruppen*, Abteilungen 1 and 2, develops in detail an analytical theory of continuous groups of projective transformations in one and two dimensions. In the domain of Projective Geometry, analytical and synthetic methods are of approximately equal power, and are always mutually supplementary. The object of this paper is to develop a synthetic theory of these groups, based on geometric construction. In the course of this development all of Lie's chief results are reached, and many new relations are seen when the subject is approached from this new point of view.

In this paper only groups of projective transformations in one dimensional space will be treated. The development of the theory for the projective groups of the plane will appear later.

§1. Construction of Projective Transformations.

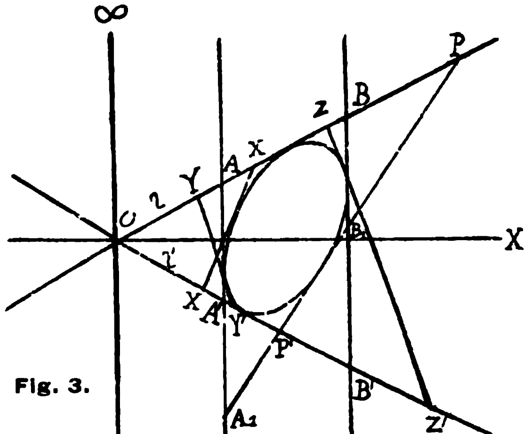


Fig. 3.

Let l and l' be two lines intersecting in O and making any convenient angle with one another; and let OX be the bisector of the angle lOl'. A projection of the line l on l' is determined by choosing three points on l and their three corresponding points on l.' Thus let x, y, z be three points on l and x,' y,' z' the three corresponding points on l.' (Fig. 1).. Draw

xx,′ yy,′ zz.′ These three lines together with l and l′ determine a conic K touching the five lines. Any other tangent to K as PP′ cuts the lines l and l′ in corresponding points of the projection.

Thus by means of a conic K *touching the two lines* l *and* l′ *a projection of the line* l *on* l′ *is completely determined.**

If now l′ be revolved about O until it coincides with l, any point P′ on l′ will be brought to P_1, so that $OP' = OP_1$. The two ranges of points are then considered as existing on the same line l. This operation of projecting a range of points on l into a new range on l′ and then by revolution about O bringing the new range back to l will be called a *Projective Transformation.* The effect of a projective transformation is therefore to shift the points of a line into new positions, so that there shall be a projective relation, i. e. a one to one correspondence, between the old and new positions of the points. A projective transformation determined by a conic K will be designated by the symbol T_K.

We observe in the first place that a projective transformation T_K usually leaves two points of the line l unaltered in position. For generally two tangents can be drawn to the conic K perpendicular to the bisector OX; these cut l and l′ in A,A′ and B,B′ respectively. A and A,′ B and B′ are therefore corresponding points, and the revolution about O brings A′ to A and B′ to B. The The points A and B′are called the double points or the *invariant points* of the transformation.

We said that generallv there are two tangents to K perpendicular to OX. This should be examined more closely. When the conic K is an ellipse, two real tangents to K can always be drawn perpendicular to OX; and hence the projective transformation determined by an ellipse always has two real invariant points. When the conic K is a parabola, there are still two real tangents perpendicular to OX; but one of them is the line at infinity; hence the projective transformation determined by a parabola always has two real invariant points, one of which is the point at infinity on l. When the conic K is a hyperbola, there are three cases to be considered. If the asymtotes to the hyperbola K make with the line OX angles which (measured in the same direction) are both less than, or both greater than, a right angle, then two real tangents to the hyperbola can be drawn perpendicular to OX, and the transformation determined by K has two real invariant points. If on the other hand

* This principle and the theorems involved in it are fundamental in all our work on the projective transformations of the straight line. The principle itself is too well known to need proof or explanation. See:
Cremona's Projective Geometry, Chap. XIV, Art. 150 (English Edition).
Reye's Geometrie der Lage, 8 Band, p. 60 and 66 (2d Edition).
Salmon's Conic Sections, Art. 329 ff. (6th Edition).

the asymptotes to K make with OX angles one less than, and the other greater than, a right angle; then the tangents to K perpendicular to OX are imaginary, and the transformation determined by K has its invariant points imaginary. But if K has one of its asymptotes perpendicular to OX, the transformation determined by K has one real invariant point. Or since the asymptote to a hyperbola is the limiting position of two parallel tangents, we may say in the last case that the transformation determined by K has two coincident invariant points.

It may happen that the three lines xx,′ yy,′ zz′ meet in a point; let Q be such a point (Fig. 2). The conic which touches these three lines and both l and l′ degenerates into the segment QO of the line QO. Any line of the pencil, whose vertex is Q, is a tangent to this conic and cuts l and l′ in corresponding points of the projection. The two projective ranges on l and l′ are now in perspective position. When l′ is brought as before into coincidence with l, one of the invariant points is O and the other is determined by the perpendicular through Q on OX.

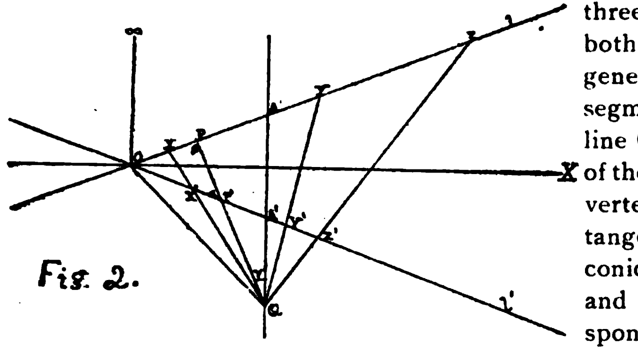


Fig. 2.

Theorem 1.—Every projective transformation of the points on a line leaves two of its points invariant; these two points may be real and distinct, or real and coincident, or imaginary.

The lines AA,′ BB,′ l, l′ are four fixed tangents to to the conic K. Any fifth tangent as PP′ cuts these four fixed tangents in four points whose anharmonic ratio is constant. The range A_1, B_1, P, P′ may be projected orthogonally on l by lines drawn parallel to AA′; and the anharmonic ratio $(A_1B_1PP') = (ABPP_1)$. But since the first anharmonic ratio is constant for all tangents to K, it follows that the second is constant for all pairs of corresponding points; hence for every projective transformation which has two real invariant points, we have the theorem that any pair of corresponding points and the two invariant points have a constant anharmonic ratio.

In the case where the two tangents to K perpendicular to OX are imaginary, it still holds that the anharmonic ratio of the four

points of intersection of any tangent to K with the lines l and l' and the two imaginary tangents AA' BB' is constant. Hence we have the following theorem which holds for two separate invariant points, whether real or imaginary:

Theorem 2.—In every projective transformation which leaves two distinct points invariant, the anharmonic ratio of any pair of corresponding points and the two invariant points is constant.

This is called the *characteristic anharmonic* ratio of the transformation T_k. It is convenient to designate this constant by the same letter k. A transformation T_k is then one determined by a conic K whose constant anharmonic ratio determined by the tangents AA,' BB,' l, l' is k; and the resulting characteristic anharmonic ratio is also k.

But when the invariant points of the transformation coincide, we no longer have a characteristic anharmonic ratio for the transformation. However another relation is found to hold for pairs of corresponding points, which relation is constant for all pairs of corresponding points in the transformation. We shall now proceed to determine this relation.

Let K be a hyperbola having one of its asymptotes as AA' perpendicular to OX, (Fig. 3). Let O'PP' be any tangent cutting l and l' in P and P,' corresponding points of the projection. Draw EP_∞ parallel to l' and touching K. Taking AA,' EP_∞, l, and l' as four fixed tangents, the anharmonic ratio determined by any fifth tangent as PP' is constant. Denoting this by k' we have

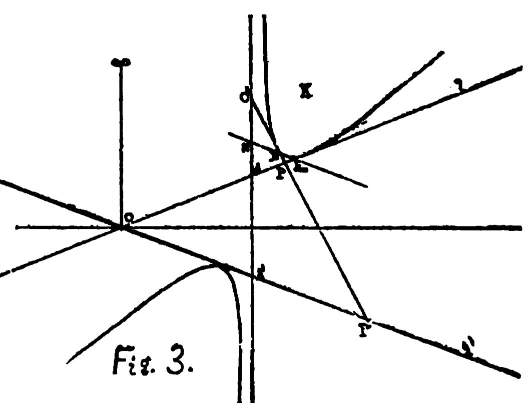


Fig. 3.

$$k' = (O'BPP') = \frac{O'P}{PB} : \frac{O'P'}{P'B} = \frac{O'P}{O'P'} \cdot \frac{BP'}{BP}.$$

But from the figure we have $\frac{O'P}{O'P'} = \frac{AP}{A'P'}$; also $\frac{BP'}{BP} = \frac{OP_\infty}{PP_\infty}$.

hence $k' = \frac{AP}{A'P'} \cdot \frac{OP_\infty}{PP_\infty}$. If the tangent PP' be moved up infinitesimally near to AA,' we see that the constant anharmonic ratio along the tangent AA' is given by $k' = (\infty EAA') = \frac{A'E}{AE}$. From

the figure we see that $\frac{A'E}{AE} = \frac{OP_\infty}{AP_\infty}$: setting these two values of k' equal to each other, we have $\frac{AP}{A'P'} \cdot \frac{OP_\infty}{PP_\infty} = \frac{OP_\infty}{AP_\infty}$. Replacing $A'P'$ by AP_1 and PP_∞ by $AP_\infty - AP$, this reduces to $\frac{1}{AP} - \frac{1}{AP_1} = \frac{1}{AP_\infty}$. This gives a constant relation between P and P_1, a pair of corresponding points. We can now state the following theorem:

Theorem 3.—A projective transformation which leaves invariant two coincident points at A *transforms any point* P *into* P_1 *such that the difference of the reciprocals of the segments* AP *and* AP_1 *is constant; this constant is equal to the reciprocal of the segment* AP_∞, *where* P_∞ *is the point which corresponds to the point at infinity on* l.

A transformation of this kind which leaves two coincident points invariant will be designated by T'_k. It is convenient to denote the above constant by the same letter k which also denotes the conic determining the transformation. Thus T'_k is a transformation determined by a conic K; it leaves one and only one point A invariant, the *characteristic constant* of the transformation, i. e., the reciprocal of the segment AP_∞ is also k.

Every conic touching the lines l and l' determines a projective transformation. It is therefore possible to construct as many different transformations of the points on the line l as there are conics touching l and l.' We know that ∞^3 conics can be drawn touching any two lines; hence we infer that there are ∞^3 projective transformations of the points on a line. Among the ∞^3 conics touching l and l' are ∞^2 hyperbolas having one asymptote perpendicular to the line OX. Hence we infer that there are ∞^2 projective transformations of the kind T' which leaves two coincident points invariant.

Connected with every transformation of the kind T are three quantities, the coördinates of the double points A and B (some point on l being taken as an origin), and the characteristic anharmonic ratio k. These may be looked upon as independent variable parameters, a consideration which again leads to the conclusion that there are ∞^3 different projective transformations T of the points on a line. Connected with every transformation of the kind T' there are two quantities, the coördinate of the invariant point A, and the characteristic constant k. These may be looked upon as variable parameters of the transformation. *

*In the analytic development of this topic Lie's equation of transformation gives three independent parameters. Lie clearly identifies two of his parameters with the

Theorem 4.—There are ∞^3 *projective transformations* T *of the points on a line; each of these leaves a pair of points invariant. Among these are* ∞^2 *projective transformations* T′, *each of which leaves one and only one point invariant.*

§2. One-termed Groups of Projective Transformations.

Our next object is to subdivide and to classify these ∞^3 projective transformations of the points on the line l. We consider first the quadrilateral ABB′A′ (Fig. 1). A system of ∞^1 conics may be described touching the sides of this quadrilateral. Call this system S. Each of these conics determines a projective transformation which has A and B for its invariant points. Each conic of the system S touches the line l at a different point; and every point of the line l is the point of contact of some conic of the system S. If C be the point of contact of any conic K, the characteristic anharmonic ratio of the transformation produced by K is given by the anharmonic ratio of the four points A, B, C, O. The points A, B, O are fixed, while the point C is movable. From the continuity of the point system on the line l we infer the continuity of the system of ∞^1 transformations which leave A and B invariant.

Let T_{k_1} be the transformation of this system which transforms P to P_1; then $k_1=(ABPP_1)$. Let T_{k_2} be the transformation of the same system which transforms P_1 to P_2; then $k_2=(ABP_1P_2)$. The two transformations T_{k_1} and T_{k_2} are together equivalent to a single transformation of the same system which transforms P to P_2. To prove this we have $k_1=(ABPP_1)=\frac{AP}{PB}:\frac{AP_1}{P_1B}$, and $k_2=(ABP_1P_2)$ $=\frac{AP_1}{P_1B}:\frac{AP_2}{P_2B}$. Eliminating the fraction containing P_1 from these two equations we have $k_1k_2=\frac{AP}{PB}:\frac{AP_2}{P_2B}=(ABPP_2)$. The conic of the system S whose tangential anharmonic ratio is k_1k_2 gives a transformation which is equivalent to the combined effect of T_{k_1} and T_{k_2}. This may be expressed symbolically by the equatiod $T_{k_1}T_{k_2}=T_{k_1k_2}$. In the same way it may be shown that the combined effect of any number of transformations of the system is equivalent to some single one of the same system. Thus $T_aT_bT_c$

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invariant points of the transformation, but does not explicitly identify his third parameter with the characteristic anharmonic ratio. This last, however, is implied in equation (4), p. 121 of "*Cont. Gruppen*," viz.: $\frac{x_1-m}{x_1-n}=k\frac{x-m}{x-n}$. It cannot be to strongly emphasized that the three parameters of Lie's equation of transformation mean geometrically the coordinates of the invariant points and the characteristic anharmonic ratio. In regard to transformations of the type T′ our result is in complete accord with Lie's equation (7) p. 123, "*Cont. Gruppen*," viz.: $\frac{1}{x_1-m}=\frac{1}{x-m}+k$.

$..T_n = T_s$; where $s = abc..n$. The characteristic anharmonic ratio of the resultant transformation T_s is equal to the continued product of the characteristic anharmonic ratios of the component transformations $T_a T_b T_c .. T_n$. The transformations of the system which leave A and B invariant have therefore the property that the combined effect of any two or more of them is equivalent to some single one of the same system. In modern mathematical language this continuous system ∞^1 transformations forms a Continuous Group of Transformations.

This is a group of one parameter, the variable parameter of the group being the characteristic anharmonic ratio. I shall denote this group of transformations by the symbol G_1. The subscript 1 indicates that there are ∞^1 distinct tradsformations in the group. Sometimes the group will be designated by G_{AB} when it is desired to call attention to the invariant points of the group.

Theorem 5.—The totality of the projective transformations which leave two distinct points of a line invariant forms a one-termed group G_1, *whose fundamental property is that the combined effect produced by two or more of the transformations of the group is equivalent to that of a single transformation of the same group. The characteristic anharmonic ratio of the resultant transformation is equal to the continued product of the characteristic anharmonic ratios of the component transformations.*

Other properties of the group G_1 will now be developed.

If we have given any transformation T_K of the group G_1, we can always find another of the same group that will exactly neutralize the effect of the first. For if T_k transforms P to P′ then $k = (ABPP')$. If $T_{k'}$ be the transformation which transforms P′ back to P, then $k' = (ABP'P)$. But we know that $(ABPP') = \frac{1}{(ABP'P)}$; hence $k = \frac{1}{k'}$, and $k' = \frac{1}{k}$. Therefore k′ exists whenever k exists. The transformations T_k and $T_{k'}$ are said to be *inverse* to one another. The inverse of every transformation belonging to the group G_1 is also a transformation of the same group.

The relation to one another of the conics K and K′ which determine a pair of inverse transformations, is easily seen to be very simple. One of them is the *reflection* of the other on the line OX. For if we project any range on l into a second range on l′ by means of a conic K, and then interchange l and l′ the conic K′ which will project the second range back into the first is obtained by revolving k about OX through 180°. The conics K and K′ are both inscribed in the same quadrilateral ABB′A′.

Property 1.—The transformations of the group G_1 *may be arranged in pairs; each pair consisting of a transformation and its inverse.*

We can also show that the group G_1 contains one *identical* transformation, i. e. a transformation that leaves every point of the line l invariant. Three conics of the system S degenerate into right lines, viz: the three diagonals of the quadrilateral ABB′A.′ The segments AB,′ BA,′ and the line joining O to the intersection of AA′ and BB′ are to be considered as conics of the system S. The last mentioned is of course the line O∞ parallel to AA.′ All lines parallel to AA′ and BB′ are tangents to the degenerate conic O∞ at the point at infinity on O∞. These parallel lines all cut the lines l and l′ in points equally distant from O, and hence all points of the line l are unaltered by the transformation. The characteristic anharmonic ratio of this identical transformation is given by k=(ABPP)=1. O is the point of contact of the conic O∞ with the line l.

Property 2.—The group G_1 *contains an identical transformation whose characteristic anharmonic ratio is unity.*

If T_k and $T_{k'}$ be a pair of inverse transformations, we have the relation $k=\frac{1}{k'}$, or kk′=1. This gives us another property of the group G_1.

Property 3.—The combined effect of any transformation and its inverse is equivalent to the identical transformation of the group. Thus $T_kT_{k'}=T_1$.

Since every transformation in the group G has an inverse belonging to the same group, the question arises can any transformation of the group be identical with its inverse. If such be the case, then must k and k′ be equal. This condition gives us $k=\frac{1}{k}$, or $k^2=1$; whence $k=\pm 1$. The first value k=1 gives the identical transformation of the group; that this is its own inverse is self-evident. The value k=−1 gives a very important transformation of the group. To say that this transformation is its own inverse is equivalent to saying that this transformation applied twice in succession will return every point on the line to its original position. Therefore a single application of this transformation has the effect of interchanging P and P,′ a pair of corresponding points; and so with every pair of corresponding points. Hence this transformation T_{-1} gives rise to an involution, and shall therefore be called the *involutoric transformation* of the group.

The position of the conic which produces the involutoric transformation of the group is such that it is its own reflection on the line OX. It therefore has the line OX for one of its axes. It is clear that there can be but one such conic in the system inscribed in the quadrilateral ABB′A′.

Property 4.—The group G_1 *contains one involutoric transformation which is its own inverse; the characteristic anharmonic ratio of the involutoric transformation is* —1.

The group G_1 contains two very noteworthy transformations, viz.: those determined by the conics AB′ and A′B. The tangents from all points on l to AB′ intersect l′ in B′. Hence the degenerate conic AB′ transforms all points of the line l into a single point B. The characteristic anharmonic ratio of this transformation is given by k=(ABPB)=0. The other degenerate conic A′B transforms all points of the line l to the point A. Its characteristic anharmonic ratio is given by k=(ABPA)=∞. Strictly speaking these are not transformations at all in the generally accepted meaning of the word; I shall call them ***pseudo-transformations***. These two pseudo-transformations form an inverse pair; this is evident analytically, and also from the fact that the lines A′B and AB′ are situated symmetrically with respect to OX.

Prof. Lie, in defining his transformations analytically, expressly states that the determinant of the transformation must not be zero. This condition excludes just these two transformations called pseudo-transformations. For Lie's equation of transformation is written

$$x_1=\frac{ax+b}{cx+d}.$$

If the determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}=0$, then $d=\frac{bc}{a}$. Substituting this value of d in the equation, we have

$$x_1=\frac{a(ax+b)}{c(ax+b)}=\frac{a}{c},$$

which shows that every point x of the line is transformed into the fixed point $\frac{a}{c}$.

The inverse of the above transformation is given by the equation

$$x=\frac{-dx_1+b}{cx_1-a}.$$

The determinant of this transformation equated to zero also gives $d=\frac{bc}{a}$. Substituting this value of d in the last equation we have

$$x=-\frac{b(cx_1-a)}{a(cx_1-a)}=-\frac{b}{a}.$$

The invariant points of either transformation are given by the equation

$$cx^2+(d-a)x-b=o.$$

Putting $d=\frac{bc}{a}$ in this it breaks up into

$$\left(x-\frac{a}{c}\right)\left(x+\frac{b}{a}\right)=o.$$

thus showing that $\frac{a}{c}$ and $-\frac{b}{a}$ are the invariant points of the transformations.

Property 5.—The group G_1 *contains a pair of pseudo-transformations, each of which transforms all points of the line* l *into one of the invariant points of the group; the characteristic anharmonic ratios of these pseudo-transformations are* o *and* ∞ *respectively.*

As stated above, the identical transformation of the group G_1 is produced by the degenerate conic $O\infty$ parallel to AA.′ O is the point of contact of this conic with l. The conic of the system S touching l at *d*O, a point infinitesimally near to O, produces an *infinitesimal* transformation; i. e. a transformation which shifts every point of the line l an infinitesimal distance. The point *d*O may be either to the right or to the left of O. In the one case the infinitesimal transformation is positive, in the other negative. The two may be expressed by the same analytical formula; and hence Lie's theorem asserts that G_1 contains one and only one infinitesimal transformation. Let $1\pm d$ represent its characteristic anharmonic ratio. If the infinitesimal transformation be repeated an infinite number of times, the result will be equivalent to a transformation of the same group, whose characteristic anharmonic ratio is $(1\pm d)^\infty$. This is a finite quantity different from unity. Hence we infer that the whole group may be generated from this infinitesimal transformation.

Property 6.—The group G_1 *contains one and only one infinitesimal transformation; the whole group may be generated from this infinitesimal transformation.*

Let us next consider transformations of the type T'_k, which leave one and only one point A invariant. It is evident that ∞^1 hyperbolas can be drawn touching l and l′ and having AA′ for an asymptote; (see Fig. 3). Call this system S.′ Every point on the line l is the point of contact of some hyperbola of the system S′; and hence we infer that the ∞^1 transformations determined by S′ form a continuous system.

Let T'_u denote a transformation of this system which transforms

P into P_1. Then $\frac{1}{AP} - \frac{1}{AP_1} = a$. Also let T_b' be another transformation of the same system which transforms P_1 to P_2; then $\frac{1}{AP_1} - \frac{1}{AP_2} = b$. Eliminating the fraction $\frac{1}{AP_1}$ from these two equations, we have $\frac{1}{AP} - \frac{1}{AP_2} = a + b = c$. But this relation is the result of a transformation T_c' which transforms P to P_2. T_c' belongs to the same system and is equivalent to the combination of T_a' and T_b'. Thus $T_a' T_b' = T_c'$. In the same way it may be proved that the combined effect of any number of transformations of the system is equivalent to some single transformation of the same system; and that the characteristic constant of the resultant transformation is equal to the sum of the characteristic constants of the component transformations.

Here again we have the fundamental property of a continuous group of transformations. The variable parameter of the one-termed group is the characteristic constant. A group of this kind will be designated by G_1'; sometimes it will be written G_A' when it is desired to call attention to the invariant point of the group.

Theorem 6.—The totality of the projective transformations which leave one and only one point of a line invariant forms a one-termed group G_1', *whose fundamental property is that the combined effect produced by two or more of the transformations of the group is equivalent to that of a single transformation of the same group. The characteristic constant of the resultant transformation is equal to the sum of the characteristic constants of the component transformations.*

Other properties of a group of this kind G_1' are easily established.

We shall show first that every transformation T_a' of the group G_1' has an inverse belonging to the same group. For if T_a' be the transformation of the group that transforms P to P_1, we have the relation $\frac{1}{AP} - \frac{1}{AP_1} = a$. The transformation of the group that transforms P_1 back to P gives the relation $\frac{1}{AP_1} - \frac{1}{AP} = \text{constant} = -a$. This latter exists whenever the former exists.

We can see in this case just as in the former case that the two conics K and K′ which determine a pair of inverse transformations are so situated that one is the reflection of the other on the line OX.

Property 1.—The transformation of the group G_1' *may be arranged in pairs; each pair consists of a transformation and its inverse.*

Two conics of the system S′ degenerate into right lines; viz.: the line O∞ parallel to AA′ and the line AA′ itself, excluding the segment included between l and l′. The degenerate conic O∞ has for tangents all lines parallel to AA′. The transformation determined by O∞ gives therefore an identical transformation. The characteristic constant of this identical transformation is given by the relation $\frac{1}{AP} - \frac{1}{AP} = k' = 0$.

Property 2.—The group G_1' *contains one identical transformation whose characteristic constant is zero.*

The transformation determined by the degenerate conic A∞A′ must now be examined. The tangents to A∞A′ from all points on l intersect l′ in A′. Therefore every point on the line l is transformed to A. This is a pseudo-transformation; its characteristic constant is given by the relation $\frac{1}{AP} - \frac{1}{AA} = -\infty$. But on the other hand the tangents to A∞A′ at A form a pencil of lines cutting l′ at all points. Therefore A is transformed into every other point on the line. This is also a pseudo-transformation; its characteristic constant is given by $\frac{1}{AA} - \frac{1}{AP} = \infty$. These two pseudo-transformations form an inverse pair.

Property 3.—The group G_1' *contains two pseudo-transformations which form an inverse pair. The characteristic constants of these two pseudo-transformations are* ∞ *and* $-\infty$.

If T_a and T_{-a} be a pair of inverse transformations, the characteristic constant of their resultant transformation is given by $a+(-a)=0$. Hence their resultant is the identical transformation of the group.

Property 4.—The combined effect of any transformation of the group and its inverse is equivalent to the identical transformation of the group. Thus $T_a T_{-a} = T_0$.

As stated above the degenerate conic O∞ produces the identical transformation of the group G_1'. The infinitesimally narrow hyperbola of the system S′ which touches O at *d*O, a point on l infinitesimally near to O, produces an infinitesimal transformation. The point *d*O may be either to the right or to the left of O, hence the infinitesimal transformation may be either positive or negative. Let *d* represent its characteristic constant. If the infinitesimal transformation be repeated an infinite number of times, the result will be equivalent to a finite transformation of the same group

whose characteristic constant is $(d+d+d\ldots \text{to } \infty)=k$. Thus any finite transformation of the group may be generated from this infinitesimal transformation.

Property 5.—The group G'_1 *contains one and only one infinitesimal transformation; the whole group may be generated from this infinitesimal transformation.*

We have seen that there are two distinct types or kinds of projective transformations of the points on a line; viz.: T_{AB} and T'_A. The first T_{AB} leaves two distinct real or imaginary points invariant; the second T'_A leaves a single point invariant. We can see likewise that there are two distinct types or kinds of one-termed groups of these transformations; viz.: G_{AB} and G'_A. The first G_{AB} leaves two distinct real or imaginary points invariant; the second G'_A leaves a single point invariant. There are no other kinds of one-termed groups. These two types of groups G_{AB} and G'_A have many properties in common, and some points of difference. We shall now summarize briefly the results of this section.

Theorem 7.—A one-termed group of projective transformations of the points on a line consists of ∞^1 *transformations having the property that the combined effect of any two or more of the transformations of the group is equivalent to that of a single transformation of the same group. Such a group contains one identical, one infinitesimal, and two pseudo-transformations; all the other transformations of the group may be arranged in inverse pairs. The combined effect of the two transformations composing any one of these pairs is equivalent to the identical transformation of the group. Any finite transformation of the group, or the whole group itself, may be generated from the infinitesimal transformation. There are two types of one-termed groups, viz.:* G_{AB}, *and* G'_A.

(1). The group G_{AB} *leaves two distinct points invariant. The variable parameter of the group is the characteristic anharmonic ratio. The product of the characteristic anharmonic ratios of the component transformations is equal to the characteristic anharmonic ratio of the resultant transformation. This group contains one involutoric transformation which is its own inverse.*

(2). The group G'_A *leaves one point invariant. The variable parameter of the group is the characteristic constant. The sum of the characteristic constants of the component transformations is equal to the characteristic constant of the resultant transformation. This group contains no involutoric transformation.*

A closer examination into the structure of these two groups G_{AB} and G'_A raises some important questions concerning the sufficiency

of our group definitions. The range of conics S touching the sides of the quadrilateral AA′B′B is composed of three distinct sub-divisions. These are separated from one another by the three line conics of the range, viz.: O ∞, AB,′ AB.′ The transformations of the group G_{AB} also arrange themselves in three corresponding sub-divisions. The hyperbolas which touch l between B and O give rise to transformations whose characteristic anharmonic ratios vary from 0 to 1. Let us call this sub-division I. The hyperbolas touching l between O and A give rise to transformations for which this ratio varies from 1 to ∞. These constitute sub-division II. The ellipses touching l between A and B give rise to transformations whose ratios are all negative and vary between —∞ and 0. These constitute sub-division III.

The combination of any two transformations of sub-division I gives rise to a transformation belonging to the same sub-division; for the product of two proper fractions is a proper fraction. The inverses of all transformations in sub-division I are in II. The combination of any two transformations in II gives a transformation also in II; but the inverses of those in II are in I. Sub-divisions I and II contain each an infinitesimal transformation. Sub-division III contains no infinitesimal transformation; the combination of any two transformations in III gives one in either I or II. The involutoric transformation divides subdivision III into two parts; all the transformations in one of these parts are the inverses of those in the other part.

In *Theorie der Transformationsgruppen*, Vol. I, page 163, Lie and Engel exhibit $x_1=ax$, where $0<a<1$, as an example of a group containing no inverse transformation. This group is identical with subdivision I above. However valid their reasoning may be from an analytic point of view, it is hardly proper from a geometric point of view to consider either sub-divisions I or II, or the combination of I and II, as a complete group.

In a similar manner the continuous group G'_A is divided by the identical and the pseudo-transformation into two parts. Each of these parts has an infinitesimal transformation and the characteristic group property. The transformations in one part are the inverses of those in the other.

§ 3. Two-termed Groups of Projective Transformations.

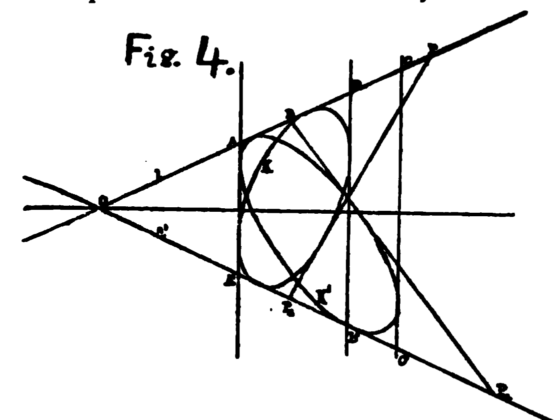
Having shown how to construct one-termed groups of the projective transformations of the points on a line, and having developed the chief properties of such groups, we now proceed to arrange and classify the groups themselves.

In the first place it is not difficult to determine the number of such one-termed groups. There are ∞^1 points on the line l; by taking each of these points with all the others, including itself, we can form ∞^2 pairs of points. Each of these pairs of points can be made the invariant points of a one-termed group G_1. Thus the ∞^3 projective transformations T_k of the points on a line fall readily into ∞^2 one-termed groups, G_1, each consisting of ∞^1 transformations which leave two fixed points invariant. Among these ∞^2 groups G_1 are ∞^1 groups of the kind G_1'. The coincident invariant points of these groups are obtained by taking each point of the line with itself. All the one-termed groups considered above are included in the formula $\infty^3 T = \infty^2 G_1 + \infty^1 G_1'$.

We can readily see from the mode of formation of these one-termed groups that every transformation T belongs to some one of these groups, and that no two of these groups can have a transformation in common, (except possibly the identical transformation which may be considered the same for all the groups).

Theorem 8.—The system of ∞^3 projective transformations of a line falls apart into ∞^2 one-termed groups. Each proper transformation of the line belongs to one and only one of these groups. There are two general types of these groups, G_1 *and* G_1'.

We shall now examine the question of *two-termed* groups of projective transformations on a line. There are ∞^3 projective transformations of the points on a line, and only ∞^1 points on the line. Hence any point can be transformed into any point on the line in ∞^2 different ways. In other words there is a system of ∞^2 transformations which leaves any point A on the line invariant. We shall proceed to show that this system of transformations has the fundamental property of a group, i. e. that the combined effect of any two or more of the transformations of the system is equivalent to some single transformation of the same system.



To show this we take a range of points on the line l (Fig. 4) and project it by means of a conic K into a second range on the line l′. Revolve l′

about O until it coincides with l; we then have a second range on l. Let A and B be the invariant points of the transformation due to K. Now project this second range into a third range on l′ by means of a conic K′ touching AA′ and CC′. If we now join the points of the first range on l with the corresponding points of the third range on l′ these joins all touch a conic K″ which determines the projection of the first range into the third. This last transformation is equivalent to the combination of the other two. AA′ is one of these joins; hence the transformation determined by K″ leaves the point A invariant. This same process may be extended to any number of transformations.

Theorem 9.—The system of ∞^2 *projective transformations that leaves one point of a line invariant forms a group whose fundamental property is that the combined effect of two or more transformations of the group is equivalent to some single transformation of the same group.*

When we come to examine the structure of one of these two-termed groups, it is easy to see that it is composed of an infinite number of one-termed groups. Take for example the two-termed group $G_{2,A}$ that leaves the point A invariant. It is made up of all the one-termed groups which have A for one of their invariant points. A may be taken with each of the other points on the line and once with itself. Thus we see that $G_{2,A}$ is made up of an infinity of one-termed groups of the kind G_1 and one of the kind G_1'. What is true of the two-termed group $G_{2,A}$ is also true of any other two-termed group.

Theorem 10.—Every two-termed group of projective transformations of a line leaving one point invariant is composed of ∞^1 *one-termed groups of the type* G_1 *and one of the type* G_1'. $G_2 = \infty^1 G_1 + 1 G_1'$.

Any two such two-termed groups have common a one-termed group. Take for example the two-termed groups $G_{2,A}$ and $G_{2,B}$. Each of them contain the one-termed group G_{AB}. We can also see that the converse of this is true; every one-termed group of the type G_1 belongs to two and only two two-termed groups.

It was pointed out above that the most general projective transformation of the points on the line involves three parameters, the coördinates of the invariant points, m and n, and the characteristic anharmonic ratio k. We have seen that when m and n are fixed quantities and k variable, the ∞^1 transformations form a one-termed group; also that when n is fixed and m and k variable we have a two-termed group. On the other hand when m and k are fixed quantities and n variable, the transformations do not form a one-termed group. Also when m and n are variable and k constant

we have ∞^1 transformations which do not form a group. These statements may readily be verified from the construction.

Every transformation of the kind T′ depends on two parameters, m and k, coördinate of invariant point and characteristic constant. If m be fixed and k variable, we have a one-termed group; but if k be fixed and m variable, we do not have a group. If m and k both vary, the resulting ∞^1 transformations of the kind T′ do not form a group. The verification is easy.

We thus recognize two distinct varieties of parameters: m and n constitute one kind and k the other. If either one or both of the parameters of the first kind are fixed quantities, we have one- and two-termed groups; but if the parameter of the second kind is fixed, we do not have a group. Thus G_1 and G_1' are the only possible variety of one-termed groups; and G_A is the only possible kind of two termed group.

Theorem II.—Two distinct kinds of parameters enter into the determination of a projective transformation, viz.: co-ordinates of invariant points, characteristic {anharmonic ratio. / constant.} We have one- and two-termed groups when and only when the parameters of the first kind are fixed quantities.

§ 4. The General Projective Group and Some of Its Special Sub-Groups.

We shall now proceed to show that all the projective transformations of a line form a group, using the word in the same sense as heretofore. Any two or more of the projective transformations of a line when carried out in succession produce a result equivalent to that produced by some single transformation. This follows immediately from the theory of projective ranges, a knowledge of which is here presupposed. For if we project any range R_1 into another range R_2, then project R_2 into R_3, and so on until we obtain n ranges; then R_n is projective with each of the preceding ranges, including R_1. Let us carry out in succession (n—1) projective transformations by means of conics touching the lines l and l′. We shall finally obtain a range R_n on l′; this being projective with the first range R_1 on l, the lines joining corresponding points on these two ranges touch a conic which also touches l and l′. Thus we see that this last conic determines a transformation equivalent to the combination of the (n—1) preceding ones. This proves that the ∞^3 projective transformations on a line form a group. We shall call this the general projective group of the line, and designate it by G_3. This general projective group is three-termed since it contains ∞^3 projective transformations.

Theorem 12.—The totality of the projective transformations of the points on a line form a three-termed group.

This general projective group does not leave any point or points of the line invariant. It is easy to see that this three-termed group G_3 falls apart into ∞^1 two-termed sub-groups, each of which leaves one point of the line invariant. The properties of these two-termed sub-groups were discussed in the last section. Since each of these two-termed groups falls apart into ∞^1 one termed groups, we see how the general three-termed group falls apart into ∞^2 one-termed sub-groups. We are now in position to arrange a scheme for the complete classification of the projective transformations of a line. $G_3 = \infty^1 G_2 = \infty^1(\infty^1 G_1 + 1G_1') = \infty^2 G_1 + \infty^1 G_1'$.

The chief properties of the general three-termed group may be readily collected from the discussions of the preceding pages. G_3 contains one and only one identical transformation; this is determined by the degenerate conic $O\infty$ perpendicular to OX. This identical transformation is contained in every two-termed and also in every one-termed sub-group. In this respect it is peculiar among all the projective transformations of the line.

G_3 contains ∞^2 pseudo-transformations; these are so distributed that every two-termed sub-group contains ∞^1 of them, and every one-termed sub-group contains two of them.

Every infinitesimally narrow conic touching l at dO, and also touching l.′ determines an infinitesimal transformation. There are ∞^2 such conics; hence G_3 contains ∞^2 infinitesimal transformations. ∞^1 of these conics touch any line as AA′ perpendicular to OX; and only one of them can touch two such lines as AA′ and BB.′ These ∞^2 infinitesimal transformations are therefore distributed so that every one-termed sub-group contains one and only one of them. There are two varieties of these infinitesimal transformations and they give rise to two varieties of one-termed groups, G_1 and G_1'.

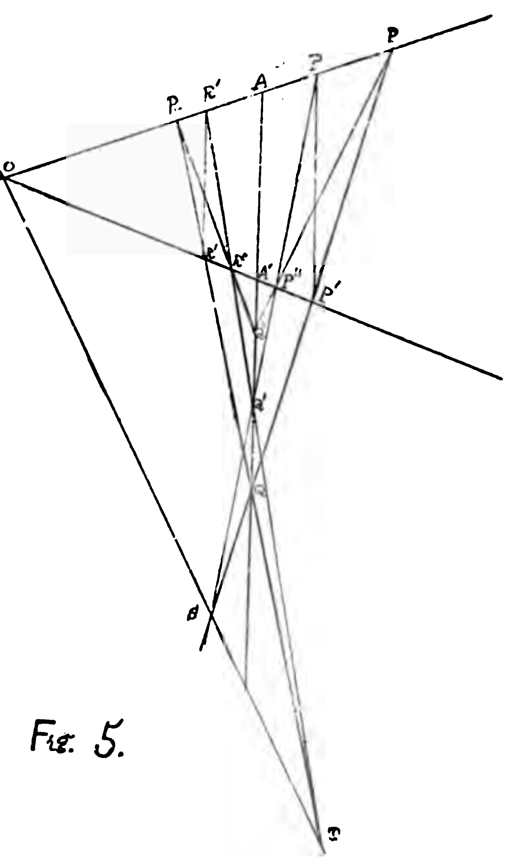
As before remarked the general projective group G_3 breaks up into ∞^1 two-termed sub-groups. These two-termed sub-groups are all very much alike and have like properties; two of them however call for special attention, viz.: the group which leaves O invariant, and that one which leaves the point at infinity invariant.

The two-termed sub-group $G_{2,0}$, which leaves the point O invariant, is made up of one-termed sub-groups each of which leaves O and some other point as A invariant. Fig. 2 shows the construction of a transformation which belongs to the one-termed sub-group G_{A0}. This transformation is determined by the degenerate conic OQ. All transformations which leave the point O

invariant are determined by conics which touch l, l,′ and O ∞: but since these three lines meet in a point, it follows that all these conics must be degenerate, and each must consist of linear segments terminating at O. Every transformation of this kind is a *perspective transformation.* The point Q from which the projecting lines are drawn may be any point in the plane. Q may therefore have ∞^2 different positions: and we see that there are ∞^2 perspective transformations, each of which leaves the point O invariant. These ∞^2 perspective transformations form the two-termed group $G_{2,0}$. This sub-group contains all the perspective transformations in the general projective group, and no others.

Theorem 13.—All the perspective transformations contained in the general projective group form a two-termed sub-group, $G_{2,0}$: *this sub-group contains no transformation which is not perspective.*

We now proceed to the consideration of the one-termed sub-groups which compose the two-termed perspective group. Each of these sub-groups has the point O and some other point as A for invariant points. For ∞^1 positions of the point Q the resulting perspective transformations leave the point A as well as O invariant. It is easy to see that these ∞^1 positions of Q must all be on the line AA,′ because the second invariant point of a perspective transformation is found by dropping a perpendicular from Q on OX. Thus the ∞^1 perspective transformations obtained by taking the center of perspective at all points on a line perpendicular to OX form a one-termed group.



This conclusion, which is thus deduced from the consideration of the line OQ as a degenerate conic, is capable of a direct geometric proof as follows: If we take on the line AA′ (Fig. 5), two points Q and Q′ as centers of perspective transformation, we can show that these two perspective transformations are together equivalent to one whose center of perspective is also a point on AA.′ With Q as the center of perspective, P and R are projected into P′ and R′ and transformed into P_1 and R_1. With Q′ as a new center, P_1 and R_1 are projected into P″ and R″. The joins of P and P″, and of R and R″ meet in Q″.

According to the construction the two triangles $P_1P'S$ and $R_1R'T$ are homologous; because the sides P_1S and R_1T meet in Q.′ P′S and R′T meet in Q, also P_1P' and R_1R' meet at infinity on AA.′ Hence the lines joining corresponding vertices must meet in a point. Thus P_1R_1, P′R,′ and ST all meet in O.

Consider now the triangles PP″S and RR″T; these are likewise homologous, for the lines PR, P″R″, ST meet in O. Hence the intersection of corresponding sides must lie on a line. But PS and RT meet in Q, P″S and R″T meet in Q′; therefore PP″ and RR″ must meet in Q″, a point on AA.′

There are ∞^1 perpendiculars to the line OX each of which determine a one-termed sub-group of the two-termed perspective group. One of these one-termed sub-groups is of the type G_1', i. e. it leaves one and only one point invariant. This particular subgroup is determined by the line O ∞ perpendicular to OX.

Theorem 14.—The ∞^2 perspective transformations, for which the centers of perspective lie on a line perpendicular to the bisector OX, *form a one-termed group.*

Let us examine a little closer the perspective transformation exhibited in Fig. 2. The characteristic anharmonic ratio k is determined by the range $(OAPP_1)$; thus $k=(OAPP_1)=\frac{OP}{PA}:\frac{OP_1}{P_1A}=\frac{OP}{OP_1}\cdot\frac{AP_1}{AP}$. But $\frac{OP}{OP_1}=\frac{\sin a}{\sin b}$; and $AP=AQ\frac{\sin\gamma}{\sin b}$; $AP_1=A'Q\frac{\sin\gamma}{\sin a}$. Hence we have $k=\frac{A'Q}{AQ}$. When the point Q is at infinity on AA,′ we have the identical transformation. The infinitesimal transformation of the group is obtained by taking Q on AA′ so far away that the angles at Q are infinitesimally small. Two points Q and Q′ equally distant from OX give a pair of inverse transformations. When Q falls at A and at A,′ we have the two pseudo-transformations. When Q falls on OX, we have the involutoric transformation of the group.

We now turn our attention to the two-termed sub-group of transformations that leave invariant the point at infinity on l. This group will be designated by $G_{2,\infty}$. The projective transformation determined by a parabola has for its invariant points the point at infinity and some finite point. There are ∞^2 parabolas touching the lines l and l′; these determine a two-termed group of transformations whose invariant point is the point at infinity on l. Every transformation belonging to this group is determined by a parabola; for every conic touching the line at infinity is a parabola.

All the transformations belonging to this group exhibit a peculiar property. Let us take for example a transformation which has A and ∞ for invariant points, and whose characteristic anharmonic ratio is k. Then we have $k=(A\infty\, PP_1)=\frac{AP}{P\infty}:\frac{AP_1}{P_1\infty}=\frac{AP}{AP_1}$. Whence $AP=k.AP_1$. If we take Q and Q_1, another pair of corresponding points, we shall have in like $AQ=k.AQ_1$. Therefore $PQ=AQ-AP=k(AQ_1-AP_1)=k.P_1Q_1$. Thus we see that the effect of such a transformation is to multiply the length of any segment by a constant, the constant multiplier being the characteristic anharmonic ratio of the transformation. This is analogous to the mechanical effect of stretching a rubber cord with one end fixed at A. We shall call this effect a *Dilation*: and shall speak of the group $G_{2\infty}$ as the group of dilations. If the multiplier k is less than unity, the effect of the transformation is contraction: but if we think of contraction as negative dilation, the word dilation is sufficient to cover all cases.

Theorem 15.—The ∞^2 parabolas touching l *and* l′ *determine a two-termed group of dilations, infinity being the invariant point.*

This two-termed group breaks up into ∞^1 one-termed groups. ∞^1 parabolas can be drawn touching l and l′ and some line as AA′ perpendicular to OX. These determine a one-termed group whose finite invariant point is A. One of these one-termed groups is of the type G_1'. This group has two coincident invariant points at infinity on l. This group is determined by the system of parabolas touching l and l′ and having the line $O\infty$ as a common axis. The only tangent to a parabola of this system perpendicular to OX is the line at infinity; and it is a common tangent to all of them. The characteristic anharmonic ratio of any transformation of this group is given by $k=(\infty\,\infty\, PP_1)=1$. Hence the constant of dilation for this group is unity; that is to say, every segment of the line l is transformed into an equal segment. Again let us take one of these parabolas touching l and l′ and having its axis per-

pendicular to OX; let a tangent be drawn cutting l and l′ in P and P.′ It is easy to show that the difference of the segments OP and OP′ is constant for all tangents to the parabola; and that this constant difference is equal to the distance from O to the point of contact with l. The effect of a transformation of this kind is to move every point of the line to the right or to the left a certain fixed distance. It is equivalent to sliding the line along itself; thereby the length of any segment is unaltered. A transformation of this kind is called a *Translation*. It is self evident that all translations form a group.

Theorem 16.—The translations of the points on a line form a one-termed group of the type G_1'; *the single invariant point of this group is the point at infinity on the line.*

We shall next examine the one-termed sub-group whose invariant points are O and ∞. This one-termed group is common to the two-termed perspective group $G_{2,0}$ and to the two-termed dilative group $G_{2,\infty}$: we shall therefore expect to find in it a combination of the properties of these two groups. Any transformation of this group is constructed by drawing a pencil of lines from a point Q on the line at infinity. The lines of such a pencil are of course parallel. The transformations effected by the systems of parallel lines form a group, which is both perspective and dilative.

Thus far we have considered the groups of transformations of the line into itself. Other one dimensional forms remain to be considered e. g. a pencil of lines through a point O and a pencil of planes through a line l. The theory for these forms is so nearly identical with that for the straight line that it is unnecessary to give the parallel development.

A perfectly obvious construction for a projective transformation of a pencil of rays is as follows: Let two planes Pl and P′l meet in a line l; and let O, a point on l, be the common vertex of two pencils of rays, one in Pl and the other in P′l. The planes determined by three pairs of corresponding lines of the two pencils together with Pl and P′l determine a cone of the second order having its vertex at O and touching both Pl and P′l. Tangent planes to this cone cut Pl and P′l in corresponding lines of the two pencils. If now P′l be revolved about l until it coincides with Pl, a projective transformation of the pencil in Pl is completed. The properties of this transformation and of groups of such transformations may easily be developed.

The Theoretical and Measured Pumping Power of Windmills.

BY E. C. MURPHY.

For more than a year the writer has devoted a large portion of his spare time to the study of windmills—trying to verify or disprove statements of windmill manufacturers of the power of their mills, adapting the mathematical theory of windmills in general to the American steel windmill of today, measuring the power of windmills, and in reconciling theoretical and measured results.

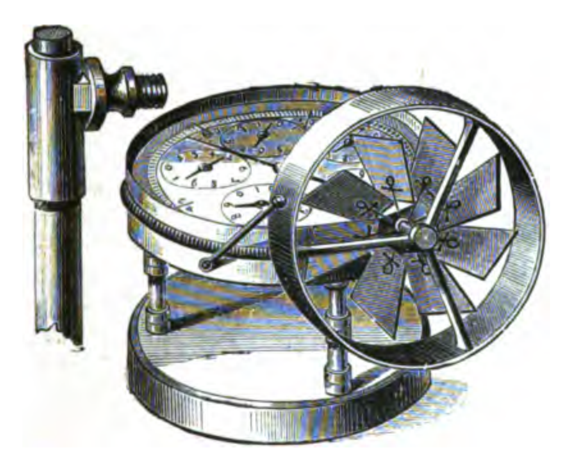


Fig. 1.

The writer hopes in the near future to supplement the present paper with a more extended set of measurements.

There is today a large demand for powerful windmills—mills that will pump a large amount of water in a short time. The cause of this demand is the need of water for irrigating purposes, and a cheap and efficient moter to raise water from water-bearing strata. The windmill has come into competition with gasoline, gas, and even steam engines for this purpose, and there is a demand for accurate information in regard to the merits of each, and especially in regard to the power of windmills.

Many improvements have been made in windmills in the last few years, and we hear rumors of other and greater improvements to be made in the near future. Doubtless engineers and inventors have made many experiments on windmills, but they have not published their results. It is surprising how few accurate data there are in regard to the actual working of windmills—motors that are in such general use and have been for centuries. There are really none with the exception of a table in *Engineering Magazine*, Vol. VIII, giving some results of measurements by Lieut. I. N. Lewis.

The measurements of power made by the writer can not be considered comparative tests. The mills were not working under the same conditions; some were heavily loaded, others lightly; still the mills tested were in good working order; the exposure was good in each case, the lift, quantity of water and wind velocity were carefully measured. The temperature and barometer pressure, though not measured each time, were found with sufficient accuracy for the purpose. The experiments show the work each mill was doing under its own conditions, not the work it might be made to do.

Let ABED, Fig. 2, represent one slat or fan of a windmill, C the axis, R_{ll} and R_o the radii or distances from the axis to the

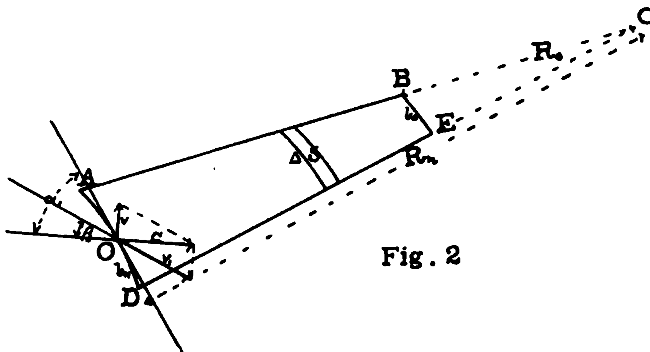


Fig. 2

ends of the fan, b_{ll} and b_o the end widths of fan. Let c be the velocity of the wind, v the velocity of the wheel and v_1 the relative velocity of the wind; that is, the velocity with respect to the moving fan. Let x be the angle which the fan makes with the direction of the wind, or axis C; y the angle which v_1 makes with axis C. Let Δs be any small element of area of fan having a length ΔR and width b_1; r the heaviness of air, K a coefficient whose value depends on curvature and friction of air on fan, L' the power of the fan, and L the power of mill.

Fig. 3 shows Δs which we assume for simplicity to be a plane.

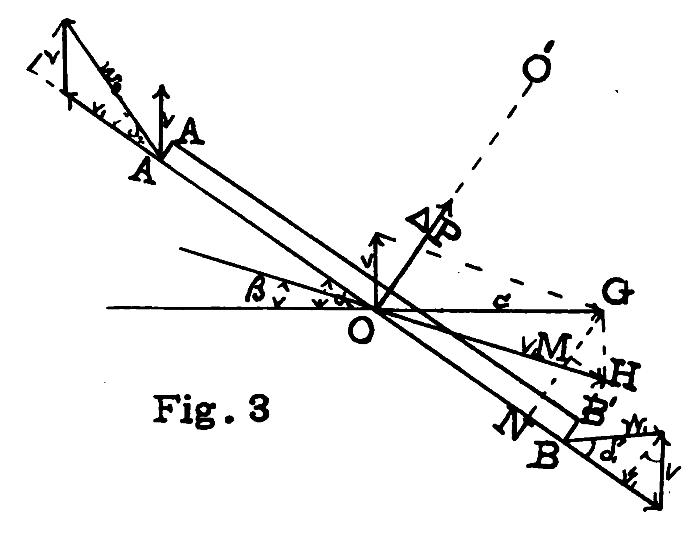
The wind pressure ΔP on Δs, Fig. 3, is perpendicular to the surface if we neglect the friction of air over the surface. This pressure

or accelerating force is equal to the mass $\triangle M$ of air flowing over $\triangle s$ in time $\triangle t$ multiplied by the acceleration in the direction of $\triangle P$. This acceleration is equal to the difference in velocity of $\triangle M$ in the direction of $\triangle P$ before and after impact, and equals $c \sin x - w_1 \cos z_1$ at B, and $c \sin x - w_2 \cos z_2$ at A.

From Fig. 3 $w_1 \cos z_1 = w_2 \cos z_2 = v \cos x$. Hence

$$\triangle P = \frac{\triangle M}{\triangle t}(c \sin x - v \cos x) \ldots\ldots\ldots\ldots (1)$$

If M is the mass of air deviated per second by $\triangle s$, then $\triangle M = M \triangle t$. The volume deviated per second is equal to the relative



velocity v_1 multiplied by the area of the projection of $\triangle s$ on a plane perpendicular to the relative velocity. Hence

$$\triangle P = K \frac{r}{g} v_1 \triangle s \sin(x - y)(c \sin x - v \cos x) \ldots . (2)$$

From Fig. 3 $v_1 \sin(x - y) = HB = NM = NG - GM = c \sin x - v \cos x$. Hence

$$\triangle P = K \frac{r}{g} \triangle s (c \sin x - v \cos x)^2 \ldots\ldots\ldots (3)$$

Resolving $\triangle P$ into two components, one perpendicular to the plane of the wheel and the other parallel to it, we have for the latter, which causes the wheel to revolve

$$\triangle P \cos x = K \frac{r}{g} \triangle s (c \sin x - v \cos x)^2 \cos x \ldots (4)$$

The other component

$$\triangle P \sin x = K \frac{r}{g} \triangle s (c \sin x - v \cos x) \sin x \ldots . . (5)$$

causes friction, and will be considered later.

The power $\triangle L^1$ of $\triangle s$ is equal to the force in (4) multiplied by v, the velocity of $\triangle s$. Hence

$$\triangle L^1 = \triangle P \cos x\, v = K\frac{r}{g}\triangle s v(c \sin x - v \cos x)^2 \cos x \quad \ldots (6)$$

Equation (6) can be put in the form

$$\triangle L^1 = K\frac{r}{g}c^3 \triangle s B \quad \ldots\ldots\ldots\ldots (7)$$

in which $B = a \cos x(\sin x - a \cos x)^2$ and $a = \frac{v}{c}$.

Table I gives values of B for values a and x.

TABLE I.

x	a=.3	a=.4	a=.5	a=.6	a=.7	a=.8	a=.9	a=1.0
45°	.0519	.0523	.0442	.0339	.0222	.0113	.0031	.0000
50°	.0633	.0665	.0636	.0557	.0449	.0329	.0203	.0096
55°	.0720	.0798	.0812	.0776	.0699	.0595	.0464	.0344
60°	.0769	.0887	.0949	.0960	.0932	.0849	.0778	.0670
65°	.0801	.0918	.1019	.1080	.1101	.1090	.1047	.0987
70°	.0718	.0883	.1011	.1105	.1173	.1214	.1229	.1221
75°	.0614	.0770	.0965	.1020	.1115	.1192	.1251	.1295

From (7) we see that for a given wind velocity c and surface $\triangle$ s the power varies with B, and, from Table I, we see for any value of a, the proper value of x for maximum power. For example: If $a = \frac{v}{c} = .8$, $x = 70°$ approximately.

The relation between x and a for maximum power is found as follows: Differentiating (6) with respect to x we have

$$\frac{dL'}{dx} = K\frac{r}{g}c^3\triangle s a[2\cos^2 x \sin x - \sin^3 x + 4a \cos x \sin^2 x - 2a \cos^3 x$$
$$-3a^2 \cos^2 x \sin x] = 0 \quad \ldots\ldots\ldots\ldots (8)$$

Simplifying (8) we have

$$-\tan^3 x + 4a \tan^2 x + (2 - 3a^2)\tan x - 2a = 0 \quad \ldots\ldots (9)$$

Solving (9) we have

$$\tan x = a \text{ and } \tfrac{3}{2}a \pm \sqrt{(\tfrac{3}{2}a)^2 + 2}.$$

The first of these values makes $\triangle L' = 0$ and gives a minimum value; of the other two $\frac{3}{2}a + \sqrt{(\frac{3}{2}a)^2 + 2}$ gives the greatest value of $\triangle L'$. Hence the relation between x and a for maximum power is

$$\tan x = \tfrac{3}{2}a + \sqrt{(\tfrac{3}{2}a)^2 + 2} \quad \ldots\ldots\ldots\ldots (10)$$

By substituting for a any desired value in (10) the corresponding value of x for maximum power is found.

For any given value of c the ratio $\frac{v}{c}$ increases as the radius of the wheel increases, and, since an angle increases with its tangent, we see that the angle which a fan makes with the direction of

the wind should increase from the inner end of the fan to the outer end; hence for maximum power the fans should be warped surfaces.

CASE I.

The fans as in Fig. 2 with x constant. This is the form of fan used in nearly all American steel mills.

$$\triangle s = b\triangle R = \frac{b_n}{R_n} R\triangle R, \quad v = \frac{v_n}{R_n} R.$$

Substituting these values in (6) we have

$$\triangle L' = K\frac{r}{g}\,\frac{b_n}{R_n}\,\frac{v_n}{R_n}\cos x\left[c^2\sin^2 x R^2\triangle R - 2\frac{v_n}{R_n}c\sin x\cos x\, R^3\triangle R - \left(\frac{v_n}{R_n}\right)^2\cos^2 x\, R^4\triangle R\right]$$

Making $\triangle R$ infinitesimal=dR and integrating R between limits R_o and R_n and dL' between corresponding limits we have

$$L' = \int_o^{L'} dL' = K\frac{r}{g} b_n \frac{v_n}{c}\cos x\, c^3\left[\sin^2 x\frac{R_n^3 - R_o^3}{3R_n^2} - 2\frac{V_n}{c}\sin x\cos x\frac{R_n^4 - R_o^4}{4R_n^3} - \left(\frac{v_n}{c}\right)^2\cos^2 x\frac{R_n^5 - R_o^5}{5R_n^4}\right] \ldots\ldots (11)$$

The ratio of $\frac{R_o}{R_n}$ is nearly constant for different sizes of wheels. Letting this ratio be t and substituting it in (11) and multiplying by n, the number of fans in wheel, we have for the power of the mill

$$L = K\frac{r}{g} n b_n \frac{v_n}{c}\cos x\, c^3 R_n\left[\frac{1-t^3}{3}\sin^2 x - 2\frac{v_n}{c}\left(\frac{1-t^4}{4}\right)\sin x\cos x + \left(\frac{v_n}{c}\right)^2\left(\frac{1-t^5}{5}\right)\cos^2 x\right] \ldots\ldots\ldots\ldots (12)$$

Example:—Six-foot steel mill No. 7, Table II.

$n=12$, $b_n=\frac{14\frac{1}{4}}{12}$, $\frac{v_n}{c}=.311$ (from Fig. 7), $x=55^o$, $\frac{r}{g}=0021$.

Substituting these values in (12) we have

$$L_6 = K\times.0021\times 12\times\tfrac{57}{48}\times.311\times.5736\times c^3[.326\times.671 - 2\times.311\times .2485\times.819 + .311^2\times.20\times.329]$$

$$=.016Kc^3[.2187 - .0725 + .0064]$$

$$=.0024Kc^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The value of the coefficient --K is difficult to determine. Professor Weisbach assumed that the suction on the leeward side of a fan is one-half the pressure on the windward side. This would make $K=\frac{3}{2}$ for a plane fan. The writer has found from experiment that the normal wind pressure on the concave side of a smooth iron surface such as is used for windmill fans is 1.15 times that on a

plane surface whose area equals that of the projection of the curved surface. This would make the value of $K=1.73$.

Substituting for K this value in (13) we have

$$L_6 = .0041c^3 \qquad (14)$$

Substituting for c the values 8, 10 and 12 miles per hour, we have for L_6 the values 6.7, 13.0 and 22.4 ft. lbs. of work per second respectively.

CASE II.

Fans rectangular and x constant.

$$\triangle s = b_n \triangle R, \quad v = \frac{v_n}{R_n} R.$$

Making these substitutions in (6) we have

$$\triangle L = K \frac{r}{g} \frac{b_n}{R_n} \frac{v_n}{R_n} \cos x \, c^3 \left[\sin^2 x \, R^2 \triangle R - 2\frac{v_n}{cR_n} \sin x \cos x \, R^2 \triangle R + \left(\frac{v_n}{c}\right)^2 \cos^2 x \frac{1}{R_n^2} R^3 \triangle R \right]$$

Making $\triangle L$ and $\triangle R$ infinitesimal and integrating between limits we have

$$L' = K \frac{r}{g} b_n \frac{v_n}{c} \cos x \, c^3 \left[\sin^2 x \frac{R_n^2 - R_0^2}{2R_n} - 2\frac{v_n}{c} \sin x \cos x \frac{R_n^3 - R_0^3}{3R_n^2} + \left(\frac{v_n}{c}\right)^2 \cos^2 x \frac{R_n^4 - R_0^4}{4R_n^3} \right]$$

Substituting $t = \frac{R_0}{R_n}$ and multiplying by n to get the power of the wheel we have

$$L = K \frac{r}{g} n \frac{v_n}{c} \cos x \, c^3 R_n \left[\frac{1-t^2}{2} \sin^2 x - 2\frac{v_n}{c} \sin x \cos x \left(\frac{1-t^3}{3}\right) + \left(\frac{v_n}{c}\right)^2 \cos^2 x \left(\frac{1-t^4}{4}\right) \right] \qquad (15)$$

CASE III.

The fans with warped surfaces.

We may find an approximate value of the power by dividing the fans into strips, x, R and v being variable, and applying (12) or (15), as the case may be to each strip, and adding the results. If the fans are rectangular we have

$$L_1 = Kn \frac{r}{g} \frac{v_1}{c} \cos x_1 c^3 R_1 \left[\frac{1-t_1^2}{2} \sin^2 x_1 - 2\frac{v_1}{c} \sin x_1 \cos x_1 \frac{1-t_1^3}{3} + \left(\frac{v_1}{c}\right)^2 \cos^2 x_1 \left(\frac{1-t_1^4}{4}\right) \right]$$

$$L_2 = Kn \frac{r}{g} \frac{v_2}{c} \cos x_2 c^3 R_2 \left[\frac{1-t_2^2}{2} \sin^2 x_2 - 2\frac{v_2}{c} \sin x_2 \cos x_2 \left(\frac{1-t_2^3}{3}\right) + \left(\frac{v_2}{c}\right)^2 \cos^2 x_2 \left(\frac{1-t_2^4}{4}\right) \right]$$

$L_3 =$ etc.

Then

$$L = L_1 + L_2 + L_3 \qquad (16)$$

Losses of Power.

I. THAT DUE TO THE AXIAL COMPONENT OF WIND PRESSURE.

Equation (5) gives the value of this component on any $\triangle s$. Multiply this pressure by f, the coefficient of friction, and by $2\pi ep$* where e equals the radius of friction and p the number of revolutions of wheel per second, we have

$$\triangle L_1 = fK\frac{r}{g}\triangle s(c\sin x - v\cos x)^2 \sin x\ 2\pi ep \ldots (17)$$

Dividing (17) by (4) after multiplying the latter by $2\pi e$ we have for the ratio of the work lost per revolution in this axis friction to the work per revolution of the mill

$$\frac{\triangle L_1}{\triangle L_2} = \frac{fK\frac{r}{g}\triangle s(c\sin x - v\cos x)^2 \sin x\ 2\pi ep}{K\frac{r}{g}\triangle s(c\sin x - v\cos x)^2 \cos x\ 2\pi Rp} = \frac{fe\tan x}{R} \ldots (18)$$

Example:—12 ft. mill with f=.07, e=¼ ft., R=4.56 ft.

From (18) $\frac{\triangle L_1}{\triangle L_2} = \frac{.07 \times \frac{1}{4} \times .8}{4.56 \times .6} = .005 = \frac{1}{2}$ per cent of power of mill. This friction loss increases with e and x, and decreases as R increases.

II. THAT DUE TO SKIN FRICTION.

From mechanics we have for this loss of power expressed in head

$$h_2 = \frac{32 f' h Q^2}{\pi^2 g d^5} \ldots (19)$$

in which h is the height to which the water is raised, Q the number of cubic feet raised per second, d the diameter of the pipe and f' the coefficient of skin friction.

III. THAT DUE TO AXLE FRICTION.

For this loss of work per revolution we have

$$\triangle L_3 = \pi e' f[G + Fr(h + h_2)] \ldots (20)$$

in which G is the weight of the wheel, piston and pump-rod, F the cross-section of the water column, r the heaviness of water, and e' the radius of axle.

Example:—12 ft. mill with G=950 lbs., double-acting pump having 4½-inch cylinder and 12-inch stroke, 30 strokes per minute, h=25 ft., back geared 2 : 1, e'=1 inch, and f=.07.

* $\pi = 3.14159$.

From (19) $h_2 = \frac{32 \times 7^2 \times .00708 \times 25 \times .11^2}{22^2 \times 32.2 \times (3/8)^5} = .03$ ft. $=.1$ of one per cent of useful work.

From (20) $\triangle L_3 = \frac{22}{7} \times \frac{1}{12} \times .07(950+173) = 20.50$ ft.-lbs.

The loss h_2 is small except when h is large or the diameter of the pipe is small. The third loss is much greater than either or both the other losses.

The Power Found by Measurement.

The quantities to be measured are the velocity of the wind, or distance traveled by it in time t, the quantity of water raised in that time and the height to which it is raised.

The velocity of the wind was measured with an air meter, Fig. 1, having a fan wheel 2¾ inches in diameter, a minute glass and a disconnector. The motion of the fan wheel is recorded on six

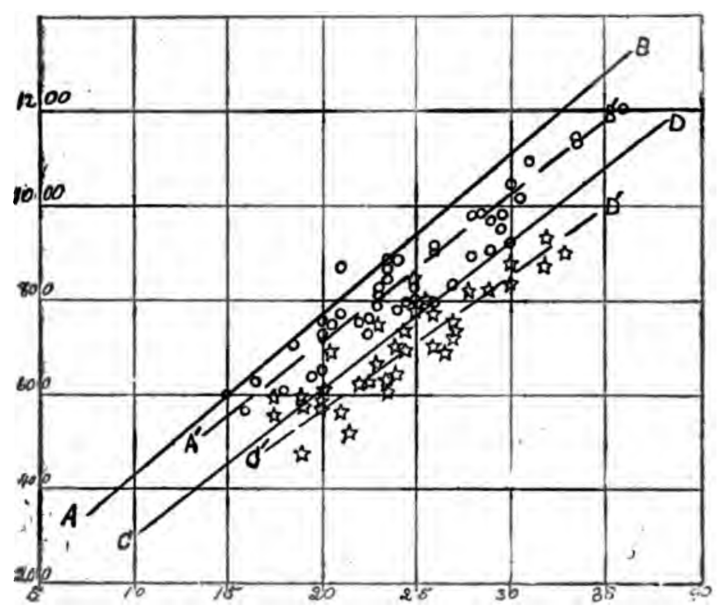


Fig. 4.—8 ft. steel mill, 78 ft.-lbs. per stroke.

dials. It will record any wind movement from one foot up to 1893 miles. The minute glass was found to have an index error of two seconds; that is, the sand runs through the glass in 58 seconds instead of 60. The meter was placed on a vane so that the axis of the fan wheel was always in the direction of the wind. The vane was placed on the end of a board and held on the platform of the mill in front of the wheel. In the case of mill No. 4, Fig. 4, th

velocity measurements were made on the top of the water tank twenty feet below the platform and the velocity v at the platform found from the formula

$$v = v_1 \sqrt{\frac{H+72}{h+72}}$$

The velocity and corresponding number of strokes of the pump were found by reading the meter when disconnected or when the

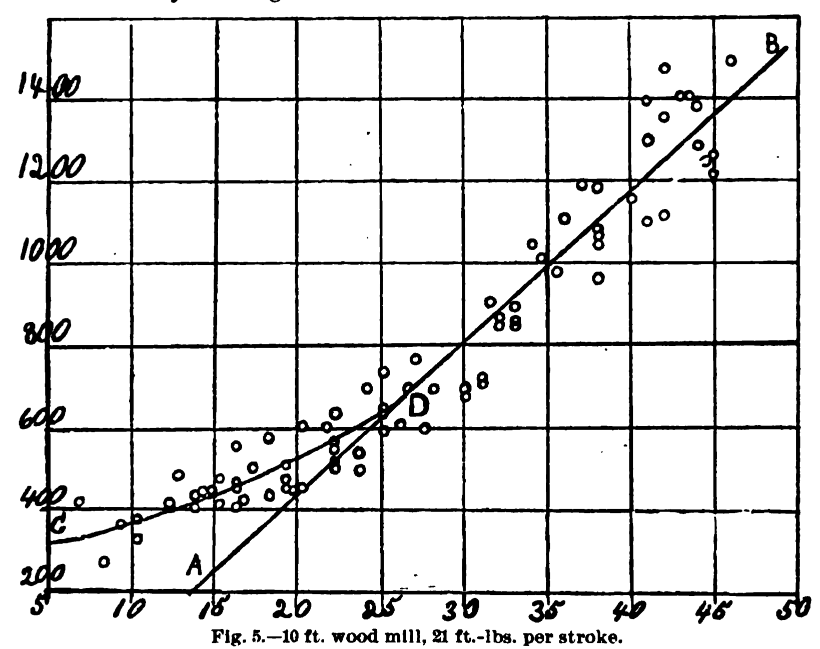


Fig. 5.—10 ft. wood mill, 21 ft.-lbs. per stroke.

minute glass was horizontal, then throwing it into gear by turning the glass upright, and counting the number of strokes while the sand was running through, or in 58 seconds, then disconnecting and reading again. The difference of the readings gave the distance traveled by the wind in the 58 seconds. From twenty to ninety such measurements were made with each mill.

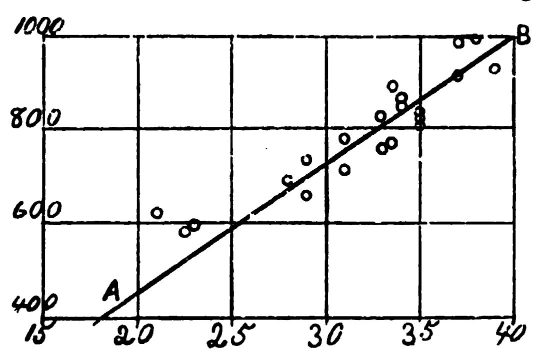


Fig. 6.—10 ft. steel mill, 9.5 ft.-lbs. per stroke.

These measurements are represented diagramatically on Figs. 4 to 11. The center of each little circle or star represents

one measurement, the velocities being laid off as ordinates and the number of strokes as abscisses.

The quantity raised per stroke was found by pumping into a vessel for a time, counting the strokes in that time, and measuring the number of quarts pumped. The number of quarts divided by the number of strokes gave the quarts per stroke. The lift was measured with a tape line. The quantity per stroke multiplied by its weight and by the lift gave the number of ft.-lbs. per stroke,

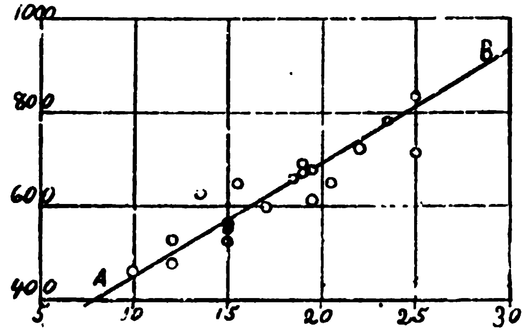


Fig. 7.—8 ft. steel mill, 13.3 ft.-lbs. per stroke.

and from the number of strokes per minute the work per minute was found.

The airmeter was compared with the Kansas University signal service anemometer and found to give readings a little less than that instrument for ordinary pumping velocities.

In Table II are given the dimensions of fans, their angle with axis, number of fans, lift, etc., of some of the mills tested.

Examining any of the figures it is seen that the distance traveled

TABLE II.

No. of Mill.	Material.	Size.	Angle of Fans.	No. Fans.	Gearing.	Lift.	Quantity per Stroke.	Ft. lbs. per Stroke.	Dimensions of Fans.	Ft. lbs. of work per sec'nd when wind is			
										6 Miles.	8 Miles.	10 Miles.	12 Miles.
4	Steel	8 ft.	62°	18	50:15	84	1-9 Gal.	78.	29x12½x5¾	17.5	23.0	30.0	37.0
5	Wood	10 "	52°	96	1:1	50	1-20 "	21.	34x3¼x1¼	6.7	9.6	11.2	12.8
6	Steel	10 "	50°	18	1:1	9	1-4 "	9.5	30½x12½x7¾	3.6	4.6	5.6	6.6
7	"	8 "	68°	18	42:17	16	1-15 "	13.8	29x10¼x4¾	2.9	4.5	6.1	7.7
8	"	10 "	71°	30	1:1	35½	1-8 "	37.	30½x10¼x5	13.4	18.5	23.6	28.7
9	Wood	23 "	51°	136	1:1	47	4-10 "	156.7	105x6x2	24.3	33.8	42.8	54.0
10	Steel	6 "	65°	12	1:1	16½	1-4 "	34.5	26x14¼x5	3.3	5.6	8.6	11.0
11	"	12 "	51°	24	1:1	28	1-8 "	27.2	36x14½x7¾	6.8	10.3	13.6	16.9

by the wind in 58 seconds, for any assumed number of strokes of the pump, varies somewhat; this we would expect on account of the sudden change in the velocity, so that a single measurement

of wind velocity and the corresponding number of strokes of the pump may be in error quite a little; but the mean of a good many such measurements cannot be far from correct. In each figure we have drawn a line AB to represent the mean, or so as to have, at every stroke, about as many circles above as below it.

For ordinary pumping velocities, that is, from 8 to 14 miles per hour, this line is straight. It represents the relation between wind

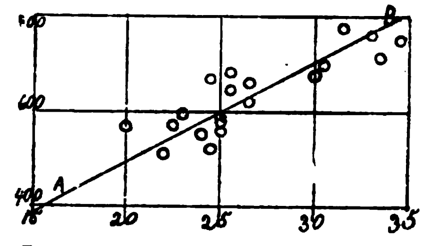


Fig. 8.—10 ft. steel mill, 37 ft.-lbs. per stroke.

Fig. 10.—6 ft steel mill, 34.5 ft.-lbs. per stroke.

vilocity and number of strokes of pump; but since the number of strokes is proportional to the power of the mill, it follows that this line represents the relation between velocity of wind and power of mill.

Here we note a difference between theory and measurement. From equation (12) or (13) it is seen that for any given mill the theoretical relation between power and wind velocity is an equation of the third degree—a cubic parabola, while from any of the

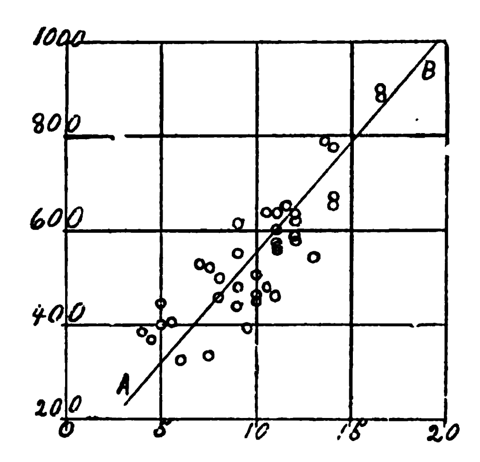


Fig. 9.—23 ft. wood mill. 156.7 ft.-lbs. per stroke.

figures it is seen that for velocities from 8 to 14 miles per hour this relation is an equation of the first degree—a straight line.

Fig. 4 represents two sets of measurements. The center of each star represents a measurement with the pump rod, disconnected 20 feet below the wheel; that is, in the set represented by the stars no water was pumped—no useful work was done. The mean line CD of this set is seen to be nearly parallel to AB and not far distant from it. From this we see that if this mill is running at a given rate and not doing any useful work, it will run at the same rate and do its useful work if the wind velocity be increased three feet per second. Or, stating this in another way, if the mill is doing no useful work at the rate of 25 strokes of the upper part of pump rod in 58 seconds, it will do its useful work in the same velocity at the rate of 20 strokes per 58 seconds.

We see from this one reason for the great variation in the work windmills do in a given wind (Table II, last four columns). The heavily loaded one will do much more work than the lighter loaded

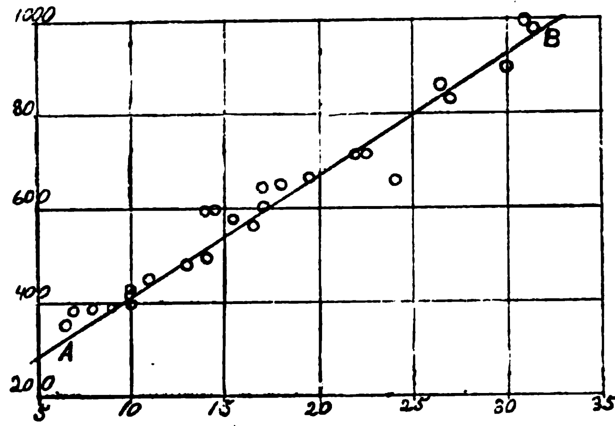
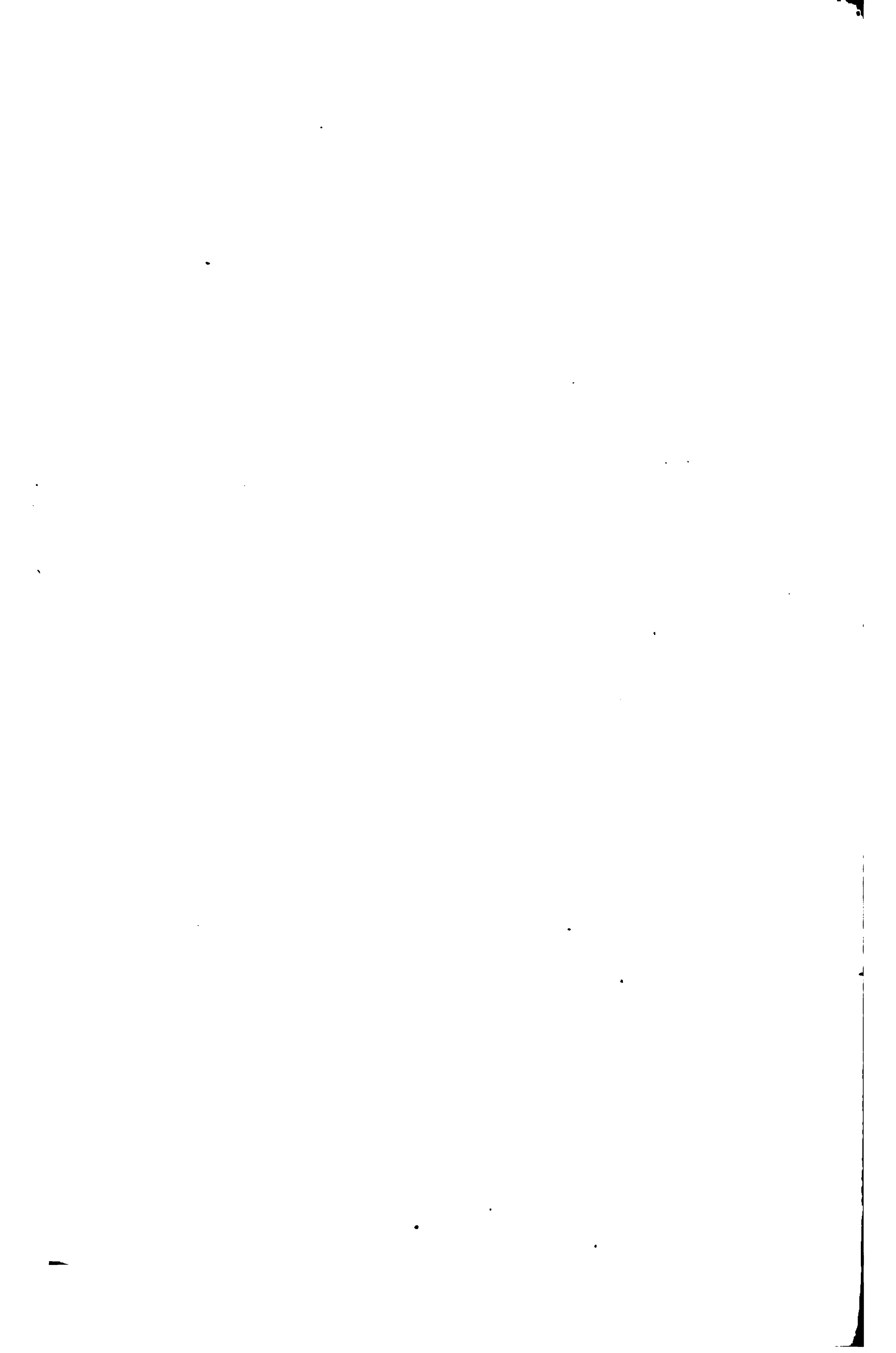


Fig. 11.—12 ft. steel mill, 29.2 ft.-lbs per stroke.

one in a wind strong enough to run it. Equation (10) expresses the relation between the ratio $\frac{v}{c}$ and angle x which the fans make with the direction of the wind for maximum power. If the angle is given, as it is for any given mill, we can find from this equation the value of $\frac{v}{c}$ for any small area of the fans, and from this, how heavy to load the mill. If, then, this ratio be kept constant, as c increases, by increasing the load, the maximum power may be obtained. One device for doing this is to automatically increase the length of stroke as the velocity of the wind increases.

In Fig. 5 the circles vary considerably from the mean line. This is due to the fact that the airmeter was not used in these meas-

urements on a vane, but was kept with its axis in the wind by movements with the hand, and in those for high velocities, due to the wheel of the mill turning partly out of the wind in gusts. The other plates need no explanation.



Two Remarkable New Genera of Diptera.

BY S. W. WILLISTON.

Townsendia, gen nov. (*Asilidæ.*)

Very small species. Head broad, much broader than high. Front very broad above, about three-fourths of the width of the head: narrow below, the sides gently convex; only moderately excavated, nearly bare, with some bristles at the vertex and on the ocellar tubercle. Face narrow, with parallel sides, flat, much receding, not at all visible in profile, with a thin row of bristles on the oral margin, otherwise wholly bare. Antennæ not as long as the head, situated near the middle of the head in profile: first joint shorter than the second, the second about as broad as long; third joint longer than the first two together, gently tapering from near its base: style slender, divaricate, about half the length of the joint. Proboscis short. Thorax moderately convex above, with bristles on the posterior part. Scutellum with a row of thin bristles on its margin. Abdomen elongate, its sides nearly parallel; moderately flattened, bare. Legs moderately stout; the first two joints of all the tarsi a little incrassate. Wings long, narrow toward the base: axillary cell narrow, the anal angle wholly wanting; alulæ wholly wanting; discal cell long and narrow, the penultimate section of the fourth vein a little shorter than the ultimate section; the third vein from the discal cell, separating the third and fourth posterior cells, wholly wanting.

This genus is remarkable for the small size of its typical species, for the broadness of the front, the narrowness of the wings at the base, the absence of the alulæ and the confluence of the third and fourth posterior cells. In one of the two specimens from which this description is drawn, there is a slight angulation of the vein at the posterior part of the discal cell where the missing vein should start from, but even this is wholly wanting in the other specimen. I can not believe that this character should have the importance that it might seem to have, as it is not at all improbable that congeneric species may be discovered in which the neuration is not abnormal. The very broad front, the receding face, and the narrowness of the base of the wings are, I believe, the most essential

characters of the genus. It gives me pleasure to name the genus in honor of Prof. C. H. T. Townsend, who has written so ably on North American diptera.

Townsendia minuta, n. sp.

Male. Black. Front and face thickly white pollinose; mystax white. Antennæ black. Mesonotum opaque yellowish white, with a broad median stripe and a spot on each side brown. Pleuræ thinly whitish pollinose. Abdomen shining black, the first segment whitish pollinose, the pile along the sides of the anterior segments white. Legs yellowish red, the front and middle femora above, the hind femora except the base and tip, a broad ring on all the tibiæ, and the tarsi for the most part, black. Wings grayish hyaline. Length 3½ millim.

Two specimens, Mexico, H. H. Smith. This species is the smallest that I know of in the family of Asilidæ.

I may mention here that the genus *Orthoneuromyia* Will. is in all probability synonymous with *Psilocurus* Loew, though the type species are very different.

Arthrostylum, gen. nov. (*Leptidæ*).

Male. Head broader than the thorax, hemispherical. Front and face narrow, of equal width. Face smooth throughout, without grooves, following the contour of the eyes and not at all visible in profile. Antennæ situated above the middle of the head in profile, with a long, terminal, jointed style; first joint a little shorter than the two following combined, second joint about as long as wide; third joint cordate; style longer than the joints preceding it, composed of five distinct segments of which the first is as wide as long and the others successively increasing in length, the last pointed at the tip and nearly as long as the four preceding it together. Thorax elongate; metanotum prominent. Abdomen long and slender, narrowed at the posterior part of the first (second?) segment, the following joints cylindrical, the hypopygium a little thicker. Legs long; the hind pair much elongated, their femora a little thickened distally; front tibiæ with a single spur, the others with two each. Wings elongate; alulæ rudimentary; fourth posterior cell and the anal cell closed near the margin.

The present genus is nearly allied to *Vermileo*, but is clearly distinct in the dichoptic male, the slender, articulated style, and the closed cells of the wings. The genus is of especial interest as adding another link between the Leptidæ and the Xylophagidæ. The style is distinctly differentiated, it is true, yet the difference

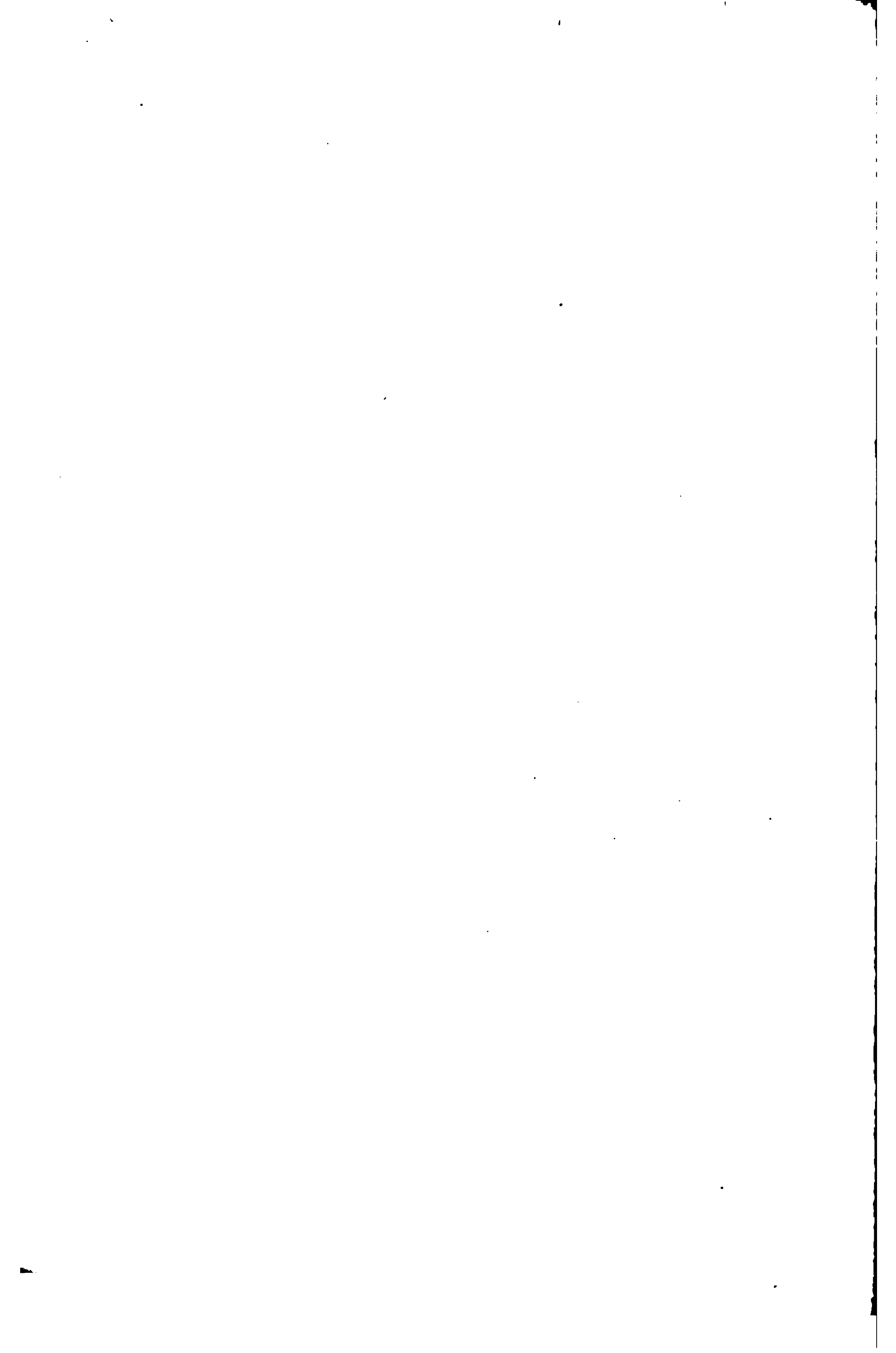
between it and the complex joint of the Xylophagidæ is one of degree and not of kind.

Arthrostylum fascipennis, n. sp.

Male. Front and face both bare, the former opaque brown, the latter opaque light yellow. Antennæ yellow: the first joint for the most part, the upper margin of the second and third joints in part, and the style wholly black. Occiput flat, gray pollinose on the sides. Mesonotum shining reddish yellow, with three black stripes, which are more or less confluent posteriorly. Pleuræ black, scarcely pollinose, yellowish in part above. Abdomen red; first (second?) segment broadly black at its base, in front of which on either side there is a yellow spot: the narrow distal end of the segment has a rounded black boss above: third segment with a smaller blackish eminence distally. Legs light yellow; the distal three joints of all the tarsi, the hind femora except their tip, and the hind tibiæ except base and tip, black; the tip of the second hind tarsal joint is blackish. Wings yellowish hyaline, with two brown cross-bands, the one running from near the costa, more narrowly behind, to the tip of the anal cell; the other filling out the tip of the wing, slightly concave on the proximal border, which crosses the third vein a little before its furcation, and less deeply colored at the immediate tip of the wing. Length 10-12 millim. (the abdomen is curved downward).

One specimen, Xucumanatlan, Mexico, July (H. H. Smith.)

Figures and full descriptions of both the above flies, will shortly appear in the *Biologia Centrali-Americana*.



Involutoric Transformation of the Straight Line.

BY ARNOLD EMCH.

A projective transformation is said to be *involutoric* if every point of the original system J can be interchanged with the corresponding point of the transformed system J_1. This relation exists, for instance, between the projective points (x_1), (x) of a straight line which are related by the equation:

$$x_1 = \frac{ax+b}{cx-a}, \quad (1)$$

where a, b, and c are real numbers. That equation (1) expresses involution, is seen by the value of x which follows from equation (1), i. e.,

$$x = \frac{ax_1+b}{cx_1-a}, \quad (2)$$

Like every general projective transformation of the straight line, the involution has two double points, i. e., places where the values for x and x_1 coincide. These values are found from equation (1) by putting $x_1 = x$:

$$cx^2 - 2ax - b = 0.$$

Whence

$$m = \frac{a + \sqrt{a^2+bc}}{c},$$

$$n = \frac{a - \sqrt{a^2+bc}}{c},$$

m and n designating the values of the double points. According as

$$\triangle = a^2+bc < 0,$$

$$\triangle = a^2+bc = 0,$$

$$\triangle = a^2+bc > 0,$$

the involution is elliptic, parabolic, or hyperbolic, i. e. has imaginary, coinciding and real, or two real double points.

Having stated these general facts we will now investigate the involutoric transformation with respect to its *group properties*.

Writing with Sophus Lie* a general infinitesimal projection of the straight line by

$$d_1x=(a+bx+cx^2)d_1t, \quad (5)$$

where x and t are independent variables, and a, b, c, constants, we can substitute in equation (5) an expression containing the values of the double points, m and n, thus we get

$$k(x-m)(x-n)=a+bx+cx^2.$$

The differential equation (5) may therefore be written in the form:

$$\frac{dx_1}{(x_1-m)(x_1-n)}=dt, \quad (6)$$

x_1 designating any variable in the transformed system and satisfying the condition, that for $t=o$, $x_1=x$. The reason for this condition is evident if it is remembered that for $t=o$ the identical transformation results.

The solution of the differential equation (6) gives

$$x_1=\frac{(me^{nt}-ne^{mt})x-mn(e^{mt}-e^{nt})}{(e^{nt}-e^{mt})x+me^{mt}-ne^{nt}} \quad (7)$$

This equation represents the *one-termed* group of those transformations which leave the points m and n invariant. Among the transformations of this group is also the involution leaving the points m and n invariant, and they are determined by the condition

$$me^{nt}-ne^{mt}=-me^{mt}+ne^{nt}, \text{ or } (m-n)e^{mt}=-(m-n)e^{nt},$$

$$\text{or } e^{(m-n)t}=-1$$

Taking the logarithms of both sides, we have

$(m-n)t=\log.(-1)$, or finally $t=\frac{(2k+1)\Pi i}{m-n}$, (8) where k may be any real and integral number. Equation (8) shows that in this case there is no differential for t, i. e., equation (6) has no meaning. Hence there is only one involution among the projective transformations leaving two points invariant. This result, well known in geometry, in connection with equation (5) shows that there is no infinitesimal transformation which by integration leads exclusively to involutions.

Thus we may say: *The system of involutions on a line has no infinitesimal transformation and, hence, does not form a group.*

Having thus shown that there are no involutoric transformations transforming involutions into involutions we may ask whether there are any general projective transformations with this property, and if so, find these transformations.

*Vorlesungen ueber continuierliche Gruppen.

In the case of involution

$$x_1 = \frac{ax+b}{cx-a}.$$

The operation of a general projective transformation,

$$x_2 = \frac{a_1x_1+b_1}{c_1x_1+d_1} \quad (9)$$

gives:

$$x_2 = \frac{(a_1a+b_1c)x+(a_1b-b_1a)}{(c_1a-d_1c)x+(c_1b-d_1a)}, \quad (10)$$

i. e., a general projective transformation. Obviously this transformation is involutoric if $d_1 = -a_1$, and $b_1c = c_1b$, or $c_1 = \frac{b_1c}{b}$.

Substituting these expresions for d_1 and c_1 in (9) the projective transformation which does not alter the involutoric transformation becomes

$$x_2 = \frac{a_1x_1+b_1}{-\frac{b_1c}{b}x_1+a_1},$$

or if we put $-\frac{b_1}{a_1} = k$,

$$x_2 = \frac{bx_1-bk}{kcx_1+b}. \quad (11)$$

In this case equation (10) assumes the form:

$$x_2 = \frac{(ab-bck)x+(b^2+abk)}{(ack+bc)x-(ab-bck)}, \quad (12)$$

and, in fact, represents an involutoric transformation. If this equation represents all involutions it must be possible to choose three arbitrary real numbers A, B, and C, such that for constant values of a, b, and c, and any real value of k between two limits, however large,

$$\begin{aligned} ab-bck &= A, \\ b^2+abk &= B, \quad (13) \\ ack+bc &= C. \end{aligned}$$

These three equations cannot exist together. We may choose any of the three numbers at random, but then the value of k is determined, and so are the values of the two other numbers. The system of involutions represented by equation (12) consists, therefore, of only a special class of involutions. This is also seen from the fact that there occurs but one parameter in (12), while the general involution has two. The system of transformations (11) has one parameter, so that we may conclude

There are ∞^1 projective transformations which transform a certain involution into ∞^1 other involutions.

These ∞^1 projective transformations form a group, for

$$x_1 = \frac{bx - bk}{ckx + b}$$

transformed by

$$x_2 = \frac{bx_1 - bk_1}{ck_1x_1 + b}$$

gives:

$$x_2 = \frac{bx - b\frac{bk + bk_1}{b - ckk_1}}{c\frac{bk + bk_1}{b - ckk_1}x + b}.$$

To each transformation can also be found the inverse in the same system

$$x = -\frac{-bx_1 - bk}{ckx - b}, \text{ or putting } k \text{ for } k.$$

$$x = \frac{bx_1 - bk}{ckx_1 + b}.$$

The invariant points of this group are

$$x = \pm i\sqrt{\frac{b}{c}}, \text{ i. e., elliptic if } b \text{ and } c \text{ are of the same sign.}$$

Taking the limiting case $k = \infty$ in equation (11) the transformation becomes

$$x_2 = -\frac{b}{cx_1}. \quad (14)$$

Transforming the general equation of involution by (14) we have:

$$x_2 = \frac{bcx - ab}{-cax - bc}.$$

Hence: *There is one and only one involutoric transformation which leaves the character of an involution unchanged.*

We will now give an example in which the connection between the analytical method in this article and the synthetical treatment of the similar subject by Professor Newson in this number of the QUARTERLY can easily be shown. Consider the special involution

$$x_1 = \frac{b}{cx}$$

with the double points $+\sqrt{\frac{b}{c}}$ and $-\sqrt{\frac{b}{c}}$.

After a transformation (11) this involution becomes:

$$x_2 = \frac{ckx - b}{cx - ck}.$$

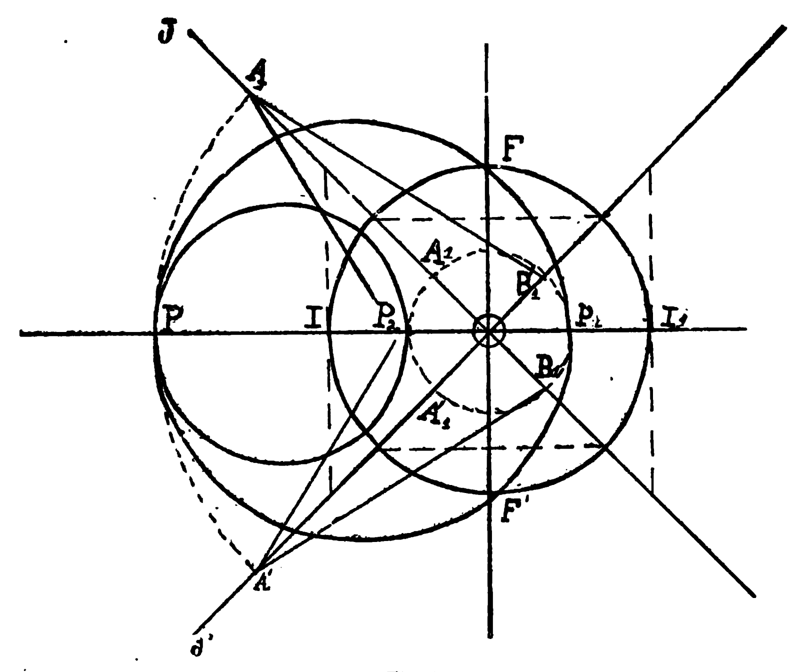


Fig. 1.

Now let us represent $\sqrt{\frac{b}{c}}$ and $-\sqrt{\frac{b}{c}}$ by the points I and I_1, respectively, and draw a circle with II_1 as diameter. Each circle normal to it and having the center on the straight line II_1, intersects this line in a pair (P, P_2) of the involution, so that

$$PO \times P_2O = \frac{b}{c}, \text{ or } xx_1 = \frac{b}{c}$$

Making $OP_1 = OP_2$ (Fig. 1), the elliptic involution

$$xx_1 = -\frac{b}{c}$$

results, and its points are obtained by the intersection of all the circles passing through F and F'.

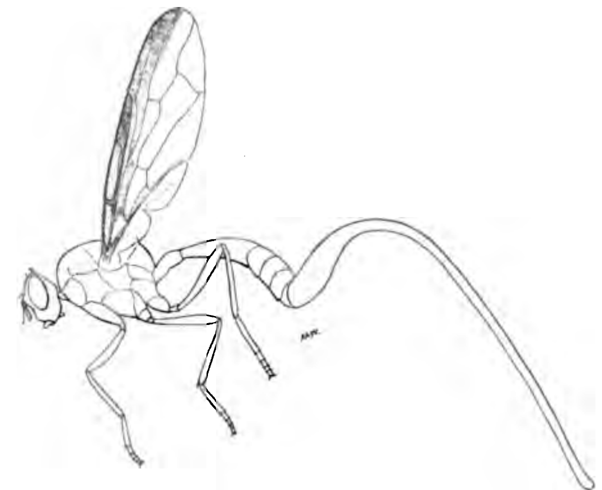
To conform with the representation in Prof. Newson's article, draw any two lines through the original O and symmetrical to the axis II_1. Let these lines be J and J'. With O as a centre and OP, OP_2, and OP_1 as radiï describe circles intersecting the rays J and J' in the points A, A'; A_1, A_1'; and B_1, B_1' respectively. A and A_1', or A_1 and A' are corresponding points in the hyperbolic involution, while the pair A, B_1', or A', B_1 belongs to the elliptic involution. It is easy to prove that the rays connecting corresponding points,

as AA_1', $A'A_1$ are the tangents to the so-called *conic of the transformation* which, in this case, is a hyperbola having I and I_1 as its vertices. The same is true of the rays connecting A and B_1', A' and B_1; they are the tangents to the conic of the elliptic involution. This conic is also a hyperbola and, like the hyperbola of the hyperbolic involution, has the rays J and J' as asymptotes. It has the chords of the circle with II_1 as a diameter included by the rays J and J' and parallel to II_1 as tangents at the vertices.

On Toxotrypana of Gerstæcker.

BY W. A. SNOW.

Concerning the true relationship of this remarkable American genus there has been much diversity of opinion. Osten Sacken, Bigot and Mik have agreed in assigning it to the Trypetidæ. Van der Wulp mentions it as occupying a place among the Ortalidæ, and this view is held by Roeder, who had the advantage, not possessed by most of the other commentators, of examining a specimen. Loew, who judged from Gerstæcker's description and figure alone, recognizes the affinities of the genus with both the Trypetidæ and Ortalidæ, and places it doubtfully in the group Pyrgotinæ of the latter family. In this group, *Toxotrypana* finds very dissimilar associates. Especially does it differ from the other forms placed here in the shape of the head and of the antennæ, the presence of ocelli, the long, drawn out anal cell, the bareness of the body and the monstrous ovipositor, and in an even more important character, the presence of lower fronto-orbital bristles.



Toxotrypana curvicauda Gerst.

In the museum of the University of Kansas are two female specimens of *Toxotrypana curvicauda*, Gerst. received in the Gaumer collection from Yucatan. One of the specimens was covered with mold and in poor condition, the front being denuded of bristles.

The other, however, has been well preserved, and shows plainly a row of lower fronto-orbital bristles. These bristles are of about the same size as the largest of the vertical ones, and only differ from the similar bristles of all Trypetidæ in not being distinctly nearer to the eyes than are the upper fronto-orbital bristles. The supposed absence of these bristles has been the principal objection to the location of the genus among the Trypetidæ. Loew has significantly remarked that it might be the case that "in conformity to the striking shortness of all the hairs of the body, the lateral bristles of the front have disappeared." Even though I had not found these bristles present in our specimen, I should have contended that *Toxotrypana* finds its most natural position among the Trypetidæ, in proximity to *Acrotoxa.* The characters which show the divergence *Toxotrypana* from the true Pyrgotinæ are links of relationship to *Acrotoxa* and its allies. The shape of the head, antennæ and palpi; the presence of ocelli; the form of the anal cell, and the not flattened ovipositor are similar in *Toxotrypana* and *Acrotoxa.* These two genera are held asunder, it is true, by several important characters, which are to be found mainly in their chætotaxy, in the course of some of the veins of the wing, in the shape of the abdomen and in the curvature of the ovipositor. The extreme length of the ovipositor in *Toxotrypana* need not be considered an important character, since in certain specimens of an undetermined species of *Acrotoxa* which I have seen in Dr. Williston's South American collection, that organ attains a length equal to that of the whole remaining body of the insect. The shape of the ovipositor in Gerstæcker's genus is therefore of higher distinctive importance than is its length.

The branching of the second longitudinal vein in *Toxotrypana* can only be regarded as a character of small importance. Mr. Roeder tells us that in his specimen one wing showed an incomplete branch, which in the other was represented by a small brown cloud only. One of the specimens before me shows no vestige of a furcation of this vein, while in the other specimen each wing has a stump extending a part of the distance between the second vein and the costa.

It is true that the auxiliary vein does not enter as steeply in the costa as it does in most Trypetidæ, but it disappears before reaching the costa and is replaced by a brown cloud as is generally the case in this family.

A comparisons of the descriptions of Gerstæcker, Bigot and Roederwith my specimens gives the assurance that the thoracic colorational differences in those descriptions are but variational, and

therefore *Mikimyia furcifera* Bigot is but a synonym of *Toxotrypana curvicauda* Gerstæcker, as suspected by Mik and Roeder.

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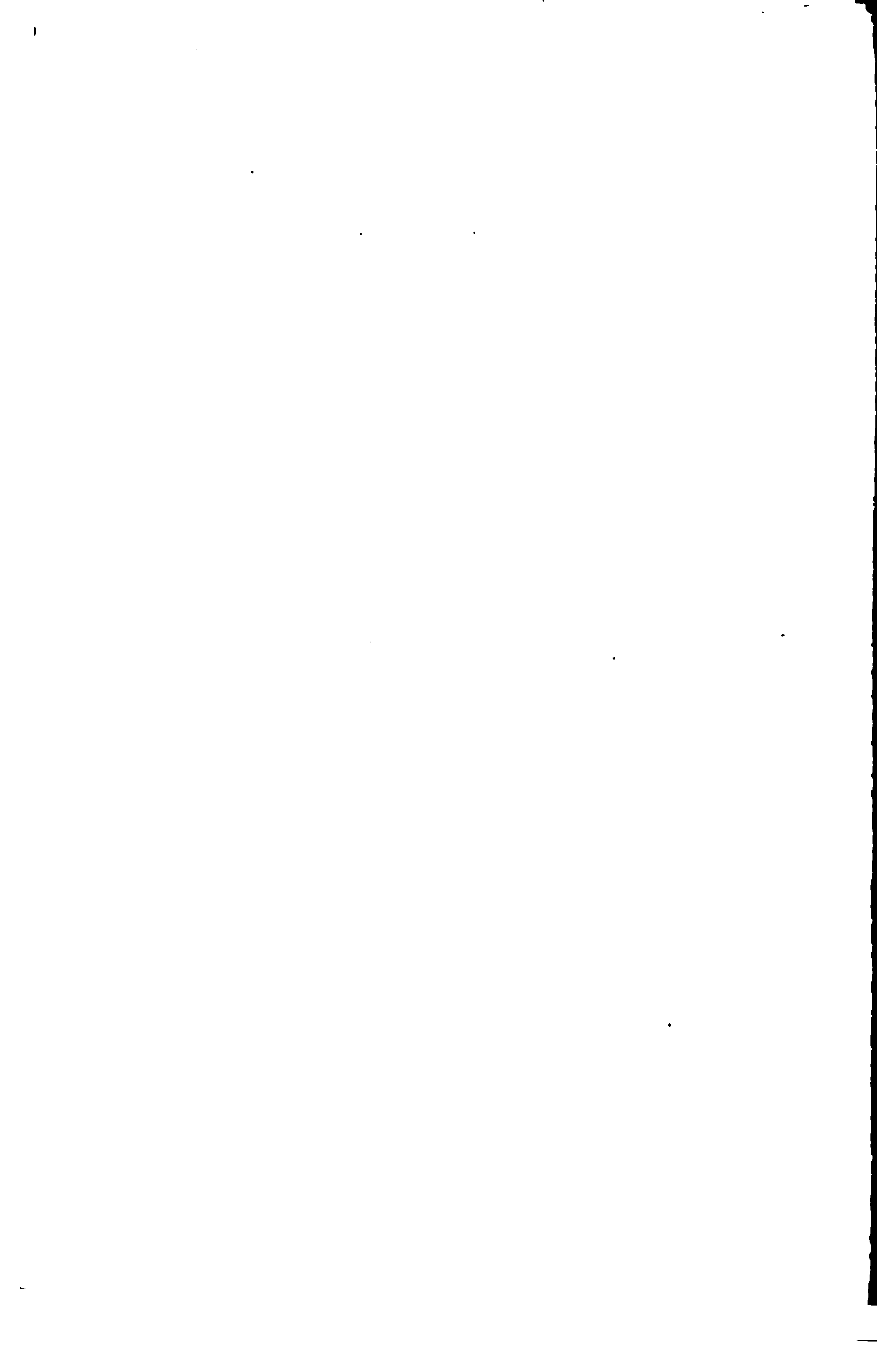
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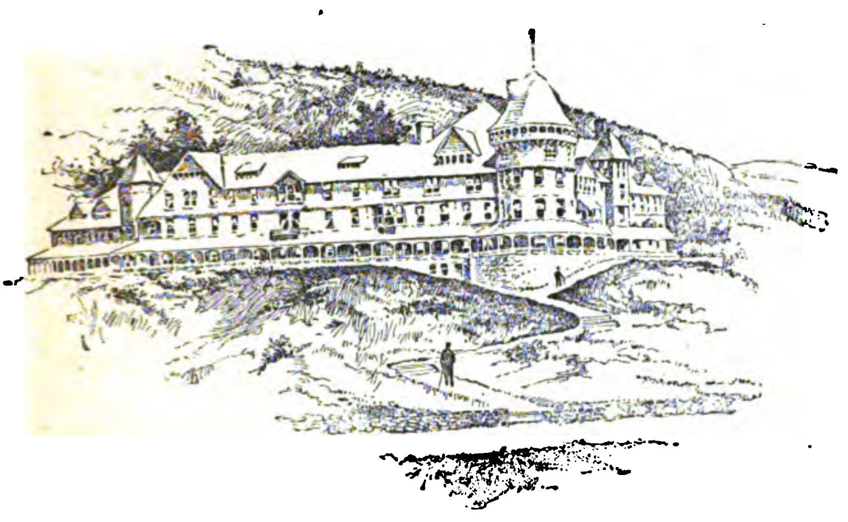
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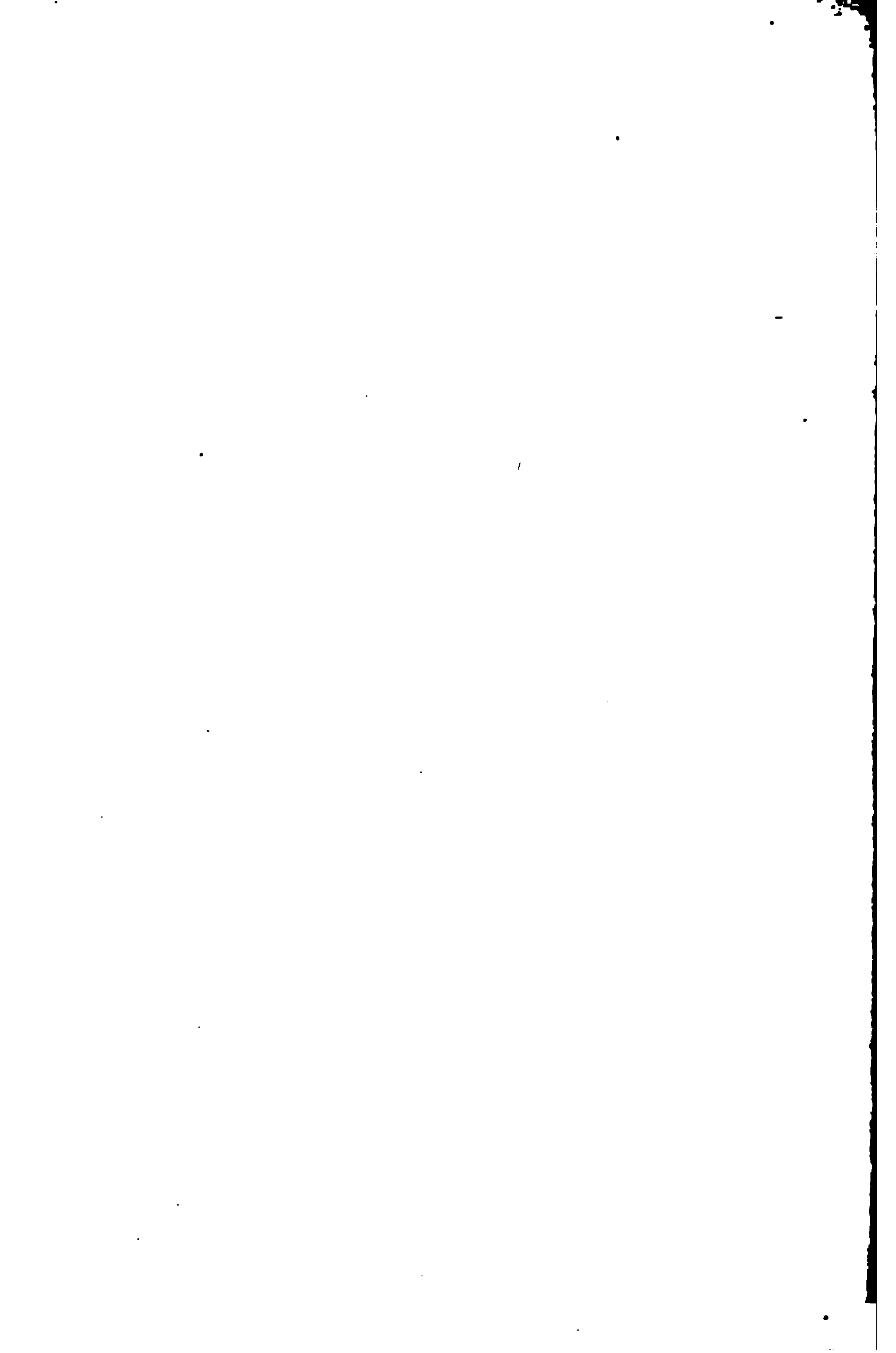
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CONTENTS

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KANSAS UNIVERSITY QUARTERLY

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A Curvimeter.

BY W. R. CRANE.

Integrating machines, or engines, as they are often called, have been in use for so many years, and have filled all the requirements so satisfactorily, that they are now considered an indispensable article for the engineer's office and the draughtsman's table. The ordinary planimeter is so well known and is so widely used that it is unnecessary to speak further of it here. There is, however, another class of integrating instruments, which is not so widely known as the planimeter, nor has the principal of the same been so thoroughly investigated. This class of instruments is commonly known as the curvimeter.

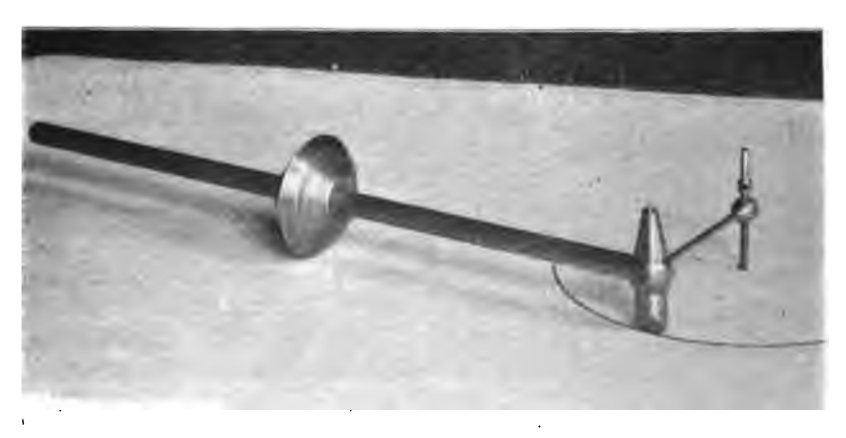
Up to the present time no accurate or trustworthy instrument has been devised for the purpose of measuring the length of figures of one dimension, such as lines and curves of every description. The possibility of such an instrument is suggested by the simple and excellent form of planimeter, known as the "stang" or "hatchet" which has recently gained such wide-spread popularity, and it was, in fact, this instrument which led to the investigations that ended in the instrument to be described.

As is well known, there are a large number of problems, such as finding the length and area of curves between their co-ordinate axes and certain prescribed limits, which up to a few years ago could be solved only by means of differential and integral calculus. At the present time there are quite a number of planimeters—area measurers—to be found on the market, the one above mentioned being the latest, but as far as I am able to ascertain there is not a single instrument, which will more than approximate the length of curves.

The following is a description of a simple curvimeter and the method of operating the same. It consists of a rigid steel shaft, accurately turned and polished, upon which a sharp rimmed steel

wheel is fitted, not so closely, however, but that it may rotate and slip from one end of the shaft to the other. To prevent any binding of the wheel upon the shaft, the wheel is made quite thick through the centre, the length of the hub being equal to the radius of the wheel. At one end of the shaft is attached a cross-bar, forming a T, each arm being equal to the radius of the wheel. The extremity of one of the arms of the cross-bar is shaped into a knife-edge, the extremity of the other arm being a point, which is furnished with a shoulder to prevent its sinking beyond a certain point in the material upon which it is worked. The knife-edge and point both lie within the plane which passes through the centre of the shaft. Passing through the cross-bar and shaft, and at right angles to both, is attached a third arm. Through the extremity of this third arm and parallel to the cross-bar is a set-screw, either end of which may slide upon the surface operated upon. The object of the set-screw is to keep the cross-bar always perpendicular to the plane upon which it moves. The upper surface of the shaft is graduated, either in the French or English system of units, as desirable; if theEnglish system is used the graduation should read at least as low as 1-64 of an inch.

The instrument as it stands complete--the sharp rimmed wheel upon the shaft, the three cross-arms, and the set-screw holding the point and knife-edge perpendicular to the plane upon which it stands, may be seen in the cut.



Crane's Curvimeter, Fig. 1.

To operate the instrument we proceed as follows:—Take the cross-bar firmly between the thumb and first two fingers, then place

the knife-edge upon the curve—a mark on the highest point of the knife-edge coinciding with the point from which measurement is to begin on the curve. The position of the wheel on the shaft being noted, the curve is then traced with the knife-edge, which is kept wholly within the curve and therefore tangent to it. The shaft would therefore be kept tangent to the curve also, the wheel rolling slowly and smoothly upon the paper, changing its direction of motion as the shaft changes its position. When the other extremity of the curve is reached the wheel is found to have changed its position, and this change of position, or the difference between the initial and final positions of the wheel on the shaft give the length of the curve plus a correction which will now be explained.

The only use of the point is in finding the correction, and after the correction has once been found for an instrument the point is no longer of any value. As will readily be seen, when the shaft is rotated about the point as a centre the wheel will tend to work off the shaft. This tendency to work off is due to the friction of the knife-edge upon the paper as it changes its direction of motion. By rotating the shaft about the point as centre through a given angle and noting the distance the wheel works off, then rotating back through the same angle, and again noting the distance the wheel has moved, we know at once, if these two distances are not equal, that the cross-bar is not perpendicular to the surface upon which it stands. This can be remedied by adjusting the set-screw until the two distances above mentioned are equal, then the cross-bar must be at right angles with the plane upon which it stands. These two distances can be made equal regardless of the size of angle through which the shaft is rotated, and when the above named condition has been reached the correction for the angle through which the instrument is rotated is known, then the correction for one degree readily follows.

We then have the correction for the instrument for one degree, a constant quantity, and in measuring a given curve, the angle through which the instrument rotates being known, we can easily find the correction for the curve. Subtracting this correction from the length noted on the shaft by the change of position of the wheel, we have the exact length of said curve.

The following table gives the results of a series of experiments performed to obtain the correction of an instrument constructed by Mr. J. E. Crosby. The first column represents the number of observations taken; columns two and three give the distance worked off while rotating through 90°; and column five represents the positions of the wheel from the cross-bar at the beginning of each observation.

Number of observations.	Rotation to the right 90°. Distance worked off.	Rotation to the left, 90°. Distance worked off.	Total distance worked off, Sum of columns I and II.	Length of shaft from cross-bar.
1	.020833	.020833	.041666	2.3 inches
2	.020833	.020833	.041666	2.9 "
3	.020833	.020833	.041666	3.8 "
4	.020832	.020832	.041664	5 3 "
5	.020830	.020831	.014661	6.2 "
6	.020826	.020829	.014655	9.3 "
7	.020829	.020327	.014756	9.5 "
8	.020831	.020830	.014661	10.2 "
9	.020832	.020832	.014664	10.6 "
10	.020833	.020833	.014666	11.3 "
11	.020833	.020833	.014666	11.6 "

The correction obtained by taking the average of all these eleven observations is .00023125 inches, for every degree through which the shaft rotates.

The correction is a constant quantity for each instrument; whether it is the same for all similar instruments regardless of size I am unable to state, as the above mentioned instrument is the only one which has been constructed as yet.

The dimensions of the instrument experimented upon are as follows:

Shaft 12 inches long, diameter of wheel, length of cross-bar, and set-screw 2 inches. Distance of set-screw from centre of shaft 2¾ inches. Width of wheel through hub 1 inch. Size of shaft ⅜ inch. Instrument not graduated.

The theory of the Curvimeter may be proved as follows:

Taking $a = x - c \sin \phi$, and $b = y + c \cos \phi$, as co-ordinates of the centre of curvature, then differentiating with respect to s, by means of the properties of the Differential Co-efficient of an arc (rectanglar co-ordinates) and the Radius of Curvature we have:

$$\frac{da}{ds} = \frac{dx}{ds} - \frac{dc}{ds} \sin \phi - c \cos \phi \frac{d\phi}{ds} = -\frac{dc}{ds} \sin \phi.$$

$$\frac{db}{ds} = \frac{dy}{ds} + \frac{dc}{ds} \cos \phi - c \sin \phi \frac{d\phi}{ds} = \frac{dc}{ds} \cos \phi.$$

And directly from these two equations

$$\left(\frac{da}{ds}\right)^2 + \left(\frac{db}{ds}\right)^2 = \left(\frac{dc}{ds}\right)^2 \text{ or}$$

$$\left(\frac{dD}{ds}\right)^2 = \left(\frac{ds}{dc}\right)^2$$ D, representing the length of the arc of evolute measured from a fixed point.

Therefore

$$\frac{dD}{ds} = \pm \frac{dc}{ds} \qquad \text{Hence } \triangle D = \pm \triangle c.$$

That is, the difference between any two radii of curvature is equal to the length of arc included between these radii on the arc of evolute.

The Sands of the Kansas River Valley.*

BY M. Z. KIRK.

Some months ago my attention was called to the great difference in character of the soils of the Wakarusa valley and those of the Kansas river valley. The former usually are black and plastic while the latter are loose and sandy and have a much lighter color. The Kansas river valley from Kansas City to Manhattan has an average width of about four or five miles and is skirted on either side by bluffs from 100 feet to 225 feet in height. This lower portion of the Kansas river traverses the extreme southern extension of the great glacial drift area of America. An examination of the country lying north of the river for several miles, even so far as Atchison and Whiting, shows that here and there throughout the whole glaciated area of the state large masses of sand are frequently found in connection with the glacial soils. In fact the great beds of loess so abundant particularly along the Missouri river valley, and in other parts of America with which the writer is more or less familiar, frequently have the fine clay and silt washed out from them by the surface waters leaving almost pure sand behind. Such conditions are often observed in central and southern Iowa and probably in many other places. Here the washing away of the clay and silt frequently leaves masses of sand on the uplands far removed from the main drainage tributaries. These are blown here and there by the wind and collect in great sand dunes often covering several square miles in extent with the sand forming so large a proportion of the soil that the country roads are almost impassable for heavily laden vehicles. With these points in mind, it occurred to the writer that possibly portions at least of the sands of the Kansas river valley had originated in a similar way. Upon investigation the following conditions were found to obtain.

On both sides of the river, but more particularly on the north side almost all the way from Kansas City to Lawrence, great masses of loess were found to exist along the bluffs, generally with a streamer reaching downward into the valley, usually forming a ridge from five to ten feet higher than the general level and from forty rods to half a mile in width and often a mile or more in

[*From a preliminary paper giving a few results obtained by the University Geological Survey of Kansas.]

length. Such streamers generally trend a little west of north and east of south, but there is not a perfect regularity in this. These conditions are very prominent in the vicinity of Lawrence. About two miles east from the Bismarck fair grounds on the left bank of Mud creek the loess bluffs are particularly prominent and represent in miniature the conditions so notable in the vicinity of St. Joseph and other points along the Missouri river where the heavy loess bluffs are weathered away leaving the sharp and rugged spires and pinnacles recalling the conditions of the Bad Lands of Dakota and Wyoming.

An overland trip was made from Lawrence to Junction City for the purpose of studying the conditions farther up stream. The glacial ridges with their streamers of sand similar to those just described as occurring around Lawrence were found in great abundance between Lawrence and Topeka. About nine miles northwest of Lawrence, at the foot of the bluff on the north side, a bed of glacial sand is found, the sands from which extend down into the valley and are very noticeable in the soil. About two miles southwest of Thompsonville the glacial clay, fragments of crystalline rocks such as greenstone and quartzite, and sand alternate and intermix. From this bluff the great sandy streamers can be traced to the valley below. Local areas of considerable extent can be seen to exist in patches in the broad level fields. Farther up the river, near Rossville, great quantities of sand and gravel were found. The hills to the northwest of St. Marys are covered with a heavy mantle of sand and gravel quite thoroughly mixed with small quantities of yellow clay. The beds contain numerous examples of small boulders and gravel, quartzite, greenstone, and flint fragments. Each year the clay and finer silt is washed away leaving the coarser gravel behind, as is well illustrated by the gravel beds at St. Marys.

Speaking in a general way it may be said that almost all of the northern bluffs of the river from Kansas City to Topeka are covered with such loess deposits, while many places along the southern bluffs also show the same conditions. Above Topeka the loess deposits gradually diminish in relative abundance, and it is an interesting fact to note that the relative proportion of sand in the river valley likewise decreases; so that before Manhattan is reached, particularly on the southern side of the valley, the sand has decreased to such an extent that its absence is very noticeable. Here and there, also, throughout the whole distance, are areas of black soil recalling the soil of the Wakarusa valley, to which it corresponds in almost every respect, and upon careful examination it

is usually found that there is a corresponding absence of loess along the adjacent bluffs. Beyond Manhattan the sand apparently becomes more abundant and farther up stream the whole valley seems to be almost entirely composed of the sandy soil. But it may be noted that as the deposits of loess decrease westward until they finally disappear we are approaching the areas of the Dakota sands and the Tertiary sands still farther to the west from which the Kansas river and its tributaries have obtained large quantities of sand to be carried downward and strewn along its banks and river bottoms.

It has long been known that the character of the soil in the Kansas river valley, particularly from Manhattan to Kansas City, is unsurpassed in productiveness. The idea so widely prevalent that a soil in order to be rich must also be black, is strongly negatived by the Kansas river soils. If they are partially composed of glacial materials it is all the more readily understood why they should be so productive. The feldspar materials brought down from the far north by the glacial movements have supplied an excessive amount of mineral fertilizers, particularly potash, and the steady wearing away of the loess banks along the bluffs has constantly added richness to the soils of the valley below; so that although they generally have an absence of the much coveted black color, yet they possess in large quantities all the ingredients and properties necessary for a wonderfully productive and lasting soil. Some years ago Prof. R. Ellsworth Call* called attention to the character of many Iowa soils which correspond so closely in origin and nature with the Kansas river soils that we may quote him as follows:

"Color, too has little to do with deciding finally whether a soil will be fertile. Usually all earths which are dark colored or black —a condition largely due to the amount of carbonaceous material derived from decayed vegetation—are considered fertile. It is true that common consent places all such samples among the fertile soils, but it by no means follows as a necessary deduction. So, too, that light drab or ashy-colored soils lack the elements of fertility, is a notion which observation and experience alike negative. The most fertile of the Iowa soils is the loess, a peculiar and very fine marl covering many hundred square miles along the Mississippi and Missouri rivers as well as the higher lands along the Des Moines. It is a soil the color of which would condemn it for agricultural purposes, but is of exceptional value for cereals,

* "The Chemistry of Soils," Science (1) Vol. XX, pp.

and is peculiarly adapted to the growth of fruit. It is finer than any other soil in the state."

It may be said, then, in conclusion, that there is abundant reason for believing that a very considerable proportion of the sands in the Kansas river valley below Manhattan, and particularly below Topeka, originated in the deposits of glacial loess so abuudant along the bluffs and uplands, and that a correspondingly small proportion has been carried downwards from the great Dakota and Tertiary sandy areas to the west. It should not be understood, however, that this article in any way opposes the generally accepted opinion that the Kansas river, with its tributaries, has carried in past times, and is today carrying, large quantities of sand from those western areas and strewing them along its banks and valleys.

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The writer will be glad to receive additions and corrections to the foregoing list for incorporation in a second part, which will contain a classification of the papers by families. It has not been attempted to include all papers published upon North American Diptera but only those of interest or importance to the systematist. Undoubtedly some important references among the Experiment Station Bulletins have been omitted.

"Horsebacks" in the Kansas Coal Measures.

BY W. R. CRANE.*

"Horsebacks," as certain peculiar formations occurring in the Kansas Coal Measures are called, are not only interesting when considered geologically, but also play a most important part in the economy of coal mining. The name "horseback" or "hogback" was probably applied to these formations on account of their peculiar rounded upper extremity, yet it might have been taken from the term "horse" as applied to an enclosed mass of "country" rock in a metalliferous vein.† At various times and in various places the following terms have been applied to these formations, "horse," "want," "trouble," "nip," etc., and most applicable of all "clay veins."‡

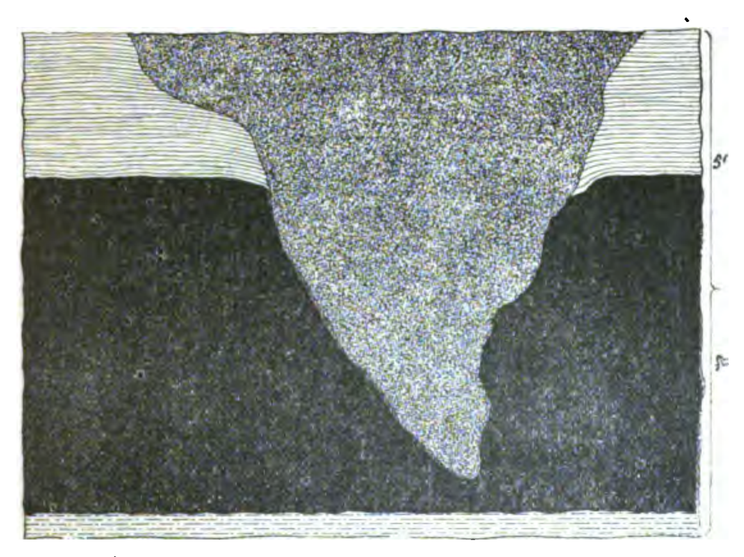


Fig. 1—Horseback protruding into the coal from above.

LOCALITIES.—"Horsebacks" are by no means confined to Kansas coal fields, but are found in great abundance in the Coal Measures of Pennsylvania, in particular, and in several adjoining

* A preliminary paper giving a few results obtained by the University Geological Survey of Kansas.
† "Ore Deposits," J. A. Phillips, p. 35.
‡ Pa. Second Geol. Sur. Rep., H., p. 27.

states. In fact there are few, if any, coal-mining localities known in America where they do not occur. In our own state they have been studied by the writer in Cherokee and Crawford counties only, so that it is not known to what extent they may be found in the other mining districts of the state.

NOMENCLATURE.—There seems to be a slight lack of harmony in the usage of the above mentioned terms, especially in different states. For instance, in Kansas the term "horseback" is applied strictly to clay-filled, almost vertical fissures which pass through the coal. In Pennsylvania such clay-filled fissures are called "clay veins." Again in the coal fields of this state a dipping down or a bulging up of the strata from above or below the coal, especially the former, is called a "roll in the slate" of the roof. This phenomenon is given the name "horseback," "nip," "want," etc., in the Pennsylvania collieries. The nomenclature adopted by the Pennsylvania miners and geologists seems the most applicable and will therefore be used in this paper.

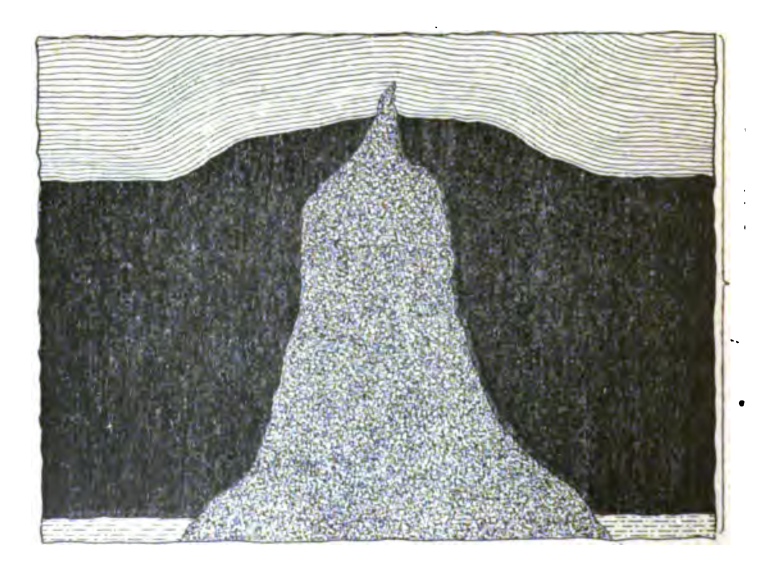


Fig. 2.—Horseback showing the upward bulging of the coal and shale.

FORMS OF THE FISSURES.—The "horsebacks" or "clay veins" of Kansas seem to be clay-filled fissures formed after the coal was consolidated. They trend in many directions with apparently no regularity. So far as has yet been observed the direction of individual fissures is wholly irregular, but a line trending northeast and southwest seems to strike a larger proportion of them than would a line in any other direction. The fissures usually are nar-

row, averaging perhaps less than five feet as they are found in the coal. They generally pass through the coal and into the shale above, often reaching almost to the surface, but sometimes thinning out to a mere fissure with no apparent thickness only a few feet above the coal. At other times they do not pass entirely through the coal from below, while still again in much rarer cases they seem to be passing downwards from above. Fig. 1. The downward extent of the fissures in most cases is entirely unknown as coal mining operations do not follow them much below the coal itself.

The fissures often bifurcate, or in some cases split into more than two branches, in both horizontal and vertical directions. Their hade is generally but a few degrees from the vertical, and perhaps a large majority of them hade less than thirty degrees from the vertical, but occasionally one is found making an angle of as high as eighty or eighty-five degrees.

NATURE OF THE WALLS.—The walls of fissures are usually rough and ragged, but sometimes are smooth and polished presenting well formed slickensides. This property is even present in the walls of the coal seam itself in some instances, although not so strongly marked as the fissure walls in the shales and clays. Usually the coal walls are rough and jagged with the irregularities

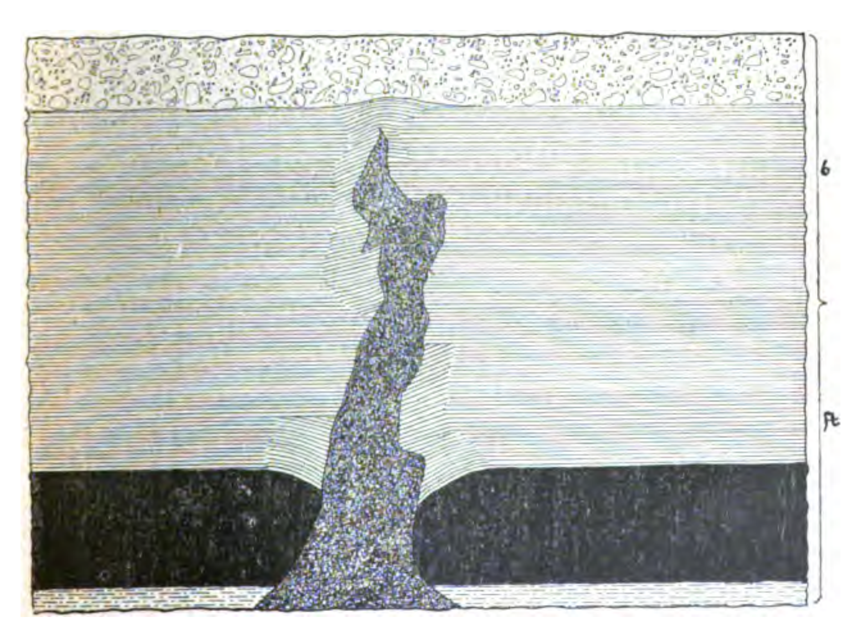


Fig. 3.—Horseback showing bulging of strata due to lateral compression.

of one side corresponding closely to those on the other implying that the coal had been broken asunder and separated horizontally, while in rare cases a vertical displacement of a few inches or a foot has taken place. Frequently angular fragments of coal have lodged in the clay filling as though it had dropped from the roof wall during the process of filling.

The horizontal position of the coal or shale strata usually has not been disturbed, but in some cases the strata adjacent to the fissure have been lifted up as though after the fissure was formed and filled with the clay a lateral compression occurred with a slight bulging upwards of the ends of the strata in contact as represented in Fig. 2.

Extent and Frequency of the Fissures.—The extent of the fissures, both vertically and laterally, can hardly be determined. The processes of mining operations are confined to so small a distance vertically that the fissures cannot be studied below the coal to any considerable extent, and the tunnels and driveways are made in such a way and the mines located in such positions that it is also very difficult, or in fact impossible, to find the lateral extent of many of the fissures. It is known that some of them extend continuously for half a mile or more, but beyond this it is largely guess work, although it is probable that many of them extend much farther.

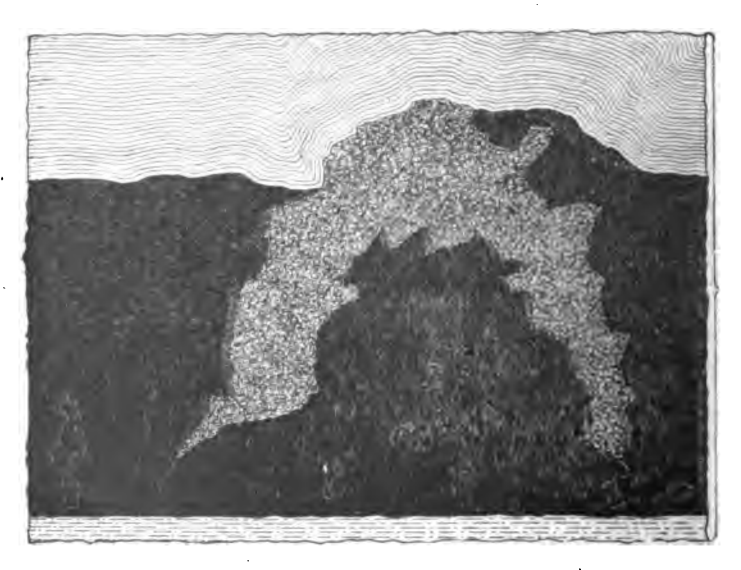


Fig. 4.—Horseback showing displacement of coal and shale, and fracture of coal.

Contents of the Fissures.—The contents of the fissures consist in most cases of fire clay which has doubtless been produced by

having the iron, and perhaps other constituents, leached out from the shales and adjacent coal. In a few instances the clay filling contains fragments of coal or of sandstone similar to that which may be seen in the shales above, implying that the coal and sandstone fragments have fallen into the fissure while the clay was accumulating.

The fire clay is very finely divided, and as a general rule has no regular structure; yet in some cases a lenticular structure is noted, the convex surfaces of the lens-shaped masses being nearly horizontal. The color of the fire clay is generally light, varying from a light yellow to pale bluish or soapstone color. The general appearance of the matrix indicates that at one time it has been in a plastic, semi-liquid, homogeneous mass from which the excess of water has gradually drained away. Before exposure to the air this clay is usually quite hard and tenacious, presenting a formidable obstacle to the miner. But on being exposed to the air it disintegrates and assumes more of the common properties of clay. Where the "clay veins" or "horsebacks" are abundant the "room and pillar" system of mining is usually employed and the masses of clay are used to as great a degree as possible to support the roof.

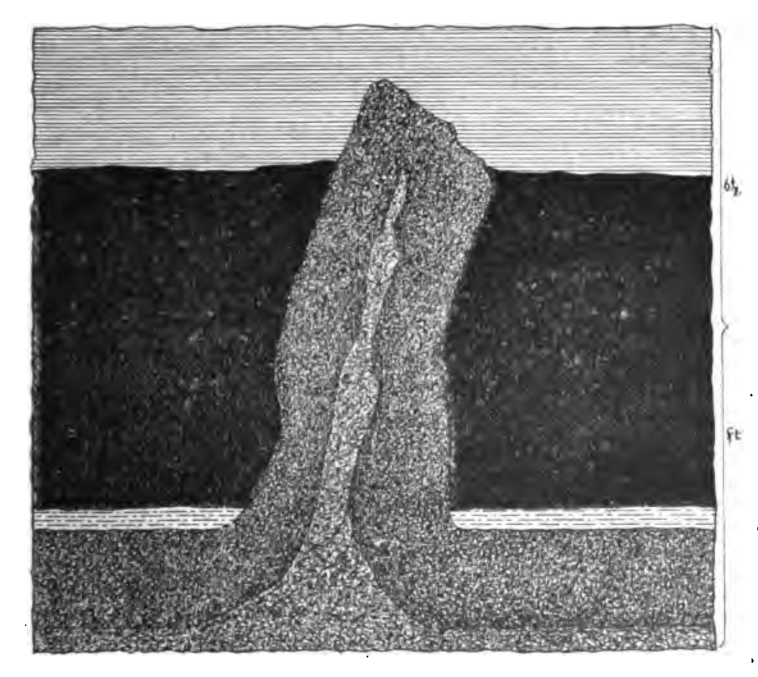


Fig 5.—Horseback made up of two varieties of fire-clay.

THEORIES OF FORMATION.—With the coal miners of this and other states different theories have been advanced to account for the existence of clay seams. One of these theories is that the fissures represent former underground waterways, and that the clay represents silt or sediment of various kinds which the stream deposited in its course, such deposition having been continued until the whole space of the fissure was filled. But how the fissure was produced in the first place this theory does not say. Another theory expressed by different miners is that the clay seams were formed contemporaneously with the coal, Neither of these views seems to correspond with all of the observed facts, consequently it cannot be concluded that either of them is correct. Before giving the view of the writer let us glance hurriedly once more at the conditions actually observed.

OBSERVED PHENOMENA. After examining a large number of mines and strip pit workings in the southeastern part of the state where the "clay veins" are quite numerous, the following facts there observed may be summarized:

1. The walls of the clay-filled fissures present a rough fractured surface as if they had been broken and torn apart by a horizontal stretching process which was greater than the coal seams could endure. In some cases the layers of coal are pressed upwards near the upper surface and downwards near the lower. Fig. 3.

2. There is always an upward displacement of the shale seams at the upper extremities of the "clay vein." Displacement, especially in the coal, is attended by fracturing. Fig. 4.

3. The fire clay in the fissure is usually homogeneous and structureless, but sometimes has an approach towards a lenticular structure. In all cases the clay in the "clay veins" is similar to that underlying the coal, and when the latter is composed of two or more varieties, as a dark and a light one, the same relation exists between them in the clay vein, as is shown in Fig. 5.

4. Angular pieces of coal are often found mixed through the clay in the fissure. These are evidently fragments of the original coal bed, for in many places their exact former position can readily be determined by their shape and the appearance of the wall of the coal. There is very little broken or finely powdered coal to be found in any of the clay seams.

PROBABLE ORIGIN. —It would seem to the writer that there is no room for doubt regarding the origin of these "clay veins." Long after the coal was formed and consolidated almost to its present state, vibratory movements of one kind or another fissured the

strata including the coal beds. The great variety of fissures as above described corresponds well with different forms of fissures observed in many parts of the world in connection with the mining for metalliferous deposits. Upon the production of such a fissure the great pressure under which the fire clay at the bottom of the coal had been existing would now be relieved on one or more sides. If the fissure passed entirely through the fire clay the surface of each wall would be relieved of pressure; if it only reached downwards to the fire clay the upper surface would likewise be relieved of pressure. Considering the exceedingly unctuous property of the clay and the softening to which it would be subjected from time to time by the underground water, it is very easily understood how it would soon move upwards sufficiently to more or less completely fill the fissure produced by the earth's tremors. This process would simply be an exaggerated case of ordinary "creeping" so commonly noted in underground workings, the upturning of the shale laminæ near the upper part of the fissure would very readily be produced by the upward movement of the clay acting under the great power which was forcing it along, while the occasional fragments of coal and sandy shale found within the "clay veins" can readily be accounted for by the occasional dropping of a block which was almost broken under the first earth movements which produced the fissures.

A Preliminary Report on the Glaciated Area of Kansas.

BY EARL G. SWEM.

(The following paper, written by Mr. Earl G. Swem, gives a few of the results of his investigations on the glaciated area of Kansas during the past summer while working for the University Geological Survey of Kansas, and is preliminary to a fuller report which it is hoped he will be able to prepare after another season's field work. E. H.)

In the summer of 1895 the writer made a tramp through the counties of Doniphan, Brown, Jackson, Atchison, Shawnee, Douglas, Leavenworth and Wyandotte for the purpose of observing glacial phenomena. The counties traversed do not constitute the whole glacial region of Kansas. This sketch, then, in so far as it deals with the glacial problem of the whole state is and must be imperfect. It is given more for presenting facts which have been preserved than for the suggestion of theories.

The whole drift region of the state has not been surveyed in detail by anyone. Of those who have been observers, Prof. Chamberlain is one. He has examined the line of the border of the drift and loess and says * "In harmony with all previous observations in the interior basin, I found in Nebraska, Kansas and Missouri, no evidence of morainic ridging on the border of the drift, and it may be safely asserted that no such morainic aggregation exists except as a local development." Dr. Wright says †in regard to this region: "It would seem that the action of water and floating ice was predominant in determining the character of the glacial deposits over that region (Eastern Kansas and Nebraska), and the theory is plausibly suggested by Prof. Todd that the extension of the ice beyond the Missouri formed glacial dams across the valleys of the Kansas and Platte rivers so as to maintain for a short period temporary lakes of considerable extent which received and distributed the bowlder fragments of ice as well as the finer elements of the glacial deposits. The most of the glaciated portion of this region is deeply covered with fine loam or loess, which is probably a water deposit and, as we shall hereafter see, is on good grounds believed by Chamberlain and Salisbury to be an assorted variety of glacial silt directly derived from glacial waters."

* 6th An. Rep. Director U. S. G., S. p. 32.
†Ice Age in N. Am., p. 344.

In the drift region of Kansas as a field of study, the student has some advantage, which he does not find in similar regions covering entire states. The drift occupies a relatively small part of the state. On the west and south it is bounded by unglaciated territory. A profitable comparison can thus be made between it and the adjacent driftless region. The fact also that in the state there are no morainic accumulations of the later glacial period removes one element of uncertainty and complexity encountered in the study of some other glacial regions.

The native rocks of eastern Kansas are carboniferous limestone, interstratified with heavy beds of shale. This arrangement has made it possible for water to show its erosive power very effectively. A topographic type has been produced which Prof. Haworth has described in Volume II of the KANSAS UNIVERSITY QUARTERLY. The shale has been easily worn away, leaving a hard capping of limestone which forms a table-topped hill, with terraces formed of layers of hard limestone, which have resisted wear more successfully than the over lying shale. This type of hill presents the appearance at a distance of steps leading up to a level surface. This is the characteristic erosion contour in that part of eastern Kansas not covered by the drift. We may safely say then that it was characteristic, previous to glacial times, of the region now lying under the drift.

The preglacial streams had eroded wide valleys. The smaller streams in some cases have valley bottoms two to three miles in width, with their former bordering limestone ridges still apparent. A good example of a preglacial valley of a small stream whose ancient bluffs still remain prominent is that of Walnut Creek, six miles southwest of Horton. The valley of Wolf river in northeastern Doniphan county it also worthy of notice. The river flows in an easterly direction as far as Severance. There it bends toward the north, and flows northerly, until it joins the Missouri. The fact that this river flows to the north in its ancient valley has seemed to me to be unusual. A closer study of it will perhaps reveal important facts in regard to the manner in which the drift was deposited. It has not been easy to secure data in regard to the depth of old river valleys. Most of the wells have been dug in drift lying on ridges, and not in river bottoms. And in those cases when wells have been made in bottom land water is usually struck before the rock is reached, so that even if we knew the depth we could not always tell the true vertical extent of drift in the valley. In the Wolf river bottom near Severance is a well twenty-two feet deep. Most of the material passed through was joint clay. Near

the surface the clay was so hard that picks had to be used. This will illustrate the information obtained from other wells similarly placed in river valleys. Big Soldier creek, which flows south through the western part of the Pottawatomie Indian Reservation, shows evidence in its banks and bed of having changed considerably its course from the preglacial channel, though it is still in its preglacial valley. At Rock Ford, about one mile west from the Agency building, this stream has for its bed a level limestone floor which reaches up the stream for 100 feet, and extends laterally from bank to bank. On the west bank a two foot layer of lead colored shale lies immediately over the limestone. In the upper part of this and rising over it is glacial debris, consisting of pebbles, bowlders (one foot in diameter,) red clay and red sand, all compactly cemented together. This is two feet thick. On the east side this debris overlies, as on the west bank, the lead colored shale. Here above it there are 18 feet of reddish clay in which pebbles, rounded and angular, are scattered. The limestone floor originally extended down stream at least 150 yards. It has been eroded back to Rocky Ford, where there is a fall of four feet, leaving still 100 feet of the floor visible. It does not seem probable that the stream originally flowed over this floor with banks of shale only two feet in height. On a small scale, I think here we have an example of falls formed through glacial agency. The computation of the age of these falls might yield interesting results.

Bowlders are not frequent near the Missouri river in Doniphan county. In the vicinity of Wathena and Iowa Point I saw none. Near White Cloud one granite bowlder two feet in diameter was observed. Bowlders are noticeably absent in the region of Troy, and between Troy and Severance, until within a mile of Severance. Here angular blocks of Sioux quartzite half a foot in diameter appear in a creek bed. Nearer Severance, where the native rock juts out upon the road, are blocks of quartzite of the same size, and also rounded blocks of limestone up to a foot and a half in diameter. Here I saw for the first time what I often observed subsequently, the arrangement of bowlders along ridges of outcropping limestone. The occurrence of bowlders in the bottom of wells 40 to 50 feet in depth is not uncommon. Bowlders commonly occur in creek beds, wherever the streams cut through native rock or are near exposures of native rock.

Approaching Horton from the east the bowlders increase in size and number. West of Horton about five miles are several large quartzite bowlders. One is 12 feet in length and 4 feet high. Speaking generally of the whole region traversed, the small

bowlders (1 to 3 ft.) are of quartzite, granite and green stone. We can safely say that 75 per cent are of Sioux quartzite. Of the bowlders over 3 feet in diameter 95 per cent are quartzite. The quartzite bowlders are much more angular than the others. Of all the bowlders observed during the summer, scratches were found upon two only. The largest isolated limestone rock observed is located 2½ miles north and 1½ miles east of Holton. This is somewhat rounded and smooth. It has some distinction in the neighborhood from the fact that John Brown's initials are carved upon it. Near it is a five foot granite conglomerate. The native rock crops out nowhere in the immediate neighborhood. Near here is a well 126 feet deep which does not reach rock. The largest bowlder observed is 6½ miles north of Topeka. It is 23 by 13 by 7 feet; near this and extending six inches out of ground is another four feet long; near this is another 19 by 8 by 1 feet. The three are granite conglomerate bowlders.

Many rock exposures were examined for striæ, but none were found. A careful search was always made wherever the conditions gave the least possibility of their occurrence.

In the greater number of cases the drift material instead of occurring directly upon the limestone is upon shale which had been weathered and disintegrated before the deposition of the drift. But the shale in all such cases, with one possible exception, was found to be undisturbed, not crumpled, crushed or to any extent visibly moved. This occurrence of the drift upon the undisturbed shale would seem to be pretty good evidence that it was not deposited as ground moraine. Two and one-half miles south of Holton is a sectionon the Rock Island R. R., which illustrates the contact of the drift and shale, though here the layer of drift is uncommonly thin. The section is 300 feet in length, 15 feet high in the middle, 4 feet high at each end. The greater part is shale. Over it all is a two foot layer of reddish soil with pebbles, which are in some places arranged in layers. Where the soil layer meets the shale the latter is disintegrated.

The great mass of glacial material in Kansas is of a pale yellowish red color. Its greatest depth except perhaps in the northern drift region is not much more than 175 feet. The deepest well of which I heard that passed entirely through drift is one northwest of Holton. This has a depth of 126 feet. A well near White Cloud, with a depth of 126 feet does not reach rock. Most of the wells examined have a depth through drift of forty to sixty feet, though depths of 70 to 100 feet were not infrequently met with. One hundred and seventy-five feet is a liberal estimate for the greater depth.

Fifty feet would be a fair estimate of the average depth. In going south from central Jackson county the drift becomes less in depth very noticeably.

I have said that the usual color of the drift is dull yellow. Below are some variations from this. More might be given, but these are typical:

Two miles from Denton on road to Everest is the following section, made by a creek.

1. Soil, 1½ feet.
2. Lead colored clay, hard and tough, at base is layer of pebbles, 4 feet.
3. Red sand with pebbles, 4 feet.
4. Very fine white sand, 3 feet.
5. Brilliant red sand to bottom of section, 1 foot.

Four miles west of Larkin is the following. The laminæ here are unusually distinct.

1. Soil and yellow clay, 3 feet.
2. Brown sand, compact, with gravel, ½ inch.
3. Brown sand, 2½ inches.
4. Deep brown coarse sand, 1½ inch.
5. Coarse red sand, 9 inches.
6. White clay, 5 inches.
7. White sand, 5 inches.
8. Fine red sand, 4 inches,
9. Black sand, compact, 2 inches.
10. Dark brown sand, 10 inches.

Below this are alternate layers of pebbles and sand. These lie upon weathered shale, which rises about 5 feet above creek bed.

A generalized section of the whole drift region would be as follows:

1. Soil.
2. Yellow clay, occasionally with pebbles, and sometimes over sand.
3. Lead colored clay passing gradually into
4. Lead colored shale.
5. Limestone.

I found the following localities of special interest:

1. Holton.

Elk Creek flows at the base of a hill perhaps 60 feet in height, and makes a very striking section. This section is almost directly north of Campbell University. It is 60 feet in length, and varies in height from two and three feet to twelve feet. At the left end can be seen five feet of brittle lead colored clay in which there are a

few pebbles. The main section shows blocks of limestone 3½ feet in length down to inch pieces. Rounded limestone bowlders are also present. Pebbles and gravel, rounded and angular limestone bowlders are mixed indiscriminately together and packed compactly. Blocks of quartzite also appear but they are as numerous as the bowlders of limestone and granite. Blocks are also found which show that they have been consolidated and hardened from rock rubbish similar to that which here lies around loose. Vertically above the bowlders there appears light gray clay, then red sand with pebbles. Ascending the bluff, all the way up to point where cave has been made by children is found the red sand with pebbles. On the walls of this cave can be seen wavy strata of sand 1½ feet long, and blotches of red sand gravel. On top of bluff there are a few bowlders. A short distance up the creek are some sand pits. These are in same bluff as preceding section, and about midway between top and bottom. They show irregular layers of white, light brown, coarse brown and gravelly sand. South of Holton about a mile are several sections of interest on the Rock Island Road. The first is about 25 feet in depth, the material is yellow sandy clay. At the base of a 9 inch layer of soil is a 3inch layer of pebbles. This stands out very conspicuously, with no similar layer above or below it. The layer follows the curve of the ridge through which the section has been made. Pebbles are not noticeable elsewhere in the section. Farther along the railroad is another section 15 feet in depth. Three and one-half feet from the top is a layer of pebbles up to seven inches in length. Below this is the sandy clay to the bottom. Farther south is another section 120 feet long. A layer of pebbles appears here as in previous sections with the clay below. The first section in the Holton region of which mention has been made shows that the hill, a part of which it discloses, is moraine in nature. It deserves more study. I went as far to the east of Holton as Larkin, about nine miles; while some interesting sections occur, there is nothing similar to that of the Elk Creekbluff in Holton. There is no evidence that the moraine, if it is properly such, as it occurs at Holton, extends to the east.

2. Pottawatomie Indian Reservation.

I found no driftless region within the territory usually defined as covered with drift until I reached the southern part of the Pottawatomie Indian Reservation in Jackson county. This is about twenty miles to the north of the southern limit of drift as sketched in geological maps of Kansas. On the Reservation appear terraced hills, the typical erosion found in the native limestone of which

mention has been made before. Typical rounded drift hills also occur. Three miles west of the Agency building, on the west side of the Big Soldier, the limestone rocks are very prominent. Here there is very little soil of any kind. On the ridges the rock forms almost a perfect floor. The blocks of limestone in this floorlike surface are somewhat rounded, but this has come undoubtedly from weathering and not glacial action. Three quartzite bowlders were found on top of one of these ridges. One of the bowlders is situated where there was no soil at all, on a limestone floor. These ridges are 1250 feet above sea level. They are higher than the land to the south and east.

3. Atchison.

The region of Atchison is interesting for its dissimilarity from Wathena, White Cloud and Troy in the adjoining county on the north. In these places only one bowlder was found lying on the ground. Near Atchison they are not infrequent. The brown flints which had been observed in other regions were very abundant in Atchison. A layer of these above the limestone can be seen in various places in the city. The following is a section exposed ½ mile north of the city on the B. & M. R. R.

1. Layer of yellow clay with pebbles and fragments of limestone, 5 feet.

2. Irregular layer of limestone bowlders from ½ foot to 1½ feet in diameter. Greenstone pebbles also are present. This is very irregular in outline, and in thickness 2 to 3½ feet.

3. Yellow clay and brown sand, 5 feet.

4. Layer of brown flint pebbles, so numerous that the layer seems to be wholly composed of them. Occassional limestone and greenstone pebbles found, 4 feet.

5. Disintegrated limestone, 2 feet.

6. Limestone.

Whole section 25 feet in depth.

Inquiries were usually made for wood found in wells. Two and a half miles north of Holton wood was found in a well at a depth of 40 feet. Wood was found in a well near Effingham at a depth of 90 feet, and at Atchison on the Midland College campus at a depth of 50 feet. Prof. Knerr of Midland College has examined the specimen taken from the college well, and says that its cells show that it is coniferous. No section was found that indicated in any way a layer of forest material.

A Geological Section at Providence, Missouri.

BY ALBAN STEWART.

On October 1, 1894, in company with Professor G. C. Broadhead of the Missouri State University, I visited the region at and surrounding Providence, Missouri. Providence is a small town, consisting only of a store, postoffice and a number of small dwellings, situated on the Missouri, Kansas and Eastern Railway, twenty-five miles northwest of Jefferson City. The bluffs, both east and west of the city offer excellent facilities for examining the various strata there exposed, having been cut down perpendicularly for the road bed of the railroad. They are capped with the Burlington limestone, below which is the Chotean limestone containing many characteristic fossils, such as *Rhynchonella missouriensis*, *Productus blairii* one or two species of *Orthoceras*, etc. The total thickness of these two limestones is one hundred and forty feet.

The following section is that existing at the town and extending one and a half miles below it.

1. Top of bluffs.
2. Burlington limestone, 90 feet.
3. Chotean limestone, 50 feet.
   Consisting of:
   a. Drab colored limestone with siliceous geodes and containing but few fossils, 20 feet.
   b. Brittle, dove colored limestone, abounding in fossils, such as *Rhynchonella*, *Orthoceras*, *Productus*, *Pernopectin shumardii*, *etc.*, 3 feet.
   c. Thin shaly limestone, including Trilobites (*Phillipsia*, spp.) Crinoids, Bryozoans, *Spirofer*, *Gomphoceras*, *Orthis*, Corals, etc., 25 feet.
4. Light green calcareaus sandstone, *Tamoeris caudagalli*, 1 foot.
5. Devonian limestone, dove colored, containing in its upper part, *Favosites alpinensis and Stomatopora expansa*, below which is found *Loxineura robusta;* four feet from the top is found a large *Turbo* like fossill and *Spirifer erriternus*, 25 feet.
6. Chert, some of which is oolitic, alternating with sandstone and limestone, 5 feet.
7. Magnesian limestone, 7 feet.

8. Sacharoidal sandstone, 10 feet.
9. Surface of river.

The strata at this point dip gradually toward the west, and the sandstone mentioned as No. 4 disappears about three hundred yards east of the town. This marks the uppermost part of the Hamilton at this place. One of the peculiarities found in this section is the Turbiform fossil mentioned in No. 5. The limestone in which it was found is very hard, and consequently, it was difficult to extract a fossil as large as this, so that its systematic characters could be accurately determined. It was referred by Prof. Broadhead to the genus Turbo, and by another paleontolgist to Pleurotomaria. It seems to be peculiar to this locality, though not abundant. The same horizon was subsequently examined in Callaway county without finding other traces of it.

The Hamilton limestone lies above the first magnesian limestone, No. 7, with only five feet of sandstone and chert intervening which must represent a long interval of time.

The following groups of the Missouri section are not represented here.

Devonian.	Corniferous. Lower Helderberg.
Upper Silurian.	Niagara. Hudson River.
Lower Silurian.	Trenton. Black River.

It seems that after the first magnesian limestone was deposited there was a slight upheaval, the limestone remaining above the surface of the Paleozoic ocean until the latter part of the Hamilton, when it was submerged and received the remaining part or the sediments of this group. Even in Callaway county, thirty miles distant, the thickness is much greater, including the Hamilton shales, which are there well developed and showing their characteristic fossils.

Just underneath the magnesian limestone the sacharoidal sandstone is found. There are but ten feet of this exposed above the surface of the river, and this for only about fifty yards; it projects outwards from the bluff and presents the evenly rounded surface characteristic of this sandstone where weathered. We were not able to determine the cause of the sudden appearance and disappearance of it. The character of the surrounding strata would not warrant the inference that it was a local upthrust. More probably it represents a small island. We were unable to examine the opposite side of the river for its occurrence there.

I am indebted to Prof. Broadhead for kind assistance in the development of the section above given.

Notes on Discharge of the Kansas River at Lawrence, Ks.. Since 1881.

BY E. C. MURPHY.

In 1880 Mr. J. D. Bowersock put in a gauge on the Kansas river at Lawrence for the purpose of comparing the height of the water on his weir,—or dam as it is called—at different dates. He has read the gauge daily since Feb. 1881, and when the water was rising or falling rapidly, two or more times a day, up to Aug. 1895, recording the reading only when there was a material change in them. He has prepared for the U. S. Geological Survey from his record, a table giving the mean weekly gauge reading or height of water on his dam during this period.

In Aug. '95, this place was chosen as one of the government river gauging stations, and the writer given charge of it. Through the kindness of Prof. F. H. Newell of the U. S. Geological Survey who has charge of all the river gauging stations in the United States, I have permission to use for this paper Mr. Bowersock's data and the discharge measurements made by the writer.

The gauge used by Mr. Bowersock was a vertical board graduated to feet and inches fastened on the east end of the south pier of the carriage bridge, with its zero on a level with a large stone in the crest of the dam. The gauge used at present is a vertical board graduated to feet and tenths, fastened to the old one with their zeros coinciding.

The crest of the dam is 595 feet long and is not quite horizontal, the south end being 4 to 5 tenths of a foot higher than the north end. When the gauge reads zero the water is on a level with the crest of dam near its center.

The flume is on the south side of the river with its gates a few feet west of crest of dam. It is 60 feet wide and has a maximum depth of 7 feet when the gauge reads zero.

The discharge measurements—except Nos. 9 and 10, were made from the west or up stream side of the bridge about 65 feet west of crest of dam. The channel has a maximum width at this place of 690 feet broken by four piers. Fig. 2 shows a plan of the dam, bridge and flume; and Fig. 1, the section where the discharge measurements were made.

From a quarter to half a mile above the bridge there is a bend in the river, which deflects the water to the north bank at the bridge, making the maximum velocity on the north side of the north pier, and causing considerable eddying on the north side of piers Nos. 3 and 4, when the water is high.

The bed of the river at this place is rock from the south abutment to a little north of pier No. 3; the rest of it is sand. The south half of the dam is cement masonry on rock, through which there is little seepage; the north half is made of timber cribs filled with rock and through these and the sandy bed there is considerable seepage. The discharge measurements were made during the months of July, Aug., Sept., and Oct., and are numbered 1, 2, 3 and 4, and c in the order in which they were made. These measurements expressed in cubic feet per second are plotted in Fig. 3, as abscissas using the corresponding gauge readings as ordinates.

Three current meters were used in measuring the velocity. Measurement No. 1, was made with Haskell meter No. 6, No. 2, with Price meter No. 19, and the other eight with Haskell meter No. 17. The depth was measured every five feet, and the velocity every five to twenty-five feet depending on the rate of its change.

Section C D being large it is impossible to measure accurately the velocity when the gauge reading is much less than one foot, so that measurements No. 9 and 10 were made at Lecompton, 11 miles up the river on the Santa Fe R. R. The only tributary of the Kansas river of any size between Lawrence and Lecompton is the Delaware. The average discharge of this stream on Oct. 25, when measurement No. 10 was made, was less than 12 cubic feet per second. Fig. 4 shows the Lecompton section on Oct. 5, when measurement No. 9, was made.

I have computed the discharge from Mr. Bowersock's record of mean weekly gauge readings by two methods. For the period from Jan. 1, 1891, to July 31, 1865, I have found the probable mean daily gauge readings by plotting the weekly means and drawing a curve similar to that for the months Aug. to Nov. '95, and so that the mean of the seven daily ordinates of this curve is the same as Mr. Bowersock's weekly mean. Entering the rating table with these probable daily mean gauge readings the discharge for each day was found. For the period Jan. 1, 1886 to Dec. 31, 1890, the discharge for each week was found by entering the rating table with the weekly mean gauge reading and multiplying the corresponding discharge by seven. The rating table gives the discharge in acre feet per 24 hours. It is computed from the rating curve, Fig. 3, by reducing the discharge in cubic feet per second, to acre feet per 24

hours. We have applied this table for the ten years from Jan. 1, 1886 to Dec. 31, 1895, assuming that the cross section where discharge measurements were made has changed very little during this time. The soundings show that the bed is hard rock or stone, except near the banks. No repairs of any magnitude have been made on the dam during this period, and those that have been made, have not—Mr. Bowersock thinks—materially changed the cross-section or velocity of approach.

Table II gives the monthly, mean monthly and annual discharge in acre feet, for the ten years 1886-95. Fig. 5 shows diagrammatically the mean monthly discharges. It is seen from this figure that the maximum discharge is in May and the minimum in December.

Table III gives some interesting facts of the different years. Column two gives the greatest gauge reading. Column three the number of weeks the gauge read zero or less. Column four the number of weeks that the gauge continuously read zero—five the time of maximum gauge reading. Column six longest period of zero gauge reading, and seven the maximum monthly discharge.

Zero gauge reading corresponds to a discharge of 787 cubic feet per second. We see then that eleven years out of these fifteen the discharge at this place was less than 790 cubic feet per second for three or more weeks at a time. From measurements made by the writer, and additional facts furnished by Mr. Bowersock in regard to the power of his dam—which it is not necessary to present in these notes, we find that the average discharge for Dec. 1894, and Jan. 1895, was not more than 550 cubic feet per second.

TABLE I.

KANSAS RIVER (LAWRENCE) RATING TABLE (1895) ACRE FEET IN 24 HOURS.

Gauge Readings	0.0′	1.0′	2.0′	3.0′	4.0′	5.0′	6.0′
0.0′	1561	5328	12670	22686	34485	48266	64032
0.1′	1711	5945	13762	23776	35773	49753	65718
0.2′	1918	6598	14674	24987	37083	51281	67422
0.3′	2177	7284	15606	26017	38414	52787	69148
0.4′	2489	8002	16558	27167	29759	54334	70804
0.5′	2849	8740	17530	28337	41128	55901	72659
0.6′	3258	9524	18720	29527	42517	57487	74442
0.7′	3712	10325	19533	30737	43924	59093	76247
0.8′	4210	11150	20564	31967	45351	60718	78071
0.9′	4749	11999	21615	33217	46799	62365	79915

TABLE II.
DISCHARGE IN ACRE FEET.

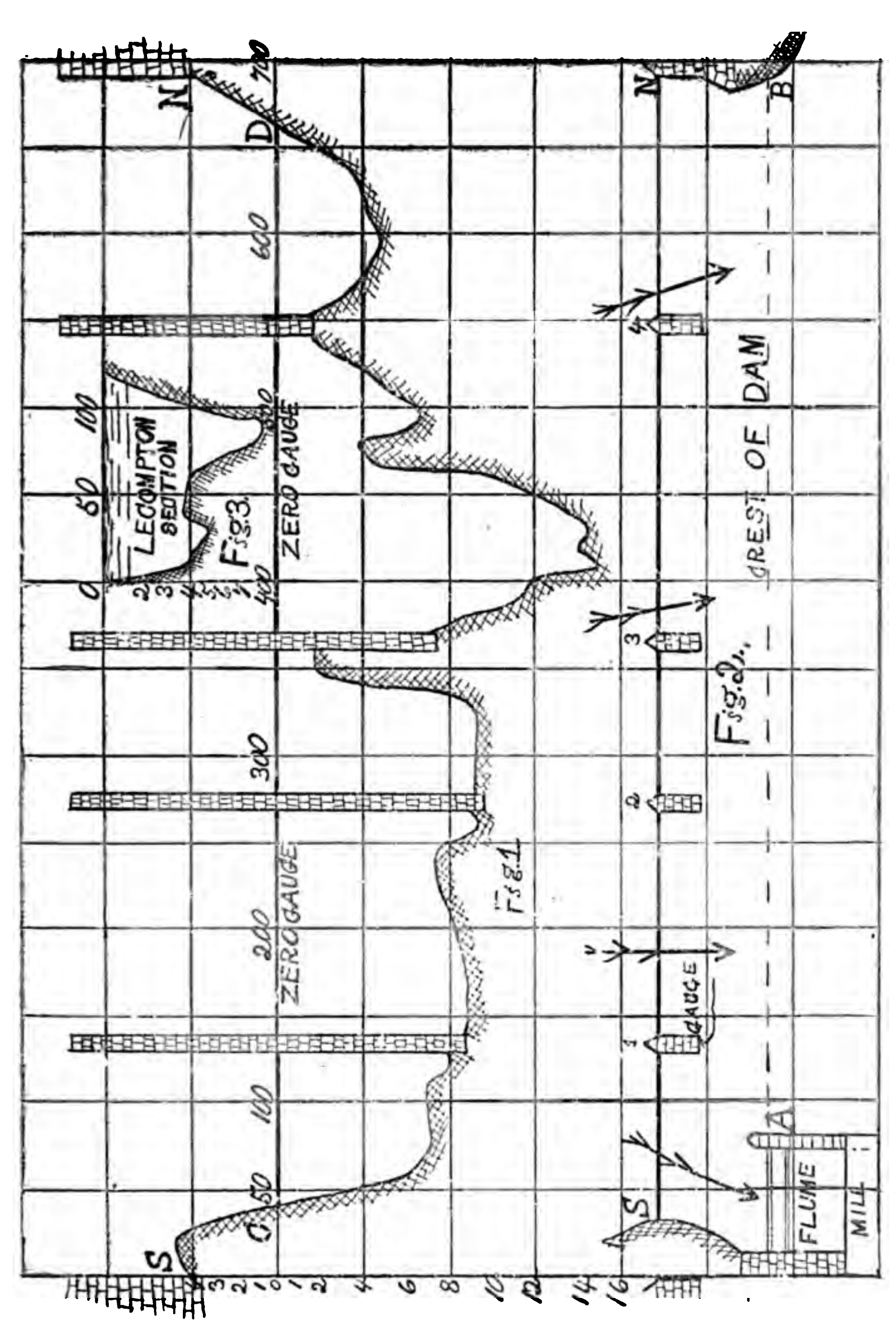
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.
1895	33,817	43,219	83.863	81 622	65,778	352,586	348,229
1894	85,633	73.017	131.179	172.707	74,270	538,044	291.401
1893	73,574	202,683	84.306	119,650	197 269	552,290	520 943
1892	102,600	293,807	783.419	756.242	2,275.348	83,895	126,543
1891	83.847	195.444	377,703	502.998	650,436	374.327	861,171
1890	332,500	193 643	285.394	152,579	212.342	194,989	69.758
1889	94.703	78,274	105.675	117.336	847,803	337,550	887.899
1888	63.637	275.797	419,900	232,756	453.620	514,589	292,712
1887	50 943	83,956	137.491	117.303	316.428	322,033	83,396
1886	53.273	620.588	750.193	694,433	767,327	237,615	114,294
Mean	93,753	203,141	313,906	294,764	586,032	355,788	339,634

TABLE II (CONTINUED)
DISCHARGE IN ACRE FEET.

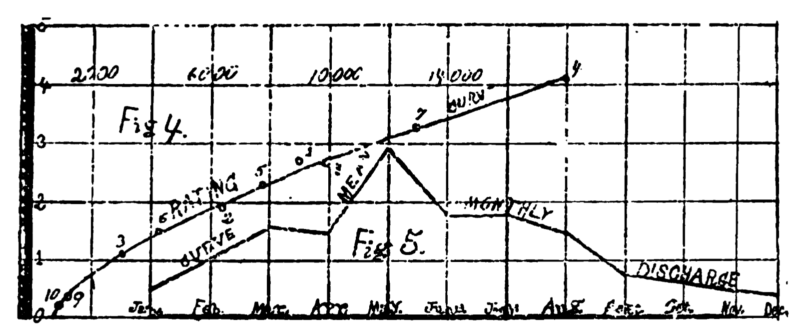
	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Mean.
1895	696.130	297,233	67.153	61,949	60,425	2,228,000	185,667
1894	90,876	85.953	73,083	40,461	33,817	1,690,441	140.870
1893	367.593	125.233	225.190	84,849	62,896	2,621,476	218,456
1892	123,543	86,133	64.477	88,450	75,302	4,859,749	404,979
1891	334,707	163,931	364.204	143,688	192,339	4,250.823	354,236
1890	63.427	93,807	74.582	79.453	57.876	1,790 215	149 185
1889	730,336	326.769	111.324	293.444	91,840	3 935,956	327,996
1888	293,071	67,438	75,185	102,823	88,319	2,879,858	239,938
1887	108,637	117.951	111,713	77.742	61,719	1.589.310	132,442
1886	143.368	81,508	60,165	83,454	76,525	3,735.733	311,311
Mean.	295,169	144,597	122,708	93,930	80,109		

TABLE III.

Years.	Greatest gauge reading.	Time of greatest gauge reading.	Weeks of zero gauge reading.	Longest zero gauge period in weeks.	Date of longest zero gauge period.	Month of greatest discharge.	Greatest monthly discharge in acre feet.
	′ ″						
1881	4—2	March 7	11	7	July 11 Aug. 24		
1882	4—3	April 10	17	7	Aug. 28 Oct. 9		
1883	4—3	June 25	4	4	Jan. 1 Jan. 22		
1884	3—9	May 5	2	1	Jan. 1		
1885	3—8	April 27	1	1	Feb. 23		
1886	4—3	May 10	7	3	Sept. 20 Oct. 4	May	767327
1887	2—6	May 23	8	3	Jan. 3 Jan. 17	June	322033
1888	3—6	June 25	6	4	Jan. 2 Jan. 23	June	514589
1889	5—0	May 13 July 22	0	0	....	July	887899
1890	2—9	January 13	9	3	Aug. 4 Aug. 18	January	332500
1891	6—0	June 1	0	0	....	July	861171
1892	8-10	May 16	5	4	Sept. 19 Oct. 10	May	2276348
1893	4—3	June 5 July 3	4	3	Dec. 4 Dec 18	June	552290
1894	5—3	June 25	15	10	Oct. 29 Dec. 31	June	538044
1895	4—0	June 10 Aug. 19	9	7	Jan. 7 Feb. 18	August	696230



Figs. 1, 2, and 3, Sections of Kansas River and Dam.



Figs. 4 and 5, Rating and Discharge Curves.

Supplementary Notes to the Article on Continuous Groups.

BY H. B. NEWSON.

The second section of the paper entitled Continuous Groups of Projective Transformation, etc., in the QUARTERLY VOL. IV, NO. 2, requires correction at one or two points and elucidation at other points. By an oversight I failed to draw one of the most important conclusions which the synthetic method yields; viz: that the real group G_1 contains a 'host' (German, *Schar*) of transformations which can not be generated from the infinitesimal transformation.

1. In the paper mentioned only real transformations are considered.

2. A pseudo-transformation transforms to one of the invariant points all the points of the line l except the other invariant point. This second invariant point is indeterminately transformed.

3. It is better to say that the group G'_1 contains only one pseudo-transformation. The distinction between plus and minus infinity is neither necessary nor proper.

4. A one-termed group G_1 of real transformations contains two infinitesimal transformations. These are given by the two infinitesimally narrow conics touching l at +dO and —dO, the two points infinitesimally near to O. The characteristic anharmonic ratios of these transformations are respectively 1+d and 1—d, where d is infinitesimally small. All transformations of the group whose characteristic anharmonic ratios are between 1 and ∞ may be generated by the first, and all between 1 and 0 by the second, of these infinitesimal transformations. The transformations of the group whose characteristic anharmonic ratios are negative can not be generated by either infinitesimal transformation.

1. A one-termed group G'_1 of real transformations contains two infinitesimal transformations whose characteristic constants are respectively +d and —d, where d is infinitesimally small. All transformations of the group whose characteristic constants are positive may be generated by the first, and all whose characteristic constants are negative by the second, of these infinitesimal transformations.

6. An exactly similar theory holds for one-termed groups of real projective transformations of the most general form in two, three and *n* dimensions. The parameter of every such group is an anharmonic ratio. Such a group contains one identical, one involutoric, two pseudo and two infinitesimal transformations. The transformations of the group whose characteristic anharmonic ratios are negative can not be generated by either infinitesimal transformation.

It would seem at first sight that these results contradict Lie's general theorem that every transformation of the projective group, in any number of variables, can be generated by the repetition of an infinitesimal projective transformation. (See "Cont. Gruppen" page 45.) Lie's theorem is correct for the group of all real and imaginary transformations; but as has just been shown it is not true that every real transformation can be generated by the repetition of a real infinitesimal transformation.

I have learned recently from a note in the Bulletin of the New York Mathematical Society for Nov. '93, page 66, that Professors Study of Marburg and Engel of Leipsic have reached a similar result for the case of Lie's special linear homogenous group in two variables; but I do not know that they are yet in possession of the complete theory. Professor Taber of Clark University has quite recently established nearly the same results in the special cases of the group of orthogonal transformations (New York Bulletin for July '94, page 258-9) and the group whose invariant is an alternate bilinear form (Mathematische Annalen, Band 46, page 581, Nov. '95.) Professor Taber has not given his results their obvious geometric meaning.

A new proof of Lie's théorem is here outlined. Let $(1+dz)$, where dz is an infinitely small complex quantity, be the characteristic anharmonic ratio of an infinitesimal transformation. If it can be shown that $(1+dz)^n = k$, where $n = \infty$ and k is any real or complex number, then Lie's theorem is proved; for the assumption of complex values of k and real or imaginary invariant space elements is equivalent to the assumption of complex values of the variables and constants in the equation $\frac{x_1-m}{x_1-n} = k\frac{x-m}{x-n}$. We may set $(1+dz) = re^{iy}$; then y is infinitely small and r is infinitesimally less or greater than, or equal to, unity. This value can be represented in the complex plane by a point infinitely near to the unit point. The locus of a point representing the value of $r^n e^{iny}$ where n becomes infinite is a spiral, which by a proper choice of dz can be made to pass through any point of the plane.

Fissicorn Tachinidæ.

BY S. W. WILLISTON.

In "Entomologica Americana," iii, 151, I described a peculiar genus of South American Tachinidæ with remarkably developed antennæ under the name *Talarocera*, in reference to the peculiar basket-shape of the organs. Figures of both male and female antennæ I give herewith. Since then two other genera of the same family with these organs abnormally developed have been described, *Dichocera* Williston and *Diglossocera* Wulp; and Brauer has recently * called attention to the fact that Walker long ago † had referred two species with divided third antennal joint to a distinct genus, *Schizotachina*. Four of the species thus made known, *Schizotachina convecta* Walker, *S. exul* Walker, *Dichocera lyrata* Williston and *Diglossocera bifida* Wulp have the third joint somewhat similar in structure in the male. In *S. convecta*, according to the description, the joint is "divided into two parts which are equal in length, slightly curved and converge toward each other at their tips:" in *S. exul*, it is "divided into two parts, which are

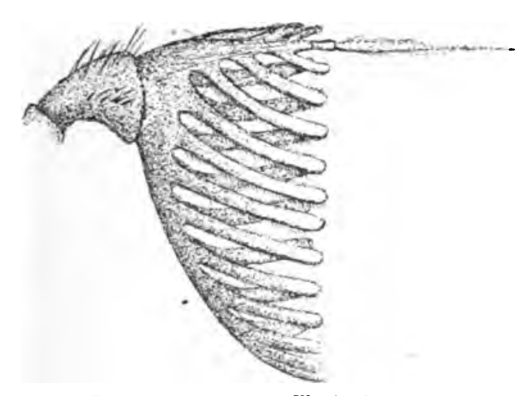


Fig. 1. *Talarocera nigripennis* Wied.; Antenna of male.

linear, rounded at the tips, and of equal length; upper part bent upward;" in *D. lyrata*, (see figure, Entom. News, Jan. 1895, p. 30), the inner division has a foot-like expansion at the tip, touching the curved outer part: in *D. bifida*, the structure is similar to that in

* Sitzungsb. kaiserl. Acad. Wissensch. clv. 600.
† Insecta Saundersia. 264.

S. convecta, and except that the latter is from Java and the former from the United States one might suspect that they belong to the same genus.

In *Talarocera nigripennis* Wied. (*T. smithii*, Will.) the structure, as will be seen from the figure, is very different and exceedingly remarkable. For their description the reader is referred to the original paper. I will only add here that all the twenty-four branches are covered with microscopic hairs or pile. In front view, I have stated, the figure presented by the ends of the rods or branches is an oval one.

Professor Mik ‡ ventures the opinion that this peculiar structure of the male antennæ in *Talarocera* and *Dichocera* may be a monstrosity, as a somewhat similar structure has been observed by Strobl and himself in the female of *Thryptocera exoleta* Meigen, or at least in specimens that differ only in that character from typical forms. That *Dichocera* can be a monstrosity is disproved by the

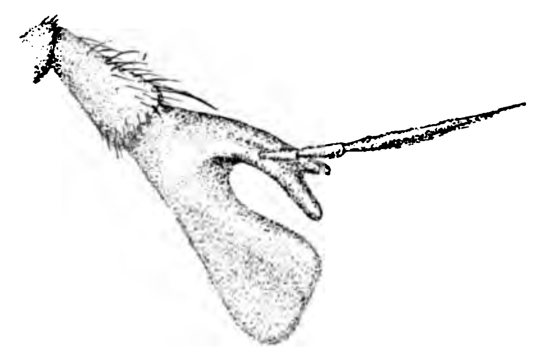


Fig. 2. *Talarocera nigripennis* Wied.; Antenna of female.

fact that numerous specimens of the species have since been found by Prof. Aldrich in Idaho, and others by Mr. Snow in eastern Wyoming the past summer. The distribution, it is thus seen, is wide.

Nor is it possible to consider the wonderfully elaborate structure in the male of *Talarocera* a monstrosity, though but a single specimen is known yet.

I may add here that *Tachina insolita* Walker belongs in the genus *Melanophrys* Will. and is possibly identical with one or the other of the described species.

‡ Wiener Ent. Zeit., May., 1895.

List of Asilidæ, Supplementary to Osten Sacken's Catalogue of North American Diptera. 1878-1895.

BY W. A. SNOW.

LEPTOGASTER.

cubensis Bigot; Roeder, **I**, 340.—Portorico.
flavipes Loew; v. d. Wulp, **X**, 1.—Massachusetts.
longipes Johnson, **D**, 273.—Jamaica.
obscuripennis Johnson, **E**, 304, 305, 323.—Florida.
pictipes Loew; Osten Sacken, **G**, 167.
scapularis Bigot, **A**, 444.—California.
sp. Osten Sacken, **G**, 167.—Mexico.

DASYPOGON.

(?) quadrinotatum Bigot, **A**, 412.—California.

SEILOPOGON.

(?) rubiginosum Bigot, **A**, 419.

ABLAUTUS.

Loew, Centur. vii, 63. 1866. *Ablautatus* Loew, Berl. Ent., Z., 1874, 377; Osten Sacken, Cat. Dipt., 67.

mimus Osten Sacken; Williston, **M**, 53.—Arizona.
trifarius ? Loew; Osten Sacken, **G**, 168.—Mexico.

DICRANUS.

Loew, Bemerk. Fam. Asil., 13, 1851; *Macronix* Bigot.

n. sp. Williston, MS.—Mexico.

STENOPOGON.

æacidinus Williston, **N**, 289.—Western Kansas.
albibasis Bigot, **A**, 422.—California.
fuscolimbatus Bigot, **A**, 421.—Mexico.

SCLEROPOGON.

ochraceus v. d. Wulp, **W**, 96.

OSPRIOCERUS.

æacus Wiedemann; Osten Sacken, **G**, 168.
sp. Osten Sacken, **G**, 168.—Mexico.

MICROSTYLUM.

fulvigaster Bigot, **A**, 410.—Mexico.

ARCHILESTRIS.

Loew; *Pseudarchilestes* Bigot, Bull Soc. Ent. Fr. 1889, 183.

magnificus Walker (*Dasypogon*); Schiner, Novara Exped., 168; Osten Sacken, G, 169; Williston, Psyche, 1889, 259.—South America.

DIZONIAS.

lucasi Bellardi; Osten Sacken, G, 170.
phœnicurus Loew; Osten Sacken, G, 170.
tristis Walker, Dipt. Saund., 93 (*Dasypogon*); Johnson, E, 323.—Argentina, Florida.

Dizonias bicinctus Loew, Cent., vii, 54; v. d Wulp, W, 96; Osten Sacken, Cat. Dipt., 68, 230; G, 169.

sp. Osten Sacken, G, 170.—Mexico.
sp. Osten Sacken, G, 169.—Mexico.

PSILOCURUS.

Orthoneuromyia Williston, S, 67.

modesta Williston, S, 68 (*Orthoneuromyia*).—South Dakota.
nudiusculus Loew; Williston, L, 10.

LAPHYSTIA.

sexfasciata Say (*Dasypogon*); Bigot, B, 87; Williston, L, 9; M, 53; Johnson, E, 323.—Montana.

Triclis notata Bigot, A, 433; B, 87. [Williston].
Laphystia subfasciata Bigot, Ann. Soc. Ent. Fr., 236, 1879 [Williston].

TRICLIS.

tagax Williston, L, 9, pl. I, figs. 6, 6a.; N, 289.—California.

CERATURGUS.

aurulentus Wiedemann; Brauer, Wien. Ent. Zeit., ii, 56.
cruciatus Say; Williston, L, 6, pl. I, fig. 2; Brauer, Wien. Ent. Zeit., ii, 56; Johnson, E, 323.—Florida.
dimidiatus Macquart; Brauer, Wien. Ent. Zeit., ii, 56.
geniculatus Bigot, A, 443.—Mexico.
nigripes Williston, N, 287.—Georgia.
rufipennis Macquart; Brauer, Wien. Ent. Zeit., ii, 56.

MYELAPHUS.

lobicornis Osten Sacken, (*Ceraturgus*); Williston, L, 7; M, 53; N, 288; Brauer, Wien. Ent. Zeit., ii, 56 (*Ceraturgus*).—California.
melas Bigot; Williston, L, 5-7, pl. I, figs. 1, 1a.—California.
rufus Williston, L, 7; N, 288.—California.

DICOLONUS.

simplex Loew; Williston, L, 11, pl. I, fig. 4.

DIOCTRIA.

albius Walker; Williston, L, 8; Slosson, J, v, 6.—Washington, Connecticut, Mt. Washington.

nitida Williston, L, 8.—Washington.

pusio Osten Sacken; Williston, N, 288.—Colorado.

sackeni Williston, L, 8.—Washington.

CALLINICUS.

calcaneus Loew; Williston, L, 33.

Dasypogon bilineatum Bigot, A, 411. [Williston].

HETEROPOGON.

Loew, Linn. Ent., ii, 488, 1847; *Anisopogon* Loew, Berl. Ent. Zeit., 1874, 377; Osten Sacken, Cat. Dipt., 68.

gibbus Loew; Osten Sacken, Cat. Dipt., 68 (*Anisopogon*); Coquillett, C, 20 (id.).

lautus Loew; Osten Sacken, Cat. Dipt., 69, (*Anisopogon*); Williston, L, 16, (id); Coquillett, C, 20 (id.)—Washington, California.

ludius Coquillett, C, 20, 21 (*Anisopogon*).—California, British Columbia.

patruelis Coquillett, C, 20-22 (*Anisopogon*).—Texas.

phœnicurus Loew; Osten Sacken, Cat. Dipt., 69 (*Anisopogon*); Coquillett, C, 20 (id.).

rubidus Coquillett, C, 20, 21 (*Anisopogon*).—California.

senilis Bigot, A, 423 (*Anisopogon*); Williston, L, 15 (id.); Coquillett, C, 20, 21 (id.); Johnson, E, 323 (id.).—California, Florida.

vespoides Bigot, A, 423 (*Anisopogon*); Coquillett, C, 20 (id.).—California.

CYRTOPOGON.

aurifex Osten Sacken; Williston, L, 11.

bimacula Say; Slosson, J, v, 6.—Mt. Washington.

callipedilus Loew; Williston, L, 12.—Wyoming.

cerussatus Osten Sacken; Williston, L, 15.

chrysopogon Loew; v. d. Wulp, W, 100; Johnson, E, 323.—Florida.

cymbalista Osten Sacken; Williston, L, 12.

dasyllis Williston, S, 66.—Colorado.

dasylloides Williston, L, 11; S, 67.—Washington.

dubius Williston, L, 13.—Oregon.

? gibber Williston, L, 14, pl. I, fig. 9.—California.

? Holopogon ? appendiculatus Bigot, A, 438. [Williston].

? nebulo Osten Sacken; Williston, L, 14.—Washington.

nugator Osten Sacken; Williston, L, 13.—Oregon.

plausor Osten Sacken; Williston, L, 12.—Colorado.
positivus Osten Sacken; Williston, L, 13, 14.
præpes Williston, L, 12, 13.—Washington.
profusus Osten Sacken; Williston, L, 13, 14.
sudator Osten Sacken; Williston, L, 13.
n. sp. Williston, L, 13, no. 18.—Washington.
n. sp. Williston, L, 12, no. 15.—California.
sp. Williston, R, pl. IX, fig. 6.
(? Lasiopogon) n. sp. Williston, L, 14, no. 22, pl. I, fig. 10.—Wyoming.
? n. sp. Williston, L, 14, no. 21, pl. I, fig. 11.—Wyoming.

PLESIOMMA.

indecora Loew; Johnson, D, 274.—Jamaica.
lineata Fabricius; Williston, P, 170.—San Domingo.

DAMALIS.

Fabricius, Syst. Antl. 1805.

n. sp. Williston, MS.—Mexico.

HOLCOCEPHALA.

abdominalis Say; Slosson, J, v, 6; Johnson, E, 323.—Mt. Washington, Florida.
calva Loew; Johnson, E, 323.—Florida.
longipennis Bellardi; Osten Sacken, G, 171.—Costa Rica.

Discocephala divisa Walker, Trans. Ent. Soc. Lond. n. ser. v, 279; Osten Sacken, Cat. Dipt., 71 (*Holcocephala*). [fide Williston].

HOLOPOGON.

nitidiventris Bigot, A, 437.—California. ["gen. nov.?" Bigot].
philadelphicus Schiner; Johnson, E, 323.—Florida.

PYCNOPOGON.

cirrhatus Osten Sacken; Williston, L, 15.—California.

STICHOPOGON.

catulus Osten Sacken, G, 170.—Mexico.
trifasciatus Say; Williston, M, 53; N, 289; Osten Sacken, G, 170; Townsend, K, 598.—New England, Kansas, Southern California, Lower California, Mexico.

Dasypogon candidus Macquart, Dipt. Exot. 1er, Suppl., 67, 48; Bellardi, Saggio, etc., ii, 78 (*Stichopogon*); Osten Sacken, Cat. Dipt., 70. [Williston].
Dasypogon galascens Walker, Trans. Ent. Soc. N. S., v, 277. [Bellardi].
Dasypogon fasciventris Macquart, Dipt. Exot., 4e Suppl., 69, 75, pl. VI, fig. 13. [Bigot].

sp. Osten Sacken, G, 170.—Mexico.

TOWNSENDIA.

Williston, Kansas University Quarterly, iv, 107.

minuta Williston, Kansas University Quarterly, iv, 108.—Mexico.

HABROPOGON.

Loew; *Dactiliscus* Rondani.

?bilineatus Williston, **L**, 11, pl. I, fig. 8.—North California.

NEOLAPARUS.

Williston, Psyche, 255, 1889; *Laparus* Loew, Bemerk. Fam. Asil., 1851 (name preoc.).

? pictitarsis Bigot, **A**, 417 (*Laparus*); Williston, **L**, 1, 4, 25 (id.).

SAROPOGON.

adustus Loew; Williston, **L**, 23; **N**, 290.—Western Kansas.

combustus Loew; Williston, **L**, 23; **N**, 290.—Western Kansas.

senex Osten Sacken, **G**, 179.—Mexico.

n. sp. Williston, **N**, 289.—California.

BLEPHEREPIUM.

Rondani, Stud. Entom. i, 89, 1848; *Planetolestes* E. Lynch A., **F**, 6, 72; Osten Sacken, **G**, 171; **H**, 418.

coarctatum Perty, Delectus, etc., 181, pl. XXXVI, fig. 4 (*Laphria*); Walker, List, etc., vi, 504 (*Dasypogon*); E. Lynch A., **F**, 6, 73; v. d. Wulp, **W**, 88; Williston, **Q**, 75.— Guatemala, Panama.

Dasypogon bonariensis Macquart, Dipt. Exot. i, 2, 194; Walker, List, etc., vi, 439 [E. Lynch A.].

? Blepharepium luridum Rondani, Stud. Entom., 89.

Dasypogon subcontractus Walker, Dipt. Saund., 455 —Amazon.

Senobasis annulatus Bigot, La. Sagra's Hist Cuba, 789, pl. XX, fig. 3; Osten Sacken, Bull. Buff. Soc. Nat. Sci., 1874, 184; Cat. Dipt., 72 (*Diogmites*).

Dasypogon secabilis Walker, Trans. Ent. Soc. (2), v, 276; Bellardi, Saggio, etc, ii, 63, pl I, fig. 4; Schiner, Verh. zool.-bot Gesellsch. xvi, 701 (*Senobasis*); Osten Sacken, **G**, 171 (*Planetolestes*); **H**, 418, (id.). [E. Lynch A.].

DEROMYIA.

Philippi; Williston, Psyche, 1889, 256; *Diogmites* Loew, Centur., vii, 35; Osten Sacken, Cat. Dipt, 72; **G**, 173; **H**, 418.

affinis Bellardi; Osten Sacken, **G**, 173 (*Diogmites*): Cat. Dipt., 72 (id.).

angustipennis Loew (*Diogmites*); Osten Sacken, Cat. Dipt., 72 (id.); Williston, **L**, 25.

basalis Walker, Dipt. Saund., 95 (*Dasypogon*); v. d. Wulp, **X**, 2.—Connecticut.

Diogmites bilineatus Loew, Cent. vii, 40; Osten Sacken, Cat Dipt., 72. [v. d. Wulp].

bigotii Bellardi; Osten Sacken, Cat. Dipt., 72 (*Diogmites*): Johnson, **E**, 324.—Florida.

brunnea Fabricius; Schiner, Verh. zool.-bot. Gesellsch., xvi, 677 (*Dasypogon*); Walker, List, etc., vi, 421 (id.); Osten Sacken Cat. Dipt., 72 (*Diogmites*).

craverii Bellardi; Osten Sacken, Cat. Dipt., 72 (*Diogmites*); **G**, 178 (id.).

cuantlensis Bellardi; Osten Sacken, Cat. Dipt., 72 (*Diogmites*); **G**, 174, 175 (id.).—Guatemala.

discolor Loew (*Diogmites*); Osten Sacken, Cat. Dipt., 72 (id.); v. d. Wulp, **V**, 77 (id.); Williston, **L**, 25.

dubius Bellardi; Osten Sacken, Cat. Dipt., 72 (*Diogmites*); **G**, 173 (id.).

duillius Walker; Osten Sacken, Cat. Dipt., 73 (*Diogmites*); **G**, 178 (id.).

jalapensis Bellardi; Osten Sacken, Cat. Dipt., 72 (*Diogmites*); **G**, 173, 174, 178 (id.).—Guatemala.

lindigii Schiner; Novara Exped., 165 (*Dasypogon*); Osten Sacken, **G**, 174 (*Diogmites*).—South America, Panama.

memnon Osten Sacken, **G**, 174, pl. III, fig. 9 (*Diogmites*).—Costa Rica, Panama.

nigripes Bellardi; Osten Sacken, Cat. Dipt., 72 (*Diogmites*); **G**, 175 (id.).

pseudojalapensis Bellardi; Osten Sacken, Cat. Dipt., 73 (*Diogmites*); **G**, 173, 178 (id.).

rubescens Bellardi; Osten Sacken, Cat. Dipt., 73 (*Diogmites*); **G**, 173, 174, 176, 177 (id.).—Costa Rica.

rufescens Macquart; v. d. Wulp, **V**, 76 (*Diogmites*); **W**, 91; Williston, **L**, 24.—Arizona.

sallæi Bellardi; Osten Sacken, Cat. Dipt., 73 (*Diogmites*); **G**, 174, pl. III, fig. 8 (id.).

tau Osten Sacken, **G**, 174, 176, pl. III, fig. 11 (*Diogmites*).—Panama.

ternata Loew (*Diogmites*); Osten Sacken, Cat. Dipt., 73; **G**, 173 (id.); Williston, **L**, 25; Johnson, **E**, 323.—Georgia, Florida.

umbrina Loew (*Diogmites*); Osten Sacken, Cat, Dipt., 72 (id.): Williston, **L**, 25.—Virginia, Connecticut.

winthemi Wiedemann, Dipt. Exot. i, 223; Auss. Zw. Ins., i, 387 (*Dasypogon*); Schiner, Verh. zool.-bot. Gesellsch., xvi, 678 (id.); v. d. Wulp, **W**, 93; **X**, 2; Osten Sacken, **G**, 177 (*Diogmites*); Williston, **L**, 24, pl. II, fig. 6; **Q**, 75; Johnson, **E**, 324.—Indiana, Kansas, Connecticut, Florida, South America.

? *Diogmites misellus* Loew, Cent. vii, 39; Osten Sacken, Cat. Dipt., 72; **G**, 173, 177; Williston, **L**, 24 (*Deromyia*); Psyche, 1889, 256 (id.).

sp. Osten Sacken, **G**, 178 (*Diogmites*).

sp. Johnson, **E**, 324.—Florida.

sp. Johnson, **E**, 324.—Florida.

LESTOMYIA.

Williston, **L**, 19.

fraudigera Williston, **L**, 21, pl. II, fig. 5.—California.

sabulonum Osten Sacken (*Clavator*); Williston, **L**, 20, pl. II, fig. 4.—California.

TARACTICUS.

brevicornis Williston, **L**, 19, 22, pl. II, fig. 3; **M**, 54 (*Aphamartania*). —Washington.

niger Macquart; Williston, **L**, 22 (*Ceraturgus*); Brauer, Wien. Ent. Zeit., ii, 54, 56 (id.).

octopunctatus Say; Williston, **L**, 22, pl. II, figs. 2, 2a, Johnson, **E**, 323; Slosson, **J**, vi, 320.—Florida, Mt. Washington.

vitripennis Bellardi; Osten Sacken, Cat. Dipt., 66 (*Ceraturgus*); Brauer, Wien. Ent. Zeit., ii, 56. [fide Williston.]

COPHURA.

Osten Sacken, **G**, 181.

sodalis Osten Sacken, **G**, 181, pl. III, fig. 13.—Mexico.

LASTAURUS.

fallax Macquart, Dipt. Exot. Suppl. i, 63, pl. VII, fig. 5 (*Dasypogon*); Schiner, Verh. zool.-bot. Gesellsch., xvii, 373; Osten Sacken, **G**, 180; Williston, **Q**, 73.—Venezuela, Mexico, Costa Rica, Panama.

? *Lastaurus mutabilis* Loew, Bemerk. Fam. Asil., 12; Osten Sacken, **G**, 180 [Schiner].

lugubris Macquart, Dipt. Exot. Suppl. i, 64 (*Dasypogon*); Schiner, Verh., zool.-bot. Gesellsch., xvii, 373; Osten Sacken, **G**, 180.—South America, Guatemala.

? *Lastaurus anthracinus* Loew, Bemerk. Fam. Asil., 12; Osten Sacken, **G**, 179, pl. III, fig. 10 [Schiner].

NICOCLES.

analis Jænnicke; Brauer, Wien. Ent. Zeit., ii, 56 (*Leptarthrus*).

abdominalis Williston, **L**, 17, pl. I, figs. 14, 14a, 14b; Coquillett, **C**, 119.

æmulator Loew; Coquillett, **C**, 119; Brauer, Wien. Ent. Zeit., ii, 56 (*Leptarthrus*).

argentatus Coquillett, **C**, 119, 120.—California.

dives Loew; Coquillett, **C**, 119; Brauer, Wien. Ent. Zeit., ii, 56 (*Leptarthrus*).

? fur Williston, **M**, 53 (*Aphamartania*).—Arizona.

pictus Loew; Coquillett, **C**, 119; Johnson, **E**, 323; Brauer, Wien. Ent. Zeit., ii, 56 (*Leptarthrus*).—Florida.

politus Say; Coquillett, **C**, 119; Brauer, Wien. Ent. Zeit., ii, 56 (*Leptarthrus*).

rufus Williston, **L**, 18, pl. I, fig. 15; Coquillett, **C**, 119.—Washington.

? scitulus Williston, **L**, 19, pl. II, figs. 1, 1a; **M**, 54 (*Aphamartania*); Coquillett, **C**, 119.—Washington.

LOEWIELLA.

Williston MS.; *Blax* Loew, Centur. x, 24, 1872; *Blacodes* Loew, Berl. Ent. Zeit., 1874, 377; Osten Sacken, Cat. Dipt., 71 (name preoc.).

bella Loew; Osten Sacken, Cat. Dipt., 71 (*Blacodes*); Williston, **M**, 54 (*? Aphamartania*); Coquillett, **C**, 33 (*Blacodes*).
clausa Coquillett, **C**, 33, 34 (*Blacodes*).—California.
cristata Coquillett, **C**, 33, 34 (*Blacodes*).—California.
trunca Coquillett, **C**, 33, 34 (*Blacodes*). —California.

PSEUDORUS.

bicolor Bellardi; Williston, **L**, 15; Psyche, 1889, 256; Osten Sacken, **G**, 183.—Guatemala.

DORYCLUS.

Jænnicke, Neue Exot. Dipt., 58; Abhand. senck. Gesellsch, vi, 366, 1867; *Ampyx* Walker, List, etc., 564, 1855 (name preoc.).

distendens Wiedemann, Auss. Zw. Ins., i, 571 (*Asilus*); Jænnicke, Neue Exot. Dipt., 366, pl. XLIV, fig. 3; Roeder, Berl. Ent. Zeit., xxxi, 77; Williston, Psyche, 1889, 256; **Q**, 77.—Brazil, Guatemala, Mexico.

Megapoda crassitarsis Macquart, Dipt. Exot. Suppl. i, 70, pl. VII, fig. 11 (male). [Roeder].
Megapoda cyaneiventris Macquart, Dipt. Exot. Suppl. i, 71, pl. VII, fig 12 (female); Osten Sacken, Cat. Dipt., 73. [Roeder].
Ampyx varipennis Walker, List, etc., vii, 564; Osten Sacken, **G**, 182 (male) (*Doryclus*). [Williston].
Doryclus latipes v. d. Wulp, Tijdschr. v. Ent., (2) v, 215, pl. IX, figs. 7-12 (female). [Roeder].

APHESTIA.

Schiner. Verh. zool -bot. Geselsch, xvi, 673, 1865.

nigra Bigot, **A**, 235.—Mexico.

CEROTAINIA.

Schiner; *Cerotolænia* E. Lynch A., **F**, 19, 1880.

dubia Bigot, **A**, 238.—Mexico.
macrocera Say; Johnson, **D**, 274. -Jamaica.
nigra Bigot, **A**, 238.

ATOMOSIA.

Macquart; *Cormansis* Walker, Dipt., Saund., 154.

beckeri Jænnicke; E. Lynch A., **F**, 19. Buenos Aires?.
eupoda Bigot, **A**, 234 (*Cormansis*).--Mexico.
mucida Osten Sacken, **G**, 184.--Mexico.
puella Wiedemann; Rondani, Truqui's Stud. Ent., 61; Osten Sacken, **H**, 418, Johnson, **E**, 323.—Florida, Brazil.
soror Bigot, **A**, 236.--Mexico.
tibialis Macquart; v. d. Wulp, **W**, 105.—Columbia.

xanthopus Wiedemann, Auss. Zw. Ins., i, 529; v. d. Wulp, **W**, 105.—Mexico.
sp. Osten Sacken, **G**, 184.—Costa Rica.

ATONIA.

Williston, Psyche, 1889, 257.

mikii Williston, **N**, 290 (*Atomosia*); Psyche, 257, 1889.

HYPERECHIA:

Schiner, Verh. zool.-bot. Gesellsch, 16, 673, 1866.

atrox Williston, **L**, 28, pl. II, figs. 7, 7a, 7b.—Pennsylvania.

POGONOSOMA.

arachnoides Bigot, **A**, 227.—Mexico.
dorsata Say; Williston, **L**, 33; **M**, 56.—Washington.
melanoptera Wiedemann; Williston, **M**, 56.—Florida.

DASYLLIS.

albicollis Bigot, **A**, 229.—Mexico.
astur Osten Sacken; Williston, **L**, 26, 27.—Washington, Oregon, Western Kansas.
columbica Walker; Williston, **L**, 27.—Washington, Oregon.
flavicollis Say; Williston, **L**, 25; Slosson, **J**, v, 6.—Mt. Washington, Connecticut.
grossa Fabricius; v. d. Wulp, **W**, 103.—Canada, Connecticut, Florida.

Laphria tergissa Say; Osten Sacken. Cat. Dipt., 75 and note 115 (*Dasyllis*); Williston, **L**, 26, 27 (id.); Johnson, **E**, 324 (id.).

posticata Say; Bigot, **A**, 218 (*Laphria*); Williston, **L**, 26 27; Johnson, **E**, 324.—Florida.
sacrator Walker; Williston, **L**, 26.—Mt. Washington.
thoracica Fabricius; Bigot, **A**, 218; Williston, L, 26.—Connecticut, Pennsylvania.
unicolor Williston, **L**, 26.—Washington.

LAPHRIA.

carbonarius nom. nov. Williston, MS.—California.

Laphria anthrax Williston (nec Meigen), **L**, 29; **M**, 54.—California.

canis Williston, **L**, 31; **M**, 54; Slosson, **J**, vi, 6.—Connecticut, Mt. Washington.
corallogaster Bigot, **A**, 227.—North America (*Lampria?*).
ferox Williston, **L**, 29; **M**, 54.—Washington.
franciscana Bigot, **A**, 225; Williston, **L**, 31; **M**, 54. Washington, California.
gilva Linn. (*Asilus*); Loew, Linn. Ent., ii, 548, 8; Schiner, Faun. Austr., i, 139; v. d. Wulp, **W**, 104.—Canada, Colorado, California.

Asilus rufus de Geer, Ins., vi, 241, 4, pl. XIII, fig. 15.

Laphria bilineata Walker, List. etc, iv, 1156; Osten Sacken, Cat. Dipt., 76; Williston, **L**, 30; **M**, 34. [Williston].

ichneumon Osten Sacken, **G**, 185, pl. III, fig. 6.—Guatemala.

numitor Osten Sacken, **G**, 185.—Nicaragua.

olbus Walker; Schiner, Novara Exped., 173.—South America.

pubescens Williston, **L**, 32; **M**, 55; **N**, 290; Slosson, **J**, v. 6.—White Mountains, Mt. Washington, Oregon, Washington.

rapax Osten Sacken; Williston, **L**, 29.

ruficauda Williston, **M**, 55; **P**, 170.—San Domingo, Cuba.

saffrana Fabricius; Bigot, **A**, 218; Williston, **M**, 54, 56; Osten Sacken, Cat. Dipt., 75 (*Dasyllis*); Johnson, **E**, 324.—Florida, North Carolina.

sericea Say; Williston, **M**, 54; Slosson, **J**, v, 6.—Mt. Washington.

ventralis Williston, **M**, 54, 55.—California.

vivax Williston, **L**, 30; **M**, 54, 55.—Washington.

vultur Osten Sacken; Williston, **L**, 29; **M**, 54.—Washington, Oregon.

xanthippe Williston, **L**, 31.—Oregon.

LAMPRIA.

aurifex Osten Sacken, **G**, 187.—Mexico, Costa Rica.

Lampria clavipes Bellardi, (nec Fabricius, Wiedemann, Macquart), Saggio, etc., ii, 13, pl. I, fig. 15. [Osten Sacken].

bicolor Wiedemann; Schiner, Verh. zool.-bot. Gesellsch., xvi, 692, 709; Bigot, **A**, 217; v. d. Wulp, **W**, 105; Williston, **L**, 32; Johnson, **E**, 324.—Brazil, Florida, Connecticut, Pennsylvania.

clavipes Fabricius; Schiner, Novara Exped., 174; Verh. zool.-bot. Gesellsch., xvi, 691; Macquart, Dipt. Exot., i, 2, 61, (non Suppl. iii, 22); Walker, List, etc., vii, 510; v. d. Wulp, **W**, 104; Osten Sacken, **G**, 186; (non Bellardi, Saggio, etc.); Williston, **Q**, 80.—Panama.

felis Osten Sacken; Williston, **L**, 32.—Washington.

mexicana Macquart; v. d. Wulp, **W**, 105; Osten Sacken, **G**, 188.

rubriventris Macquart; Williston, **L**, 32.—Georgia.

spinipes Fabricius, Syst. Antl., 162 (*Laphria*); Wiedemann, Dipt. Exot., i, 240 (id.); Auss. Zw. Ins., i, 525 (id.); Schiner, Verh. zool.-bot. Gesellsch, xvi, 692; Osten Sacken, **G**, 187.—Brazil, Central America.

Laphria affinis Fabricius, Syst. Antl., 163. [Williston].

NUSA.

Walker, Dipt. Saund. 108, 1854; Williston, Psyche, 1889, 256; *Andrenosoma* Rondani, Dipt. Ital. Prodr., i, 160, 1856; Osten Sacken, Cat. Dipt., 77.

chalybea Williston, **M**, 56 (*Andrenosoma*); **P**, 170 (id.). —San Domingo, Cuba.

cincta Bellardi; Osten Sacken, Cat. Dipt., 77 (*Andrenosoma*); **G**, 188 (id.).—British Honduras.

cineria Bellardi; Osten Sacken, Cat. Dipt., 77 (*Andrenosoma*); **G**, 188 (id.)—Panama.

formidolosa Walker; Osten Sacken, Cat. Dipt., 27 (*Andrenosoma*); **G**, 188 (id.).—Guatemala, Nicaragua, Panama.

fulvicauda Say, Jour. Acad. Phil. iii, 53, pl. VI; Compl. Wr., i, 12 (*Laphria*); Williston, **L**, 33; **M**, 56 (*Andrenosoma*).—California, Maine, Florida.

Laphria pyrrhacra Wiedemann, Auss. Zw. Ins., i, 517; Schiner, Novara Exped., 175 (*Andrenosoma*); Osten Sacken, Cat. Dipt., 77 (id.); **G**, 188 (id.); Johnson, **E**, 324 (id.).

xanthocnema Wiedemann; Schiner, Verh. zool.-bot. Gesellsch., xvi, 691; v. d. Wulp, **W**, 104 (*Andrenosena*); Osten Sacken, Cat. Dipt., 77 (id.).

sp. Osten Sacken, **G**, 189 (*Andrenosoma*).—Guatemala.

OMMATIUS.

marginellus Fabricius; Roeder, I, 339.—Portorico.

peregrinus Osten Sacken, **G**, 210.—Panama.

saccas Walker; Johnson, **D**, 274.

tibialis Say; Williston, **M**, 76; Johnson, **E**, 324.—New England, Georgia, Florida.

ATRACTIA.

marginata Osten Sacken, **G**, 212.—Nicaragua.

PROCTOCANTHUS.

arno Townsend, **K**, 599.—Lower California.

brevipennis Wiedemann, v. d. Wulp, **W**, 108; Williston, **M**, 73; Osten Sacken, **G**, 206; Johnson, **E**, 324.—Argentina, Georgia.

craverii Bellardi; Osten Sacken, **G**, 206.

exquisitus Osten Sacken, **G**, 206, pl. III, fig. 12.—Mexico.

fulviventris Macquart; Johnson, **E**, 324.—Florida.

heros Wiedemann, v. d. Wulp, **W**, 108; Williston, **M**, 73, 74; Johnson, **E**, 324.

milbertii Macquart; Williston, **M**, 73, 74; **R**, pl. IX, fig. 7; Osten Sacken, **G**, 206.

philadelphicus Macquart; Williston, **M**, 73, 75; Johnson, **E**, 324.—New England, Florida.

rufiventris Macquart; Roeder, **I**, 339.—Portorico.

rufus Williston, **M**, 73, 74.—North Carolina, Massachusetts.

virginianus v. d. Wulp, **W**, 109, pl. X, figs. 5, 6.—Virginia.

ECCRITOSIA.

amphinome Walker, List, etc., ii, 387; Osten Sacken, **G**, 207.—Mexico, Lower California.

Proctocanthus zamon Townsend, **K**, 600 [fide Williston].

ERAX.

æstuans Wiedemann (non Linneus); Osten Sacken, Cat. Dipt., 234; Williston, **M**, 64, 65, 72.—Eastern States, Brazil.

Erax rufibarbis Macquart, Dipt. Exot., I, 2, 116, 22; Osten Sacken, Cat. Dipt., 79. [Williston].

affinis Bellardi; Osten Sacken, **G**, 202.

anomalus Bellardi; Williston, **M**, 64, 65, 69; **U**, 137; Osten Sacken, **G**, 198-201; Coquillett, **C**, 175 (*Efferia*).—Arizona.

aridus Williston, **T**, 254.—California.

bastardii Macquart; Williston, **M**, 64, 65, 71; **O**, 419, fig. 525; Johnson, **E**, 324.—Atlantic and Central States, Portorico.

? *Asilus æstuans* Linn. (non Wiedemann), Syst. Nat., ii, 1007, 5; Amœn. Acad., vi, 413, 95; Fabricius, Syst. Ent., iv, 379, 8. (For other references see Osten Sacken, Cat. Dipt., 79). [Williston].

? *Asilus macrolabis* Wiedemann, Auss. Zw. Ins., i, 458, 51; Osten Sacken, Cat. Dipt., 79 *(Erax)*; v. d. Wulp, **W**, 113. [Williston].

Erax femoratus Macquart, Dipt. Exot., i, 2, 115, 20; Osten Sacken, Cat. Dipt., 79; Roeder I, 339. [Williston].

Erax incisuralis Macquart, Dipt. Exot., i, 2, 117, 24; Osten Sacken, Cat. Dipt., 79. [Williston].

Erax tibialis Macquart, Dipt. Exot., i, 2, 118, 27; Osten Sacken, Cat. Dipt., 79. [Williston].

bellardi Schiner; Osten Sacken, Cat. Dipt., 81 (*Neocristicus*); Williston, **Q**, 85.

candida Coquillett, **C**, 175-177 (*Efferia*); Williston, **U**, 137.—California.

carinatus Bellardi; Osten Sacken, **G**, 203, 204; Townsend, **K**, 598.

?*Erax* sp. Osten Sacken, **G**, 205, no. 11. [Osten Sacken].

cinerascens Bellardi; Osten Sacken, **G**, 198, 202; Johnson, **E**, 324; Townsend, **K**, 599.—Lower California, Florida, Washington, Western Kansas, Arizona, Connecticut.

Erax furax Williston, **M**, 64, 65, 67. [Williston].

Erax albibarbis Macquart; Osten Sacken, Cat. Dipt., 79. [Osten Sacken].

costalis Williston, **M**, 64.

comatus Bellardi; Osten Sacken, **G**, 203.

completus Macquart; Osten Sacken, **G**, 199.

dubius Williston, **M**, 64.—Washington.

Erax n. sp. Williston, l c., 68.

flavofasciatus Wiedemann; v. d. Wulp, **W**, 113.

halæsus Walker; Johnson, **D**, 274.—Jamaica.

jubatus Williston, **M**, 64-66; **U**, 137; Osten Sacken, **G**, 203.—New Mexico.

lascivus Wiedemann; Schiner, Verh. zool.-bot. Gesellsch., xvi, 687; xvii, 394; Williston, **Q**, 86.

latrunculus Williston, **M**, 64, 65, 67; **T**, 254.—Arizona, Montana.

leucocomus Williston, **M**, 64, 65, 69.—Western Kansas, New Mexico.

maculatus Macquart; Osten Sacken, **G**, 198, 200, 201; Williston, **Q**, 86.—North Carolina, Texas, Georgia, Florida, Mexico, Guatemala, South America.

Erax lateralis Macquart; Osten Sacken, Cat. Dipt., 79; Williston, **M**, 64, 65, 70; Johnson, **E**, 324. [Williston].

Erax ambiguus Macquart; Osten Sacken, Cat. Dipt., 79. [Osten Sacken].

Asilus interruptus Macquart, Hist. Nat. Dipt., i, 310. [Osten Sacken].

Erax (*Eristicus*) *villosus* Bellardi; Osten Sacken, Cat. Dipt., 81 (*Neoeristicus*). [Osten Sacken, Williston].

nigrimystaceus Macquart; Osten Sacken, **G**, 203.

parvulus Bellardi; Osten Sacken, **G**, 203, 204.

pernicis Coquillett, **C**, 175-177 (*Efferia*).—California.

pogonias Wiedemann; v. d. Wulp, **W**, 114.—Arizona.

prolificus Osten Sacken, **G**, 197, 198, 202-205.—Mexico.

quadrimaculatus Bellardi; Osten Sacken, **G**, 197.

Erax bimaculatus Bellardi, Saggio, etc., ii, 45, pl. II, fig. 11; Osten Sacken, Cat. Dipt., 80. [Osten Sacken, Williston].

rapax Osten Sacken, **G**, 198, 201.—Mexico.

rava Coquillett, **C**, 175, 176 (*Efferia*); Williston, **U**, 137.—Texas.

rufitibia Macquart; Roeder, **I**, 329,—Portorico.

similis Williston, **M**, 68.—Arizona.

stamineus Williston, **M**, 64, 68; **U**, 137; Osten Sacken, **G**, 201.—Montana.

stylatus Fabricius; Schiner, Verh. zool.-bot. Gesellsch, xvi, 686; v. d. Wulp, **W**, 112.—Wisconsin, Brazil.

tagax Williston, **M**, 64, 65.—Arizona.

tricolor Bellardi; Osten Sacken, **G**, 202; Townsend, **K**, 599.—Lower California.

triton Osten Sacken, **G**, 198, 200.—Mexico.

unicolor Bellardi; v. d. Wulp, **W**, 114; Osten Sacken, **G**, 203, 205.

varipes Williston, **M**, 64, 71.—Arizona, Western Kansas.

sp. Johnson, **E**, 324.—Florida.

sp. Osten Sacken, **G**, 203, 204, no. 9.—Mexico.

sp. Osten Sacken, **G**, 205, no. 10.—Mexico.

sp. Osten Sacken, **G**, 198, 202, no. 7.—Costa Rica.

sp. Osten Sacken, **G**, 198, 199, no. 2.—Mexico.

MALLOPHORA.

bergii E. Lynch A., **F**, 35; An. Soc. Cien. Arg., xv, 10.—South America, Mexico.

Mallophora pica Macquart Dipt. Exot. Suppl. iv, 78; Osten Sacken, Cat. Dipt., 78. [E. Lynch A.].

bomboides Weidemann; Williston, **M**, 57, 58; Coquillett, **C**, 118; Johnson, **E**, 324.—Florida.

clausicella Macquart; Williston, **M**, 59; Coquillett, **C**, 118.—Pennsylvania.

fautrix Osten Sacken, **G**, 191, pl. III, fig. 14; Coquillett, **C**, 118.—Mexico, California.

fulvi-analis Macquart; Osten Sacken, **G**, 191.

fulviventris Macquart; Osten Sacken, **G**, 191.

guildiana Williston, **M**, 60; Coquillett, **C**, 118.—Western Kansas, Montana, North Carolina, California.

infernalis Bellardi, Saggio, etc., ii, 21 (Wiedemann?); v. d. Wulp, **W**, 106; Osten Sacken, **G**, 189.—Mexico, Panama.

laphroides Wiedemann; Bigot, Ann. Soc. Ent. Fr., 542, 1857 (*Megaphorus*); Williston, **L**, pl. II, fig. 11; **M**, 59; Coquillett, **C**, 118; Johnson, **E**, 324.—Georgia, Florida.

Mallophora heteroptera Macquart; Bigot, l. c. (*Megaphorus*); Schiner, Verh. zool.-bot. Gessellsch, xvii, 387.—Brazil.

macquarti Rondani, N. Ann. di Bologna, 1850 (Sep. Dipt. etc., Osculati, 13); Loew in litt. in Osten Sacken, Cat. Dipt., 78, 233; Osten Sacken, **H**, 420.—Brazil, Cuba.

Mallophora scopifer Macquart (non Wiedemann), Dipt. Exot. i, 2, 89; Bigot, in Sagra's Nat. Hist. Cuba, 790.

Mallophora scopipeda Rondani, Arch. per la Zool., iii, 1863 (Sep., Dipt. Exot., 46); Williston, **Q**, 83.

megachile Coquillett, **C**, 118.—California.

nigra Williston, **M**, 58; Coquillett, **C**, 118.—Minnesota.

orcina Wiedemann; Williston, **M**, 58; Osten Sacken, **G**, 191; Coquillett, **C**, 118; Johnson, **E**, 324. Virginia, Florida, Arizona.

pluto Wiedemann, Auss. Zw. Ins., i, 477 (*Asilus*); Schiner, Verh. zool.-bot. Gesellsch., xvi, 688; Novara Exped., 176; v. d. Wulp, **W**, 106; Osten Sacken, **G**, 190, 191.—South America, Guatemala.

robusta Wiedemann; w. d. Wulp, **W**, 106.

sp. Osten Sacken, **G**, 190, no. 4.—Guatemala.

sp. Osten Sacken, **G**, 190, no. 3. Guatemala.

PROMACHUS.

albifacies Williston, **M**, 60, 63, 64; Osten Sacken, **G**, 192, 195.—Arizona, Mexico.

anceps Osten Sacken, **G**, 192, 194.—Panama.

? *Promachus fuscipennis* Bellardi (non Macquart), Saggio, etc., ii, 24, pl. II, fig. 1 [female; the male belongs to a different species.—Osten Sacken.]

bastardii Macquart; Williston, **M**, 60, 63; Osten Sacken, **G**, 192, 193.—New England, Pennsylvania.

Asilus ultimus Walker, Dipt. Saund., 136. [Osten Sacken].

cinctus Bellardi; Osten Sacken, **G**, 192, 193.—Guatemala, Nicaragua.

fitchii Osten Sacken, Cat. Dipt., 234, note 121; **G**, 192; Williston, **M**, 60, 61; Johnson, **E**, 324.—Kansas, Connecticut, Florida.

Trupanea apivorus Fitch: Osten Sacken, Cat. Dipt., 78.

forfex Osten Sacken, **G**, 192, 194.—Costa Rica.

Promachus quadratus Bellardi, Saggio, etc., ii, 27, pl. II, fig. 3; Osten Sacken, Cat. Dipt., 78. [Osten Sacken].

fuscipennis Macquart; Osten Sacken, **G**, 192.

nobilis Osten Sacken, **G**, 192, 196.—Costa Rica.

princeps Williston, **M**, 60, 62; Osten Sacken, **G**, 192.—Washington.

Promachus n. sp Williston, **L**, pl. II, fig. 14. [Williston.]

pulchellus Bellardi; Osten Sacken, **G**, 192.

quadratus Wiedemann; Osten Sacken, **G**, 192, 193.

rufipes Fabricius; Wiedemann, Auss. Zw. Ins., i, 487, 93; v. d. Wulp, **W**, 107; Williston, **M**, 60-62; Osten Sacken, **G**, 192.—Florida, Illinois.

trapezoidalis Bellardi; Osten Sacken, **G**, 192, 193.

truquii Bellardi; Osten Sacken, **G**, 192.

vertebratus Say; Williston, **M**, 60, 62; Osten Sacken, **G**, 192, 193.—Kansas.

n. sp. ? Townsend, **K**, 598.—Lower California.

sp. Osten Sacken, **G**, 193, no. 2.—Mexico.

sp. Osten Sacken, **G**, 192, 195, no. 5.—British Honduras.

STENOPROSOPUS.

arizonensis Williston, **S**, 76.—Arizona.

HELICMONEURA.

Bigot, Thoms. Arc. Ent. ii, 352, 1858; Williston, Psyche, 1889, 255; *Moctherus* Loew, Linn. Ent. iv, 58, 1849 (preoc.); *Neomoctherus* Osten Sacken, Cat. Dipt., 82, 1878.

plebeius Osten Sacken, **G**, 209, (*Neomoctherus*).—Mexico.

truquii Bellardi, v. d. Wulp, **W**, 116 (*Moctherus*).

NEOITAMUS.

affinis Williston, **S**, 73.—California.

distinctus Williston, **S**, 73; Slosson, **J**, vi, 320.—New Hampshire, Connecticut.

TOLMERUS.

notatus Wiedeman; Williston, **S**, 74.—North Carolina, Michigan, South Dakota, New England.

callidus Williston, **S**, 75.—Oregon, Washington.

ASILUS.

angustifrons Williston, **S**, 71.—Washington.

annulatus Williston, **S**, 70.—New Hampshire, Massachusetts, Connecticut, South Dakota, Kansas.

astutus Williston, **S**, 70.—California.

auratus Johnson, **E**, 305.—Florida.

chrysauges Osten Sacken, **G**, 208.—Guatemala.

flavipes Williston, **S**, 72.—Pennsylvania, Connecticut.

midas Brauer, Sitzb. d. k. Acad. d. Wiss., 387, pl. II, 1885; Williston, **S**, 69; Osten Sacken, **G**, 208, 209.—Mexico, New Mexico.

novæ-scotiæ Macquart; Williston, **S**, 68; Slosson, **J**, vi, 320; Johnson, **E**, 324.—Connecticut, Mt. Washington, Florida.

sericeus Say; Osten Sacken, **G**, 208; Williston, **S**, 68.—New England, Pennsylvania, Indiana, Kansas.

RHADIURGUS.

leucopogon Williston, **S**, 75.—South Dakota, Nebraska.

EPITRIPTUS.

niveibarbus Bellardi; v. d. Wulp, **W**, 117.

albispinosus Bellardi; v. d. Wulp, **W**, 117.

ARNAMOSTUS.

iopterus Wiedemann, Auss. Zw. Ins., i, 438 (*Asilus*); Loew, Diptf. Südafr., 142; Schiner, Verh. zool.-bot. Gesellsch. xvi, 684; Osten Sacken, **G**, 211.—Brazil, Honduras.

ADDENDUM TO PAGE 175 UNDER DIOCTRIA.

albius Walker; Coquillett, **C**, 80.

nitida Williston; Coquillett, **C**, 80.

parvulus Coquillett, **C**, 80.—California.

pusio Osten Sacken; Coquillett, **C**, 80.

resplendens Loew; Coquillett, **C**, 80.

rubidus Coquillett, **C**, 80.—California.

sackeni Williston; Coquillett, **C**, 80.

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Editorial Notes.

Prof. W. H. Carruth has just published, through Holt & Co., an annotated edition of Scheffel's Ekkehard, with introduction, illustrations and map.

Professor E. H. S. Bailey, who has spent the last six months working with masters in Germany, will return to his post in the department of chemistry about February 1st.

Dr. S. W. Williston has passing through the press a volume entitled "Synopsis of the Families and Genera of North American Diptera." The Bibliography in this number of the QUARTERLY will appear as an Appendix.

The *Scientific American* for January 4th and January 11th, has interesting articles on the bicycle, with many illustrations of parts and methods of manufacture.

Prof. F. H. Hodder has published through Eldredge & Bro., Philadelphia, a text book on the governmental machinery of Kansas, state and local, entitled "The Civil Government of Kansas." The introduction consists of a sketch of the history of the state.

Dr. E. C. Franklin, acting professor of chemistry, has spent some time during the autumn over the newly discovered simple elements argon and helium. He has successfully isolated both gases, preparing a large number of specimen tubes, and used them in demonstrations before his classes.

The department of Palæontology has for sale or exchange a magnificent slab, of about twenty square feet in area, of Uintacrinus Socialis from the Kansas Cretaceous. It has numerous complete heads in excellent preservation. Address Prof. S. W. Williston, Lawrence, Kansas.

The department of Physical Geology and Mineralogy at the University now has passing through the press volume I of the reports of the University Geological Survey of Kansas, and hopes that it will be ready for distribution about the first of April. It will contain about 100 pages of text with many figures and plates, and is devoted almost exclusively to the stratigraphy of the carboniferous area of Kansas.

A discovery of much interest has recently been made in western Kansas of an extinct species of Bison, the skull having an expanse of nearly four feet. Embedded below the humerus of the skeleton was a small, but perfectly formed arrow head. The Bison has not yet been identified with certainty, but seems closely allied to *B. antiquus*, though evidently larger. The formation is apparently the same as that which yielded the skeletons of Platygonus recently obtained by the University. The Bison skeleton, that of a bull, will be mounted shortly in the University Museum.

Prof. L. L. Dyche returned about November 1st from a six month's trip to Greenland. As the fruits of his expedition the University museum received ten walrus skins; two walrus skeletons; two narwhale skeletons; a group of four barren ground caribou or Greenland reindeer, five polar bears, includiug two old males, one female, one yearling and a cub, a series suitable for a fine group; ten seal skins, representing three species; a collection of one hundred and thirty skulls of North Greenland mammals; a collection of North Greenland fowls; a collection of rocks characteristic of the country; three hundred bird skins, more than twenty-five hundred eggs, and a number of nests; and finally, a considerable ethnological collection.

The new Physics and Electrical Engineering Building is designed for the most advanced original research, as well as for complete undergraduate instruction. Besides offices, lecture and class rooms, there are two general laboratories of 5,200 sq. ft. floor surface, fitted with all laboratory conveniences; a chemical room, balance room, suitable repair and supply rooms, storage battery room, and individual research rooms, fitted with stone piers, gas and water. The building is constructed without iron, and wired to deliver any current to any room. An instrument maker is constantly constructing new apparatus. The building is heated and ventilated throughout by the Sturtevant Fan system, with automatic electric regulating service, There is now almost $27,000 of apparatus for demonstration and experiment, and a sufficient corps of assistants to keep the laboratory in use all day. A system of circuits connects with the Engine House, where eight different dynamos can deliver into the laboratory different kinds of currents aggregating over 100 H. P.

Mr. David H. Holmes, a bachelor of arts from Ohio Wesleyan, and Doctor of Philosophy from Johns Hopkins, was elected in December to the chair of Latin Language and Literature in this institution, to succeed the late Professor Robinson. Mr. Holmes spent three years teaching in Massachusetts, first in high schools and then in Wilbraham Academy as instructor in Latin. Mr. Holmes' post graduate work was in Greek, Latin and Sanscrit. He held at Johns Hopkins a scholarship in Sanscrit, later a fellowship in Greek and the last year was assistant instructor in Sanscrit. In 1893 he was elected Professor of Latin in Alleghany College, Meadville, Pa., but resigned a year later to go abroad for further study, residing one semester at Berlin and one at Bonn. While abroad he published his doctor's thesis, "Verbs Compounded with Prepositions, in Thucydides." He also published at Bonn this year, in Latin, an "Index Lysiacos" of the Greek orator, Lysias, which has received favorable notice from the reviews.

Dr. Holmes is a native of Indiana, and thirty years of age.

The QUARTERLY is in receipt of a work which has a double interest for the members of the University, as coming from two former instructors, whose names are synonyms for thoroughness and energy; this is "The Elements of Physics," by Edward L. Nichols, professor of Physics in Cornell University and William S. Franklin, professor of Physics and Electrical Engineering at the Iowa Agricultural College. It is to appear in three volumes; the present one treating Mechanics and Heat, while the two subsequent volumes will treat Electricity and Magnetism and Sound and Light. It has been written with a view to providing a text-book which shall correspond with the increasing strength of the mathematical teaching in university classes. To quote from the introduction; "The present writers

having had occasion to teach large classes, the members of which were acquainted with the elementary principles of the calculus, have sorely felt the need of a text-book adapted to their students. The present work is an attempt on their part to supply this want. It is believed that in very many institutions a similar condition of affairs exists, and that there is a demand for a work of a grade intermediate between that of the existing elementary texts and the advanced manuals of physics. No attempt has been made to produce a complete manual or compendium of experimental physics. The book is planned to be used in connection with illustrated lectures, in the course of which the phenomena are demonstrated and described. The authors have accordingly confined themselves to a statement of principles, leaving the lecturer to bring to notice the phenomena based upon them. In stating these principles free use has been made of the calculus, but no demand has been made upon the student beyond that supplied by the ordinary elementary college courses on this subject. Certain parts of physics contain real and unavoidable difficulties. These have not been slurred over, nor have those portions of the subject which contain them been omitted It has been thought more serviceable to the student and to the teacher who may have occasion to use the book to face such difficulties frankly, reducing the statements involving them to the simplest form which is compatible with accuracy." The book has received at the hands of the publishers, MacMillan & Co., New York, a worthy and substantial form such as befits a work destined to be standard.

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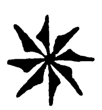
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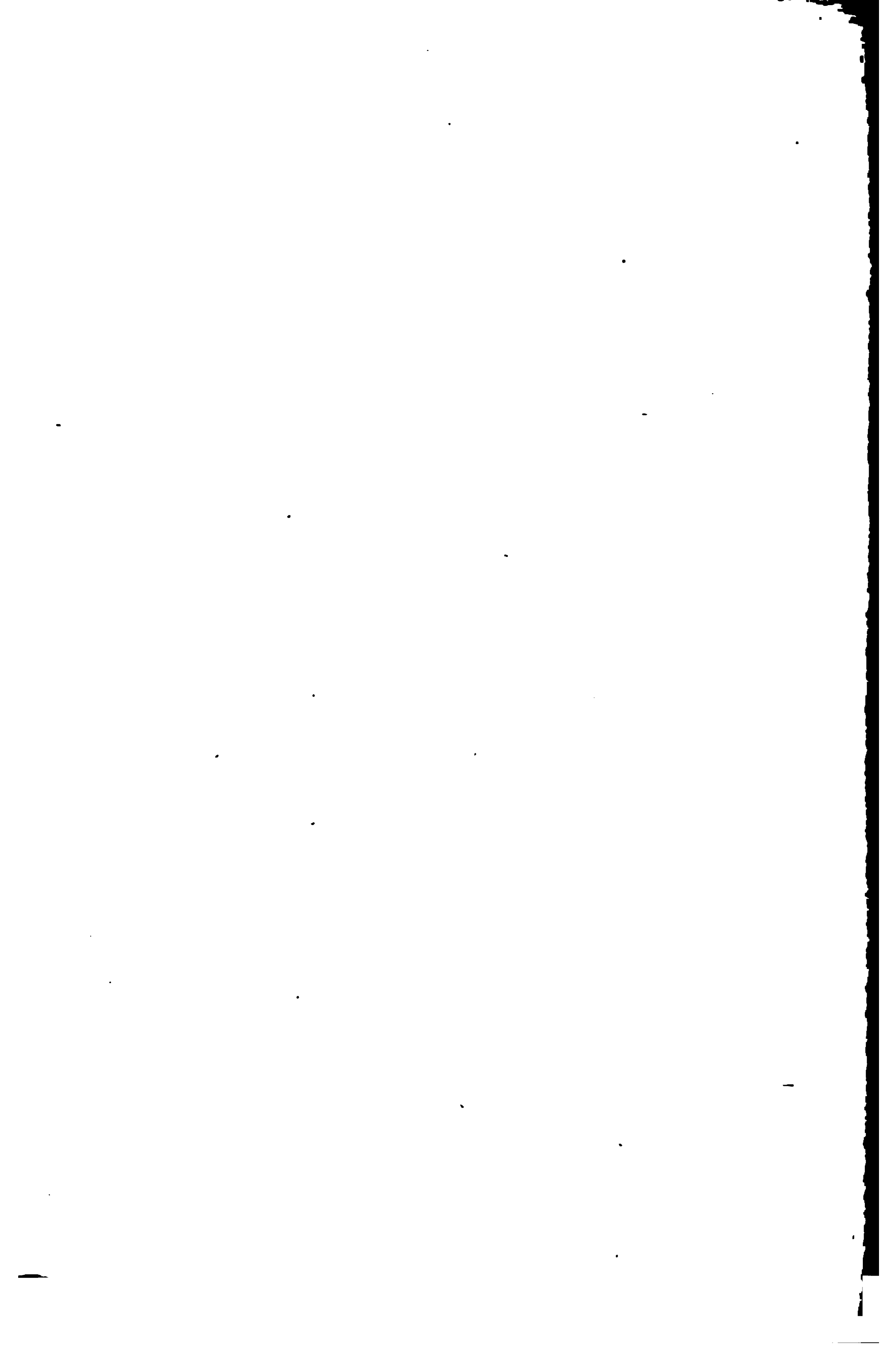
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KANSAS UNIVERSITY QUARTERLY

VOL. IV. APRIL, 1896. No. 4.

On the Skull of Ornithostoma.

BY S. W. WILLISTON.

(With Plate I.)

In the first volume of this QUARTERLY, the writer described briefly a skull of *Ornithostoma* (*Pteranodon*), correcting some important errors which had been made by Marsh. The University of Kansas has recently come into possession of another skull, collected by Mr. C. H. Sternberg in western Kansas, which adds some interesting facts to those hitherto known. It is to be regretted that the very faulty figure of *Pteranodon* published by Marsh stills finds expression in different works, and especially that the recent edition of Dana's Manual should be disfigured by it. The injured condition of the posterior part of the specimen from which the figure was made had obliterated the supratemporal fossa, and in consequence the orbits are represented as directed obliquely backward. One need not call attention to the absurdity of representing an animal with such a long and slender maxillary portion having eyes that would look towards the back of the head.

In the specimen from which the figures in Plate I are drawn, the skull has been somewhat compressed obliquely from above downward, and not laterally as is almost invariably the case. By a careful comparison of the two sides the relative and natural conditions of the parts have been made out with tolerable certainty. From them it will be seen that the orbits are directed, as would be expected, anteriorly, rather than posteriorly. The supratemporal fossa is of considerable size and the occipital crest is limited to the posterior part of the skull, not extending as a sharp ridge through the whole length, as stated by Marsh.

As is always the case with the Kansas specimens of these creatures, sutures of the skull are for the most part wholly obliterated, and therefore but little can be stated of the exact relations of the different elements.

The writer has already shown* that the lower jaws do not have the sharply triangular shape with concave upper surface, as originally described. The buccal surface is flat and limited on either side by a thin narrow parapet of bone. A similar structure apparently obtains in the upper jaws; the roof of the mouth is not arched, but flat, and it does not seem possible that the mouth cavity could have been more than an eighth of an inch in depth in life when closed, necessitating an extremely thin tongue, if it projected at all forward. The conjoined ant-orbital and narial opening is elongate, subtriangular in shape, and with all the sides gently concave. In length it measures 60 mm. with a greatest width of 30 mm. a little in front of the glenoid articulation. Its margins are thin and smooth. At the posterior part a thin triangular plate of bone descends to unite with the anterior ramus of the jugal, showing a distinct line of union with it, which runs obliquely to below the middle of the anterior margin on the orbits. On its front, concave border, as it overlaps the jugal, may be seen the beginning of a sutural line which probably separates the lacrymal. Projecting from the superior anterior angle of the orbit is a slender process, extending apparently directly outward or at the most with a downward curvature. It has a length of 18 mm. and a width near the extremity of 5 mm. Its posterior border is concave, and forms part of the orbit. If this is the prefrontal extending over the orbit, as in *Mosasaurus horridus* Will., I do not know of what the descending process connecting the jugal is composed.

The roof of the orbits is nearly horizontal above, where the least distance between the margins of the two sides is about 25 mm. The orbits are triangular in shape, with the angles rounded. Posteriorly the roof is directed outward and downward with its superior surface sloping into the supratemporal fossa. At its extremity, where it unites with the postfrontal elements, it is narrowed. The distance between this extremity on the two sides measures 60 mm. The anterior and inferior borders of the orbit are thin and sharp. The jugal is broad and smooth below. The anterior ramus has a width of 18 mm, the posterior one at its upper part a width of 7 mm. The squamosal descends downward and backward nearly to the head of the quadrate and then turns upward along the occipital margin. Below it there is a thin, flat bone, which arches backward to the head of the quadrate and forward along the under margin of the jugal, and which seems to be the quadratojugal. There is no interval between it and the

*Kansas University Quarterly IV, 61, plate.

bones above, but there can be little doubt that they are distinct elements. The more thickened squamosal measures 6 mm. in width at the middle, while the conjoined width of the squamosal and quadratojugal is nearly 20 mm. The infratemporal opening left between these bones above and the quadrate below is 45 mm. in length and 15 mm. in width at the upper part. It is directed downward and forward, its anterior extremity reaching as far forward as the middle of the orbit. The supratemporal opening if of considerable size. The crushed condition of the brain capsule renders it difficult to ascertain its width accurately, but its length directly backward from the extremity of the orbital roof is 25 mm. The width of the completely enclosed cranium across the middle of the fossæ can not be over 20 mm. The occipital surface ascends very obliquely upward and backward and was evidently somewhat concave from side to side. The hemispherical occipital condyle, 9 mm. in diameter, looks obliquely downward and backward. The basioccipital is quadrangular in shape, is gently concave and has two pairs of small, pit-like depressions. The suspensorium, of whatever elements it is composed, is broad and flat, passing directly outwards and apparently somewhat downwards to the head of the quadrate. The quadrates are very long and are directed very obliquely downward and forward. At its upper part the union is very firm with the opisthotic or supratemporal, the squamosal and quadratojugal. The lower extremity is also firmly and broadly fixed with the squamosal. Running inwards and a little backwards, as well as upwards from the glenoid articulation is a flattened process which evidently is the pterygoid. The glenoid surface has a transverse diameter of 15 mm. and an anteroposterior one of 10 mm. It has an oblique trochlear surface directed from within outwards and backwards. In front of the glenoid articulation there is a slender opening between the narrow palatines and the maxilla. The palatines at the front end of this opening have a width of 10 mm. The posterior nares have apparently an incomplete division. They measure together about 30 mm. in width and have a length of about 90 mm. Their margins are everwhere thin.

Bibliography of North American Diptera.

BY S. W. WILLISTON.

PART II.*

Owing to illness in the writer's family, during the printing of the first part of this paper, the manuscript of a considerable number of references was unintentionally omitted. They are given below, together with a few additional ones discovered since. Following them will be found condensed references to the different papers under each family. There has been an endeavor to make this part as complete as possible, yet doubtless some references have been overlooked.

Although only indirectly related to the present dipterological fauna of North America, it will repay the student to give attention to the fossil forms, as it is not at all unlikely that some of the genera hitherto known only from the later North American geological deposits may be found living. For this reason references to Mr. Scudder's papers on fossil diptera have been added, a list of which he has very kindly given at the writer's request.

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Syrphidæ. **2**-1; **3**-1; **6**-8, 9, 14, 15, 18, 37, 38, 39, 40, 41; **11**-37, 39; **16**-4, 5, 6, 8, 9; **25**-1, 6; **29**-4, 10; **30**-22, 26, 27, 28; **32**-7; **33**-1, 3; **35**-1, 5, 6, 8; **38**-38, 48; **42**-1, 7, 21, 25, 43, 45, 48, 49, 51, 52, 60, 65; **45**-2, 3; **47**-2.
Tabanidæ. **1**-1; **6**-2, 31; **28**-2; **30**-10; **38**-38, 40, 48; **41**-1; **42**-3, 36, 58; **45**-1, 6.
Tachinidæ. **6**-2, 13, 23, 42, 43, 45; **7**-6; **8**-1; **10**-1; **16**-8½, 9; **29**-7, 9; **11**-3, 4, 21, 29, 30, 31, 38, 40, 41, 42, 45; **30**-18; **31**-1; **33**-3, 10, 13; **35**-7; **38**-1, 3, 4, 5, 7, 8, 10, 11, 13, 15, 16, 18, 21, 24, 25, 26, 27, 29, 32, 33, 34, 35, 36, 38, 46, 48, 50, 51, 52, 57, 58; **44**-1; **42**-9, 16, 32, 35, 47, 56, 64, 66, 68; **45**-2, 5, 7; **49**-1; **52**-9.
Therevidæ. **6**-26; **11**-17, 18, 27; **30**-10. **42**-47.
Tipulidæ. **1**-3; **4**-1; **26**-1; **29**-3; **30**-6, 7, 9, 10, 20; **33**-3, 12, 14; **38**-37; **39**-1; **42**-21; **45**-1.
Trypetidæ. **11**-19; **16**-8½, 9; **25**-2; **27**-1; **29**-5, 6; **31**-6; **33**-8; **35**-9; **38**-20, 47; **45**-1; **48**-8.
Xylophagidæ. **6**-6; **30**-1, 10; **42**-

Involutoric Collineations in the Plane and in Space.

BY ARNOLD EMCH.

In No. 2, Vol. 4, of this Quarterly I have treated the involutoric transformation of the straight line and made an attempt to show the usefulness of the elementary conceptions of the *theory of groups* in the solution of geometrical problems. In this article I propose to do the same thing with regard to the plane and space, and shall confine myself to transformations with real variables and coefficients.

I. The Plane.

1. Among the collineations of the plane *involutions* occur only in the perspective collineation. Referring to cartesian coordinates their equations may be written in the general form

$$x_1 = \frac{ax}{bx+cy-a},$$

$$y_1 = \frac{ay}{bx+cy-a}.$$

The invariant elements are: the origin ($x=y=0$), every ray $y=mx$ through the origin, and every point of the straight line

$$bx+cy-2a=0.$$

The origin and this line are known as the center and the axis of the perspective involution (collineation). To the points (x_1, y_1) of the line at infinity corresponds the so-called *counter-axis*, in German *Gegenaxe*,

$$bx+cy-a=0.$$

The other counter axis, or the line whose corresponding points (x, y) are at infinity, is

$$bx_1+cy_1-a=0$$

The counter-axes of an involution in a plane coincide, as is seen from their equations.*

*See W. Fiedler, Darstellende Geometrie. Vol. III. 3 ed, page 573-77.

The effect of the general projective transformations in the plane

$$x_2=\frac{a_1x_1+b_1y_1+c_1}{a_3x_1+b_3y_1+c_3},$$

$$y_2=\frac{a_2x_1+b_2y_1+c_2}{a_3x_1+b_3y_1+c_3},$$

upon the perspective involution (1) is the new transformation

$$x_2=\frac{(a_1a+c_1b)x+(b_1a+c_1c)y-c_1a}{(a_3a+c_3b)x+(b_3a+c_3c)y-c_3a},$$

$$y_2=\frac{(a_2a+c_2b)x+(b_2a+c_2c)y-c_2a}{(a_3a+c_3b)x+(b_3a+c_3c)y-c_3a}.$$

This will be an involution if

$$b_1=a_2=c_1=c_2=o,$$

and

$$a_1=b_2=c_3.$$

In this case the transformations which do not change the character of an involution are (we change the indices 3 into 1):

$$x_2=\frac{a_1x_1}{b_1x_1+c_1y_1+a_1},$$

$$y_2=\frac{a_1y_1}{b_1x_1+c_1y_1+a_1}.$$

The resulting involution is

$$x_2=\frac{a_1ax}{(b_1a+a_1b)x+(c_1a+a_1c)y-a_4a},$$

$$y_2=\frac{a_1ay}{(b_1a+a_1b)x+(c_1a+a_1c)y-a_1a},$$

Putting in (2) $b_1=c_1=b_2=c_2=o$, $a_2=a_1$, the perspective collineation leaving the center invariant arises. Each ray through this center C intersects the axis of collineation in an invariant point S, such that for two corresponding points A and A_1 on this ray the so-called *characteristic anharmonic ratio of the perspective collineation** is

$$\triangle=(CSAA_1=\frac{a_1}{c_3}.$$

Taking an other transformation of the three-termed group of perspective collineations, with the characteristic anharmonic ratio $\triangle_1$, the combined effect of the two collineations is a collineation of the same group with the characteristic anharmonic ratio

$$\triangle_2=\triangle\triangle_1.$$

In the involution $\triangle=\frac{a}{-a}=-1$, while in the transformation (3) $\triangle_1=\frac{a_1}{a_1}=+1$, so that, in fact

*The first time introduced into geometry by W. Fiedler.

$$\triangle_2=\frac{aa_1}{-aa_1}=-1,$$

as is also seen from the resulting involution (4).

Transformation (3) being a perspective collineation in which the center lies in the axis of collineation, $b_1x_1+c_1y_1=0$, and where $\triangle=+1$, is sometimes called an *elation.** All the elations which leave the center invariant form a two-termed group, so that we now have the result:

The effect of the two-termed group of elations upon the involutions that leave a certain center invariant is the same system of involutions.

We will investigate how this result is to be modified if we apply the two-termed group of elations to a certain involution with the coefficients a, b, c. Evidently we may put

$$\begin{gathered}a_1a=A\\ b_1a+a_1b=B\\ c_1a+a_1c=C\end{gathered}$$

where A, B, and C are three independent arbitrary real numbers. In fact, we can choose the three coefficients a_1, b_1, and c_1, or an elation out of the group, in such a manner that these equations of condition are satisfied. This is the case if

$$a_1=\frac{A}{a},$$

$$b_1=\frac{aB-Ab}{a^2},$$

$$c_1=\frac{aC-Ac}{a^2},$$

Hence:

The effect of the two-termed group of elations upon a certain involution is the system of all involutions, that leave the center invariant.

The axes of the elation, and the original and the resulting involutions, are represented by

$$\begin{gathered}b_1x_1+c_1y_1=0,\\ bx+cy=2c,\\ (b_1a+a_2b)x+(c_1a+a_1c)y=2a_1a,\end{gathered}$$

respectively, and intersect each other in the point

$$x=\frac{2ac_1}{bc_1-b_1c},\quad y=\frac{-2ab_1}{bc_1-b_1c},$$

If, therefore, the axes of the elation and the original involution have fixed positions, this point is invariant, which implies the theorem:

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*Sophus Lie, loc. cit. page 262.

The combined effect of a one-termed group of elations and a certain involution is the system of all involutions whose axes intersect each other in one and the same point. This point is determined by the intersection of the axes of the original involution and the one-termed group of elations.

That really *all* involutions of the system are obtained is seen from the trigonometric tangent of the resulting axis of involution, which follows from (4),

$$-\frac{b_1 a + a_1 b}{c_1 a + a_1 c}.$$

In this expression a, b, c, b_1, c_1, are fixed quantities; but it may assume any real value if a_1 varies,

We have now discussed the the essential parts of our problem and without entering into further details we will finally treat two important special cases.

2. SPECIAL CASES.

(*a*) *Orthogonal and oblique*, or *axial symmetry*, may be considered either as a linear, or as a special perspective involution in which the center is at infinity. The analytic representation will be identical in both cases if we choose the origin in the axis of symmetry, and let this be the X-axis, and let the Y-axis be parallel to the axis of elation. The center of the perspective involution and of the one-termed group of elations is the infinitely distant point of the Y-axis. Using either orthogonal, or oblique coordinates the equations of the axial symmetry may be written

$$x_1 = x,$$
$$y_1 = -y,$$

and those of the elation

$$x_2 = x_1,$$
$$y_2 = mx_1 + y_1.$$

Applying the elation to the symmetry the result is the involution and especially the axial symmetry $x_2 = x$,

$$y_2 = mx - y,$$

having as its axis

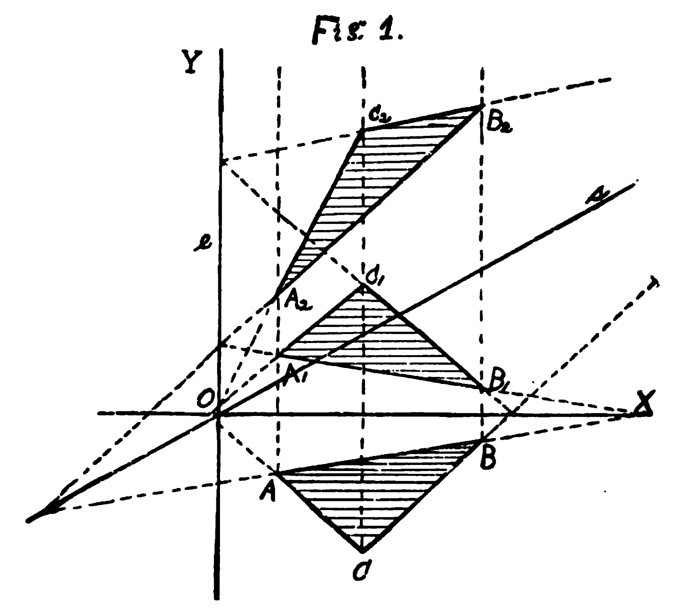
$$y = \frac{m}{2} x,$$

or in words:

The effect of a one-termed group of linear elations upon a certain axial symmetry having the same infinite center is the system of all axial

symmetries whose axes pass through the point of intersection of the axis of the one-termed group of linear elations and the axis of the symmetry.

Fig. 1.



In Fig. 1 these relations are all illustrated in the case of the orthogonal symmetry, or the *reflection* on the X-axis.

The original triangle ABC is reflected into $A_1B_1C_1$. An elation having its center at an infinite distance on the axis of Y, transforms $A_1B_1C_1$ into a triangle $A_2B_2C_2$, such that it forms an oblique symmetry with the original triangle ABC. The axis S of the new symmetry passes through the origin O.

The purely geometrical proof is easily obtained from the figure. It is well known that the areas of the three triangles ABC, $A_1B_1C_1$, $A_2B_2C_2$ are equal.

(*b*) *Central symmetry* is a special case of the perspective involution. It is represented by the equations

$$x_1 = -x,$$
$$y_1 = -y.$$

The two-termed group of elations transforms the central symmetry into the system of involutions

$$x_2 = \frac{a_1 x}{b_1 x + c_1 y - a_1},$$
$$y_2 = \frac{a_1 y}{b_1 x + c_1 y - a_1},$$

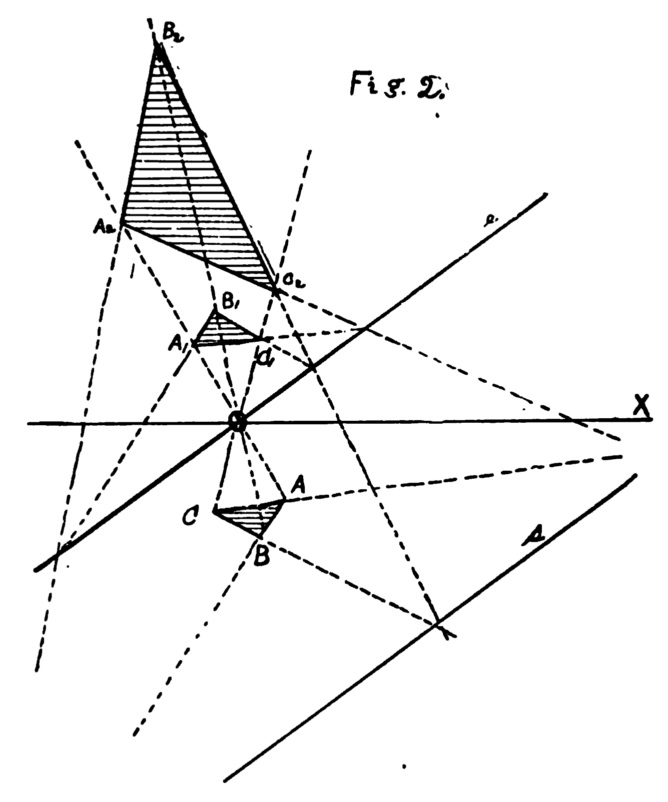
consisting of all involutions that leave the center invariant.

Thus we may say:

The effect of the two-termed group of elations upon the concentric symmetry is the system of all involutions that leave the same center invariant.

From the form of the above equations we also deduce the fact:

The effect of the one-termed group of elations upon the concentric symmetry is the system of all involutions whose axes are parallel to the axis of the one-termed group of elations.



We illustrate this theorm in Fig. 2. The central symmetry with the centre O transforms any original triangle ABC into $A_1B_1C_1$. An elation with the axis e transforms the triangle $A_1B_1C_1$ into the triangle $A_2B_2C_2$, such that it forms an involution with the triangle ABC whose axis is S is determined by the points of intersection of the pairs of sides AB and A_2B_2, BC and B_2C_2, CA and C_2A_2.

The purely geometrical proof for this is easily found by means of the *theorem of Menelaos* concerning the transversals of a triangle, so that we may leave it to the reader.

It may be well to add the remark that every *linear involution*

$$x_1 = -ax + by,$$
$$y_1 = cx + ay,$$

with
$$a^2 + bc = 1$$

is a perspective involution with its center at infinity, or an axial symmetry. The axis of symmetry is

$$y = \frac{1+a}{b} x,$$

and the center is at infinity in the direction of the pencil of parallel rays

$$y = \frac{-1+a}{b} x + m,$$

where m is a parameter.

2. Space.

3. In space there are two different kinds of involutions. The first is comparable to the one in the plane and the theorems found there may be generalized for space; their nature is the same. The equations are

$$x_1 = \frac{ax}{bx+cy+ez-a},$$
$$y_1 = \frac{ay}{bx+cy+et-a},$$
$$z_1 = \frac{az}{bx+cy+et-a},$$

and do not change their character by the three-termed group of elations that leave the origin invariant, i. e.,

$$x_2 = \frac{a_1 x_1}{b_1 x_1 + c_1 y_1 + e_1 z_1 + a_1},$$
$$y_2 = \frac{a_1 y_1}{b_1 x_1 + c_1 y_1 + e_1 z_1 + a_1},$$
$$z_2 = \frac{a_1 z_1}{b_1 x_1 + c_1 y_1 + e_1 z_1 + a_1},$$

4. The second kind may be represented by the formula*

*W. Fiedler loc. cit. page 573-77.

$$x_1 = \frac{ax+by}{z},$$
$$y_1 = \frac{cx-ay}{z}, \qquad (5)$$
$$z_1 = \frac{a^2+bc}{z},$$

To the plane $z=o$ corresponds the plane at infinity; thus every pencil of rays having its apex in the XY-plane is transformed into a pencil of parallel rays, and a pencil of planes with its *carrier* in the XY-plane is transformed into a pencil of planes. The invariant elements of the involution are

$$\frac{y}{x}=\frac{c}{a+\sqrt{a^2+bc}},\quad z=\sqrt{a^2+bc};$$

$$\frac{y}{x}=\frac{c}{a-\sqrt{a^2+bc}},\quad z=-\sqrt{a^2+bc}.$$

Both lines intersect the z-axis and are parallel to the xy-plane.

Designating by A and B the angles which these rays make with the positive part of the xz-plane there is

$$tgA=\frac{c}{a+\sqrt{a^2+bc}},$$

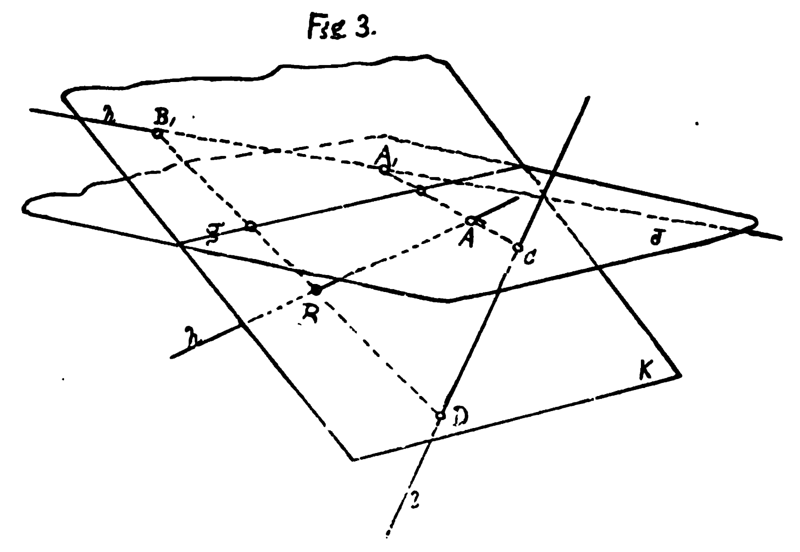
$$tgB=\frac{c}{a-\sqrt{a^2+bc}};$$

therefore the trigonometric tangent of the angle under which the lines cross each other

$$tg\ (A-B)=\frac{2\sqrt{a^2+bc}}{b-c}$$

The condition for a right angle is $b=c$.

Fig 3.



Designating in Fig. 3 the lines of invariant points by g and l, it is clear that a ray connecting any point of g with any point of l is

an invariant ray with those two points as double-points. Every point of the ray is transformed into a point of the same ray, such that the *cross-ratio* of the two corresponding points and the double-points is harmonic. Passing a plane through g it will intersect l in an invariant point and conversely every plain through l intersects g in an invariant point. Planes through g and l are therefore invariant planes. Taking such a plane J through either g or l, it will intersect the other in an invariant point C. Every ray through C in J is invariant; the involution in the plain J is therefore a perspective involution with C as a center and with that one of the lines g and l through which the plane passes as an axis.

If any ray h in space is given, the corresponding ray h_1 in the involution is found by the following construction: Through g pass any two planes J and K intersecting l in C and D and h in A and B. In the involutions of the planes J and K determines the corresponding points to A and B, respectively, which in the figure are designated by A_1 and B_1. The ray through these points is the corresponding ray to h. From this is seen immediately that, *in the involution of the second kind in space, to any ray in space corresponds a ray such that they are on the same ruled hyperboloid with the rays of invariant points.*

Since the rays g and l and a third line determine a ruled hyperboloid it is also seen that *in this involution the generatrices of any hyperboloid through the rays of invariant points are transformed into generatrices of the same hyperboloid; or in other words, the involution leaves such hyperboloids invariant.*

As ∞^2 rays pass through a point in space there are also ∞^2 hyperboloids of this kind passing through a point in space. The points of a line determine all the hyperboloids through g and l, their number is therefore $\infty^1 \times \infty^2 = \infty^3$. Thus we may say:

The second kind of involutions in space leaves the points of two rays and the complex of hyperboloids through these rays invariant.

As a corollary we may add:

There are ∞^1 invariant hyperbolic paraboloids through any point in space.

(Through a point ∞^1 rays may be drawn which with the rays of invariant points are parallel to the same plane).

Again:

In the involution of the second kind in space there are ∞^2 invariant hyperbolic paraboloids

It must be remarked that the above propositions are true for a

general projective transformation leaving the points of two rays invariant.

5. In this chapter we have to investigate whether there are any projecctive transformations in space which leave the character of the involution of the second kind unchanged.

Writing again the equations of these involutions

$$x_1=\frac{ax+by}{z},\qquad y_1=\frac{cx-ay}{z},\qquad z_1=\frac{a^2+bc}{z}, \tag{6}$$

we deduce the ratio

$$\frac{x_1}{y_1}=\frac{ax+by}{cx-ay},$$

or

$$\frac{x_1}{y_1}=\frac{a\frac{x}{y}+b}{c\frac{x}{y}-a}.$$

Designating the ratios $\frac{x_1}{y_1}$ and $\frac{x}{y}$ respectively by u_1 and u, this relation assumes the form

$$u_1=\frac{au+b}{cu-a}, \tag{7}$$

which shows that the ratios u_1 and u are in the relation of an involution in the straight line. From this we conclude *that the pencil of planes through the axis z is transformed like the point-range in the involution of a straight line.*

The double-elements of this involution are

$$u=\frac{a+\sqrt{a^2+bc}}{c}=\frac{x}{y},$$

$$u=\frac{a-\sqrt{a^2+bc}}{c}=\frac{x}{y},$$

which with $z=\pm\sqrt{a^2+bc}$ confirms the previous result.

In my article in the QUARTERLY* I have shown that the one-termed group of transformations

*Loc. cit.

$$u_2=\frac{bu_1-bk}{kcu_1+b} \tag{8}$$

with the invariant points $\pm i\sqrt{\frac{b}{c}}$, where k designates a parameter and b and c the same constants as in (6), leaves the character of the involution (7) unchanged. The transformation (8) applied to (7) gives

$$u_2 = \frac{(ab-bck)u+(b^2+abk)}{(ack+bc)u-(ab-bck)}. \qquad (9)$$

Assuming $\frac{1}{z}$ as the factor of proportionality, we obtain as the resulting involution in space

$$\begin{aligned} x_2 &= \frac{(ab-bck)x+(b^2+abk)y}{z}, \\ y_2 &= \frac{(ack+bc)x-(ab-bck)y}{z}, \\ z_2 &= \frac{(ab-bck)^2+(ack+bc)(b^2+abk)}{z}. \end{aligned} \qquad (10)$$

The transformation in space which produces this result is

$$\begin{aligned} x_2 &= bx_1 - bky_1, \\ y_2 &= kcx_1 + by_1, \\ z_2 &= z_1(b^2+bck^2). \end{aligned} \qquad (11)$$

These equations are obtained from (8) and (9) by noticing that the determinant of the transformation (9) is

$$(a^2+bc)\ (b^2+bck^2).$$

The transformations as represented by equations (11) is linear and the determinant is

$$\begin{vmatrix} b & -bk & 0 \\ kc & b & 0 \\ 0 & 0 & b^2+bck^2 \end{vmatrix} = (b^2+bck)^2,$$

i. e., always positive.

The invariant rays are

$$\frac{x}{y} = \pm i\sqrt{\frac{b}{c}},$$

and $$z=0, \text{ or } \infty.$$

These lines are real or imaginary according as b and c are of apposite or same sign. Thus, the system of linear transformations (6) leaves a tetrahedron invariant which has one of its sides at infinity, and as its finite edges

$$x=y=0; \ \frac{x}{y} = i\sqrt{\frac{b}{c}}, \ z=0; \ \frac{x}{y} = -i\sqrt{\frac{b}{c}}, \ z=0.$$

The system, however, does not form a group.

6. The next question is now as to the relation of the involution of the second kind in space and the system of transformations which leave its character unaltered. For this purpose equation (8) may be written

$$u_2 = \frac{u_1 - k}{k\frac{c}{b}u_1 + 1}. \qquad (12)$$

Designating the invariant elements of (7) by m and n, and those of (8) or (12) by p and g, there is

$$mn = -pg,$$

or

$$\frac{m}{p} = -\frac{g}{n},$$

or since $g = -p$, $\frac{m}{p} = \frac{p}{n}$,

This proposition may be written

$$\frac{m+p}{m-p} = \frac{p+n}{p-n},$$

or

$$\frac{\frac{m+p}{m-p}}{\frac{n+p}{n-p}} = -1,$$

which shows that the anharmonic ratio of the four double-elements is -1. Hence, *the cross-ratio of the invariant planes of the involution* (7) *and the transformation* (12) *is harmonic.*

If in the expressions for m and n, b and c are of opposite sign, and $|bc| < a^2$, then m and n are of the same sign and p and g real; when b and c are of the same sign, then m and n are of opposite sign and p and g imaginary.

Equation (9) may be written

$$u_2 = \frac{u + \frac{b^2 + abk}{ab - bck}}{\frac{b^2 + abk}{ab - bck} \cdot \frac{c}{b} \cdot u - 1}$$

or by putting $\frac{b^2 + abk}{ab - bck} = \frac{b + ak}{a - ck} = -k^1$,

$$u_2 = \frac{u - k^1}{-k^1\frac{c}{b} - 1}, \qquad (13)$$

The parameter k^1 may, like k, assume any real value, since from the expression above we deduce

$$k=-\frac{b+ak^1}{a-ck^1}.$$

Considering the transformations (12) and (13) separately it is seen that the planes through the z-axis are transformed into two certain systems of planes through the z-axis by the transformations (12), and (13), such that the one system is the reflection *of the other on the* yz *plane.*

The double-elements of (13) are

$$u=\frac{-2\pm\sqrt{4+k^{12}\frac{c}{b}}}{k^1\frac{c}{b}}$$

Designating them by r and s, there is

$$rs=-\frac{b}{c},$$

or $rs=mn=-pg=-\frac{b}{c}$,

or since $g=-p, \frac{r}{p}=\frac{p}{s}$,

and

$$\frac{\frac{r+p}{r-p}}{\frac{s+p}{s-p}}=-1,$$

i. e., *The invariant planes of the resulting involutions* (13) *form a harmonic ratio with the invariant planes of the transformation* (7).

If $a^2+bc=1$, the involutions in the planes $z=\pm 1$ become axial symmetry and assume the form

$$x_1=ax+by,$$
$$y_1=cx-ay,$$

and

$$x_1=-ax-by,$$
$$y_1=-cx-ay.$$

Every straight line intersecting the planes $z=+1$ and $z=-1$ in two points A and A^1 is transformed into a line passing through the correspoinding points of A and A^1. For intermediate points the relation $z^1=\frac{1}{z}$ must be added. Thus the geometrical interpretation of these involutions is easily obtained.

In case that b=c, $a^2+b^2=1$, and the involution becomes

$$x_1=ax+by,$$
$$y_1=bx-ay,$$
$$z_1=\frac{1}{z},$$

Geometrically this means that every straight line parallel to the z-axis is first reflected on the xz-plane, so that $x=x^1$, $y=y^1$, $z=z^1$ and then rotated about the z-axis,

$$z_1=ax^1-by^1,$$
$$y_1=bx^1+ay^1,$$

which results into the involution

$$x_1=ax+by,$$
$$y_1=bx-ay,$$
$$z_1=\frac{1}{z}.$$

A Study of the Type of the Greek Epitaphios with Special Reference to the Oration in Thucydides.

DAVID H. HOLMES.

This study is an outgrowth of a more special investigation into the genuineness of the Epitaphios of Lysias, a much disputed subject, but none the less interesting because it has received more or less attention throughout the history of classical study. There is a feeling now-a-days that it is bad classical form to treat a subject which has once been treated. Indeed, this feeling has been so alive as to have induced a search for the "new and novel", which has at times been followed by results either heterodoxical or quite wide of the mark. At any rate, keenly conscious of this classical cant, I have felt the necessity of creating an excuse for daring to study again a subject so much studied, and in my *Index Lysiacus* (Bonn 1895) I have endeavored to furnish such an excuse in the means which it affords of more accurately deciding some internal questions as to whether or not Lysias wrote the speech which bears his name. One of the outgrowths of this study is the present paper, and, like most outgrowths, it is lacking somewhat in unity of purpose. This is especially visible in the partial stylistic comparison based on participle and finite verb, a comparison which was not carried out further by reason of the limitations of space as well as of inclination. Yet so far as I know there is no study which exactly covers the same ground with this, and while it is in a certain sense a compilation of facts, it will be found not wholly lacking in originality of treatment. The literature bearing on the subject is too extensive to admit of enumeration here.

I shall, therefore, first consider the different representatives of the Epitaphios historically, its significance and its type. I shall next consider the oration given in Thucydides in particular. Finally, I shall endeavor to draw a conclusion as to the relation which this oration sustains to the history of Thucydides and to the oration actually delived by Pericles.

The Public Funeral.

The manner in which the Greeks conducted a public funeral is described by Thucydides (2, 35): "The relics of the dead were exposed in a tent, erected for the purpose, for three days, during which the relatives might bring funeral offerings. When the time came for burial, the wagons of each tribe bore a coffin of cypress wood in which the bones of its slain were deposited, and one bier covered with a pall was carried in commemoration of the missing. Any one, citizen or alien, might join the procession, and women who were related to the dead were present to lament them. The remains were placed in a public tomb in the most beautiful suburb of the city—the Ceramicus—and an orator expressly chosen by the senate, pronounced the funeral speech." From the epitaphioi of Lysias, Plato and Demosthenes, we learn that sacrifices were offered and games celebrated in honor of the event.

The Funeral Orations.

If Demosthenes is to be believed, the Athenians were the only people who honored those who fell in the service of their country with funeral orations (Lept. p. 499):

μόνοι τῶν πάντων ἀνθρώπων ἐπὶ τοῖς τελευτήσασι δημοσίᾳ ποιεῖτε λόγους ἐπιταφίους ἐν οἷς κοσμεῖτε τὰ τῶν ἀγαθῶν ἀνδρῶν ἔργα.

Although this is not true to the letter, yet the delivery of such orations was *customary* only at Athens, while elsewhere it was *occasional*, and doubtless in imitation of the Attic example. The origin of their institution is very ancient and consequently very uncertain. The Scholiast on Thuc. 2, 35, understands Pericles to ascribe the institution to Solon, upon whom the later Greeks were accustomed to father almost any law or usage which could not otherwise be accounted for. (Dionys. Halic. Ant. Rom. 5, 17; Diog. Laert. Sol. 1, 2, 8.) Thirwall and Grote believe that they had their origin in the Persian wars. (Thw. Hist. Gr., vol. 3. p. 54; Grote Hist. Gr., vol. 6, p. 41. See also Diod. Sic. 11, 33, who says expressly: *τότε πρῶτον*; and Dionys. Halic. Ant. Rom. 5, 17.) It is not improbable that both views are correct; they clearly do not necessarily conflict. They could not have been instituted much later at any rate, since we have an actual example of one within forty years afterwards, and since about nine years later still, at the beginning of the Peloponnesian war (B. C. 431), the practice was so firmly established that a regular course of ceremonies was prescribed for the occasion.

The specimens of this kind of composition during the classical period, of which any mention has come down to us, are as follows:

1. The earliest is the oration of Pericles in honor of the citizens who fell before Samos in a war which was concluded 440 B. C. Stesimbrotus quoted by Plutarch in his life of Pericles (p. 156 D) has preserved a fine sentiment from this speech, which is alluded to by Aristotle (Rhet. 1, 7, 34) as the work of Pericles, where he compares the loss of the slain to the abstraction of the spring from the year. (I differ from Roscher and agree with Weber here as to Aristotle's quotation being from the Samian oration. See also Grote Hist. Gr., vol. 6. p. 41.)

2. The second in order is the speech which is the especial subject of our study, reported by Thucydides in the second book of his history.

3. The next in order, though of uncertain date, is the oration composed by Gorgias the Sicilian. The type of the epitaphios was not set until the time of Gorgias, who unfortunately was the first to give the stamp to this style of composition. It appears from Philostratus who considered it a specimen of ὑπερβάλλουσα σοφία, that it was delivered at Athens over those who fell in the Persian wars. It was about this time, not earlier than 427 B. C., that Gorgias, then advanced in years, first came to Athens. The stilted and unnatural style of this sophist exerted but too powerful an influence at Athens, and formed the model for subsequent epitaphioi.

4. The funeral oration of Lysias was written *ostensibly*, to commemorate the valor of the Athenians, who, under the command of Iphicrates, went to the aid of the Corinthians, B. C. 394; but *actually*, (as I endeavored to show in a paper prepared several years ago) it was intended for a practice-speech. Its genuineness is questioned by some scholars, and asserted by others, and this difference of opinion ought to be an excuse for further study of the question. I cannot forego the somewhat irrelevant but brief introduction here of the immature conclusion I then reached in a study of this question. I give it only for what it is worth and by no means as my final conclusion: There is no external evidence against the genuineness of Lysias' epitaphios, unless the silence of Dionysius concerning it is to be so construed. What external evidence there is, is in favor of its Lysianic origin. In respect to internal evidence, the deviations from Lysias' norm outweigh the parallelisms by reason of the greater weight which naturally attaches to differences. But when we consider the exigencies of the panegyric style our scales swing into equilibrium. Put on the side of the similarities the testimony of antiquity and the scale

turns in favor of the genuineness of our epitaphios. But the question is to me still an open one.

Thirwall (Hist. Gr., vol. 3. p. 131) calls this oration "a noble oration, a worthy rival to that of Thucydides". Blass says he is choked by the antitheses and has to get into the atmosphere of the Olympiacus before he can breathe with ease. Grote says it is a very fine composition (Hist. Gr., vol. 6. p. 191), while Dobree (Adv. 1. p. 8) calls it "non modo Lysia sed quovis oratore indignam." Hence, we see, its merit is largely a matter of personal taste. For my part, I like it the more I study it. So far as its chronological inaccuracies are concerned, these considerations have been known to give way before the exigencies of rhetoric even in modern times.

Photius and Suidas (quoted at length by Sauppe, Fragm. Oratt. Att., p. 170) speak of the funeral orations of Lysias in the plural number.

5. We have the authority of Photius (Ed. Bekk., p. 487) for saying that Isocrates was guilty of plagiarism for having introduced into his Panegyricus many things which had been said by Archinus and Thucydides and Lysias in their funeral orations. The funeral oration of Archinus, therefore, comes next, being somewhat earlier than 380 B. C., the date of the Panegyricus. Plato also in his Menexenus (p. 234 B) indicates that he is acquainted with some funeral oration of Archinus. The date and circumstances are unknown, but a fragment of Archinus, (on the mortal condition of man) is preserved by Clement of Alexandria, which seems to belong to it. (See Sauppe, Fragm. Oratt. Att., pp. 166-7). Plato also makes reference to an oration of an entirely unknown Dion. (Menex. 234 B; 236 A).

6. Next in order comes the Menexenus of Plato. Socrates is represented as the speaker, and professes to have been taught the oration by Aspasia, but he talks about events which occurred thirteen years after his own death, which happened B. C. 399. The ironical and the serious are inseparably blended throughout the whole. Plato here indulges in his sarcastic propensities at the expense of the orators, more especially those of the Sicilian school.

7. We now come to an instance of the ἐπιτάφιος λόγος among the Asiatic Greeks. On the death of Mausolus, king of Caria (B. C. 352), his queen Artemisia offered a prize for the best literary production in his honor. Theodectes, a Lycian, but a pupil of Plato and of the Apollonian Isocrates, Theopompus of Chios (both of whom were likewise disciples of the Athenian Isocrates), as well as

Naucrates the Erythræan, are mentioned among the candidates. The prize is reported by some to have been awarded to Theodectes, by others to Theopompus. The oration of Naucrates is mentioned by Dionysius of Halicarnassus, as among the models of this kind of composition.

8. Demosthenes has left on record an express testimony that he was appointed to deliver a speech in honor of those Athenians who fell at the battle of Chæronea, B. C. 338. (Dem. de corona, p. 320): *χειροτονῶν ὁ δῆμος τὸν ἐροῦντ' ἐπὶ τοῖς τετελευτηκόσι . . . . . ἐχειροτόνησεν . . . . ἐμέ.*

The epitaphios logos given in editions of his works manifestly refers to the battle of Chæronea. Dionysius of Halicarnassus (de adm. vi dicendi in Dem., c. 13) speaks of this speech with great contempt and considers it spurious, which is likewise the general opinion of ancient critics. The question has been carefully discussed by Westermann. Its bungling imitations of the Menexenus, the un-Demosthenic manner of its treatment of Philip, its ridiculous falsification of history so far as the Thebans are concerned, are little less than conclusive against it, though the imitator has successfully reproduced the Demosthenian rhetorical whip-crack in several instances. The Epilogus indicates that the writer, whoever he was, was acquainted with the epitaphios of Hyperides, and had imitated it.

9. The last extant funeral oration which belongs to the Hellenic period is that of Hyperides (B. C. 322). As a work of art it may be placed on a level with the speeches of Pericles and Aspasia as these are delivered to us by Thucydides and Plato. It deals more in historical allusions than any extant epitaphios. It enjoys the distinction of having the most simple and the most pleasing style.

Literary Origin of the Epitaphios Logos.

The Epitaphios Logos claims for itself, like the early inhabitants of Greece, an autochthonic origin. Homer, in the twenty-fourth book of the Iliad, commencing with line 720, gives a brief account of the threnos, "lamentation", which attended the reception of the body of Hector. From other authorities also, (for whom consult Becker's Charicles and Guhl and Kohner) we learn that the threnos was an essential feature of the funeral rites of the early Greeks. The best detailed account of the funeral ceremonies is, perhaps, that given by Lucian, (de Luctu, 10). Plato in his laws (947 B) gives the regulations for the burial of a *ἱερεύς*. From these descriptions of later writers as well as of Homer, we learn that an *ἀοιδὴ* of a mournful character was sung by the *θρηνῳδοὶ* (the minstrels

or hired mourners), participated in by the female relatives and a chorus of women. The imperfect tense, θρήνεον, in the passage of Homer cited above, seems to warrant the conclusion that the ἀοιδὴ consisted of *strophes*, and that at each pause some mourner broke in with some passionate lament. The *antistrophe* is easily provided for by the chorus of women. Such an explanation at least utilizes all the material and accords well with the previous and subsequent history of the chorus. Thus we have here again an illustration of the μὲν and δὲ principle entering into the Greek life as well as into the Greek language. The threnos without doubt contained many allusions to the valor and virtue of the dead, and this fact gave to the threnos a personal and aristocratic character which was in exact keeping with the times in which it flourished. It was then that the government of Greece most nearly approached the patriarchal form. It was then that the family was a unit for both political and religious purposes. Later, when the tyrants came in, the threnos was changed to meet the new political and social conditions. In the lyric odes to the dead, of Pindar and Simonides, we see the next step in this development. The ode of Simonides on the heroes of Thermopylæ is especially significant in this connection as forming the connecting link between the personal and aristrocratic threnos and the impersonal and democratic epitaphios. It would be interesting to trace the line of resemblance between the growth of the Epitaphios Logos and that of the tragic drama, did it not carry us beyond the limits of our present purpose. Suffice it to say that in the dithyramb from which the tragic drama was developed we have the same three classes of performers: the minstrels, the leaders of the chorus, and the chorus itself—as we had in the early threnos. The prose of the Epitaphios may be said to correspond to the colloquial iambics (Arist. Poet., c. 4) of the tragedy, the orator to the mourner and the bard. The incorporation of legends is common not only to the threnos and the dithyramb, but is a very marked feature of the Epitaphios. In fact, the presence of ἤθη is to be looked for as much in the Epitaphios as in the tragedy.

The transformation of the threnos into the epitaphios was due to the reforms introduced into the society of Athens by the establishment of the Athenian democracy near the close of the sixth century. The exact time at which public funerals began is not known. But, without discussing the weight of inferences to be drawn from the statements of Lysias (Epit. 3-20), of Thucydides (2, 35), and of Herodotus (5, 78), we can say with safety, at least, that the custom did not exist before the time of Cleisthenes.

The Significance of the Epitaphios.

The meaning of the Epitaphios is revealed by a study of the times in which it flourished. We have already said that it was impersonal and democratic. The family had become the city. No more of the superiority of birth as an element of government. Yet the Athenian could not easily forget the pleasant taste of the old aristocracy. And while at Athens absolute equality must be insisted upon, the orator who would praise the **ἐυγένεια** of their dead relatives and by so doing indicate their own noble origin, was loudly applauded. If they could not give vent to their feeling of aristocracy, as toward other families, it could find expression for itself as toward other cities. All Athenians were equal, but Athens was a good deal better than Sparta. The medium of this expression was the Epitaphios or the Panegyric. So the Epitaphios was one of the ways the democracy had of expressing its aristocratic sentiments. In this respect we have a close parallel to the Epitaphios in our Fourth of July oration. On every Fourth of July, we Americans endeavor to stifle our individual aristocracy, and adopt the motto "every man is as good as another and a good deal better," and become nationally very aristocratic as toward other nations.

The Type of the Epitaphios.

The Epitaphios in its later development came to be regarded as a distinct species of epideictic oratory. The earlier epitaphioi were, no doubt, much less formal, but the differences chiefly to be noted between the earlier and the later representatives of this type must have been in point of diction rather than in subject-matter. And so, for the Sicilian school made many changes in this art, and the epitaphioi that are extant bear very conspicuously the stamp of this school. That the Epitaphios is epideictic and therefore very near the Panegyric in its nature, is not only evident internally from the similarity in diction and to a less extent in subject-matter, but there are certain external coincidences, a few of which it may not be amiss to mention. Thus, races and athletic contests attended the delivery both of the Epitaphios and the Panegyric. Again, the same writers produced the same kind of literature. Gorgias who set the pattern for the Epitaphios also wrote a Pythicus and an Olympicus. Lysias has also an Olympiacus. The chief difference between the Epitaphios and the Panegyric seems to be in the greater range of subjects of the later. Nor was there the same liberty in the choice of subject-matter peculiar to each. The Epitaphios was much more restricted also in this respect. "They

conformed", says Professor Jowett, "to a regular type. They began with gods and ancestors and the legendary history of Athens, to which succeeded an almost fictitious account of later times. The Persian war formed the center of the narrative. In the age of Isocrates and Demosthenes the Athenians were still living on the glories of Marathon and Salamis." They closed with the threnos, the lamentation of the orator over the slain, and the paramythia, the consolation of the bereft.

Notwithstanding the restricted limits which hampered the orators in this style of composition, there is not a little difference to be observed in content in a comparison of the different epitaphioi. Thus, the epitaphios of Lysias is distinguished from similar compositions by the fullness and enthusiasm with which he dwells on the old Attica of legend. The language of Demosthenes suggests the perfunctory discharge of a tiresome duty (cf. Epit., p. 1390). Pericles (Thuc. 2, 36) and Socrates (Menex., 239 B) dismiss these prehistoric achievements with the briefest possible notice, though they are gravely cited by the Athenians at Platæa (cf. Hdt., 9, 27), and Aristotle assures us (Rhet., 2, 22, 6) that without such allusions no epitaphios would be complete. Hyperides, who in other respects shows himself independent, omits altogether to mention them. Isocrates (Panegyr., 10) declares himself content to rest his claims rather on the treatment than on the novelty of his subject. It would be interesting to note other differences and especially the more than ordinary personal character of the epitaphios of Hyperides, and how from this was developed another style of composition of which the funeral sermon of to-day is its legitimate offspring, but here space, unity and inclination again prevent.

Summary of Pericles' Funeral Oration.

(Thuc. 2, 35-46).

I. EPAINOS—CHAPS. 35–42.

(a) Prooimion. Ch. 35.

Ch. 35. Heretofore, orators on like occasions have commended the institutional origin of the funeral panegyric. To myself however a public burial would have seemed preferable to entrusting the virtues of many men to the eloquence of one. There is danger to the orator of overstating the case so far as that auditor is concerned who is ignorant of the facts, while to the well-informed hearer justice does not seem to be done to the valor of the dead. Disbelief and envy are excited in those whose own valor is surpassed by that of the eulogised. I shall, however, conform to law and custom and endeavor to meet your approbation.

(b) Prothesis. Ch. 36.

Ch. 36. I shall begin with just praise of our ancestors as is but fitting, seeing that they have handed down to us a country always inhabited by the same race—an empire which our immediate forefathers augmented and to which we ourselves have made additions, until it is now in good trim for either peace or war. But our present and past victories over Barbarian or Greek are too well known to need rehearsal here. I shall rather point out by what institutions we have risen to empire—and our civil policy which is the cause of our greatness. Such topics, I take it, are not unsuitable to the present occasion.

Τὰ τῶν 'Αθηνῶν. Chaps. 37-41.

Ch. 37. In our form of government we are not imitators but set the pattern for others. We are a democracy. Equality is at the basis of our laws, merit at the basis of our public preferment. The poor and the rich have an equal chance to contribute to the public weal, as well as to enjoy the honors in the gift of the state. The same spirit pervades throughout the private life of our citizens and we render cheerful obedience both to the written laws of the state and to the unwritten laws of society.

Ch. 38. We are public spirited also, in that the celebration of games and festivals, the public and private entertainments, lighten the public heart. Nor are we limited in the enjoyment of luxuries. Such is our greatness that the best productions are brought to us from every quarter of the globe (world).

Ch. 39. We differ from our enemies in military matters. We do not deny to strangers free access to our city in order to conceal our resources. Courage in action and not cunning in strategem is our defense. The Lacedaimonians rely upon the severity of their military discipline to develop manly courage. An easy mode of life does not unfit the Athenian courage for valorous deeds. They form a confederacy to attack us; we defeat them unassisted and on their own ground. No enemy has yet encountered our united forces—yet they complain of defeat as if caused by our whole strength, and boast of victory as if over our entire armament. The inborn courage of our disposition and not that acquired by institution frees us from apprehension for the future and makes us prepared for all exigencies.

Ch. 40. Our city is also to be admired for its cultivation of philosophy unmixed with effeminacy. We regard riches as a means to an end, not as an occasion for boasting. We account shiftless-

ness and not poverty disgraceful. Private business and domestic cares do not prevent our citizens from being well informed in public affairs. He who neglects the state is useless to the state. The measures we adopt are the result of discussion and political sagacity. In the case of other states, ignorance is the basis of their courage only to be undermined by reflection. We gain our friends not by receiving benefits, but by conferring obligations. He who does you a favor will be more likely to do you another than he whom you yourself have favored, for the kindness he returns will not be esteemed a favor but regarded as a debt. Our generosity springs not from the calculations of interest but from the confidence of liberality.

Ch. 41. In short Athens is the school of Greece. Every Athenian possesses that individuality and versatility which enables him to adapt himself to whatever circumstances, and that with grace. The truth of this is attested by the present power of our city which is the result of these very qualities. Our state alone is greater than report. Our enemies experience no chagrin when defeated by such opponents, while our subjects do not complain that we are unworthy of empire. We need no Homer to attest our power to future ages to which we shall always be the theme of admiration—made so by monuments of our deeds left on every land and sea. It was for such a country that these men fought and fell, and in such a cause they well deserve the emulation of us all.

Ἔπαινος τῶν ἀποθανόντων. Ch. 42.

Ch. 42. I have thus praised Athens to show that the contest between us and our enemies is not for equal stakes; and so indirectly to establish the worth of our fallen heroes whose valor has adorned the city with all that makes it the theme of my encomiums. Their courage is evinced by their glorious death. Their faults as private citizens are effaced by their public services. They did not hesitate to meet danger that they might enjoy their wealth or escape their poverty. To the enjoyment and attainment of riches they preferred vengeance on their country's foes. They preferred the safety of the state purchased by their death to personal safety at the price of submission. Thus with their bodies they bore the brunt of battle and perished at the hight of glory.

II. THRENOS—CH. 43.

Ch. 43. You who have survived them may pray for a safer career, but greater courage you need not desire. It is yours to become enamoured of your city's grandeur and to be mindful of

that valor by which it was attained. The sepulchers of the dead are in the memories of the living. The whole earth is the tomb of the illustrious. Their virtues are not alone inscribed on perishable stone in their own country, but are written on the eternal tablets of the heart in all lands. Emulate their noble example, account happiness liberty, and liberty valor. Remember that it is not the unfortunate that should be most unsparing of their lives, but the prosperous as well who have most at stake. For to the high-minded death is less grievous than adversity.

III. PARAMYTHIA—CH. 44-46.

Ch. 44. To the parents of the departed, I do not offer condolence but consolation, for as theirs was the noblest death, so yours is the noblest sorrow. Yet it is difficult to comfort those who have learned by experience to prize the blessings they have lost. To those of you who are young comes the hope of other offspring with which to bury your sorrow and enrich the state. To the old is offered the remembrance of past happiness and the lustre given to the remainder of their lives by the glory attained by their children. For honor never grows old. In the declining years of life, it is not so much gain that gladdens, as honor and respect.

Ch. 45. To you, the sons and brothers of the slain, belongs the contest of emulation. For no one refuses justice to departed merit, and though you surpass them you will not be thought equal. The envy of competition ceases only with the death of its object, whereas the merit which obstructs no one is honored with a zeal unmixed with jealous rivalry. To the widows, let me say, it will be your greatest glory to guard the virtue of your sex and to become the theme of conversation among men as little as possible.

Ch. 46. The tribute of words has now been offered to the dead. The tribute of deeds is this public funeral, and the maintainance and education of their children at the country's expense will be a just and liberal reward. Where the rewards of virtue are the most liberal, there are found the best citizens. And now let each of you take a sad farewell of the deceased and depart in peace.

The Relation which our Speech Sustains to the History of Thucydides and to the Oration Actually Delivered by the Orator Pericles.

Technically speaking the funeral oration of Thucydides here put into the mouth of Pericles is the only example of the epideictic class in the history of Thucydides.

Since we have it from Thucydides himself, that he had, from the commencement of the war, formed the purpose of writing its

history, we may fairly suppose that he heard most of the discussions which took place in the Ecclesia between 433 and 424 B. C., the latter year being the year when his twenty years exile from Athens began. Such discussions would be the addresses of the Corcyrean and Corinthian envoys in 433 B. C., the speeches of Pericles, the debate on Mitylene in 427 B. C., and the speech of the Lacedaimonian envoys in 425 B. C. These, then, are the speeches which must form the basis of a consideration as to whether he treated the speech historically or artificially.

Confining our discussion to the funeral oration, we find that it gives rise to three queries: (1) Does Thucydides here represent the style of Pericles? (2) Does Thucydides here faithfully portray the policy of Pericles? (3) Does Thucydides here give the words of Pericles?

Professor Jebb's theory is that Thucydides does here represent the style of Pericles and for three reasons: (a) Thucydides must have repeatedly heard Pericles whom he pronounces the first of Athenians—most powerful in action and in speech (1, 139), and it would therefore be strange if he did not give some traits of the eloquence which was so stirring in those times. (b) The bold imagery and striking phrases which are attributed to him by Aristotle and Plutarch are parallelled by certain portions of his Thucydidean speeches. Thus, Arist. Rhet. 3, 10, 7:

ὥσπερ Περικλῆς ἔφη τὴν νεότητα τὴν ἀπολομένην ἐν τῷ πολέμῳ οὕτως ἠφανίσθαι ἐκ τῆς πόλεως, ὥσπερ εἴ τις τὸ ἔαρ ἐκ τοῦ ἐνιαυτοῦ ἐξέλοι......ib. *τὴν Αἴγιναν ἀφελεῖν ἐκέλευσε τὴν λήμην τοῦ Πειραιέως.* Plut. Per. 8, 5: *τὸν πόλεμον ἤδη καθορᾶν ἀπὸ Πελοποννησοῦ προσφερόμενον*, and of those who fell at Samos: *ἐγκωμιάζων ἐπὶ τοῦ βήματος ἀθανάτους ἔλεγε γεγονέναι καθάπερ τοὺς θεούς· οὐ γὰρ ἐκείνους αὐτοὺς ὁρῶμεν, ἀλλὰ ταῖς τιμαῖς ἃς ἔχουσι καὶ τοῖς ἀγαθοῖς ἃ παρέχουσι ἀθανάτους εἶναι τεκμαιρόμεθα.* With such expressions as the foregoing are compared the following in the epitaphios: ch. 43: *τὸν ἀγήρων ἔπαινον κάλλιστον ἔρανον προϊέμενοι.* ch. 41: *μνημεῖα κακῶν κἀγαθῶν ἀΐδια ξυνκατοικίσαντες.* ch. 43: *ἀνδρῶν ἐπιφανῶν πᾶσα γῆ τάφος*, and others. Cf. also ch. 62 in his speech to the Athenian Ecclesia: *κηπίον καὶ ἐγκαλλώπισμα πλούτου.*

(c) There is a majesty in the rhythm of the whole, a certain union of impetuous movement with lofty grandeur which Thucydides gives to Pericles alone. Thus, Professor Jebb. But, if I may be so bold as to have an opinion in the presence of such authority, I think it unlikely that there is any conscious imitation of the style of Pericles in the speech as given by Thucydides, and for the following reasons:

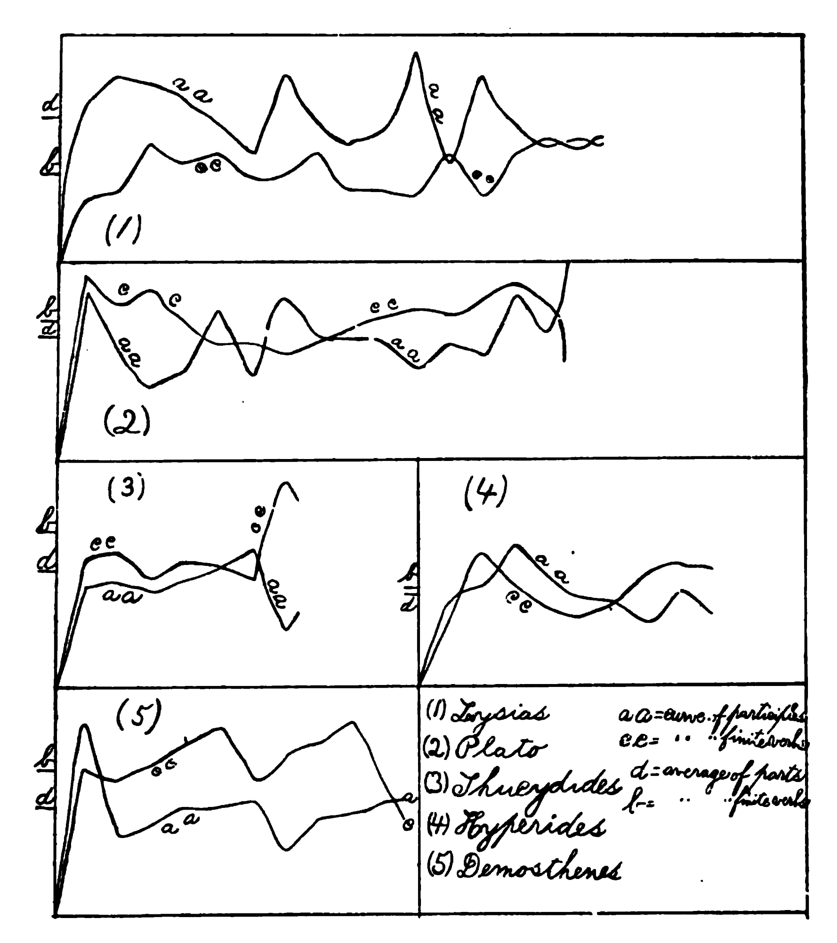
If we were to try to conceive what sort of a speech Thucydides himself would have written for his history, keeping in view the spirit and style of the historian as seen in the other portions of his writings, no speech could fit the conception better than this funeral oration. True, it is also a fact that if we were to try to conceive what sort of a speech Pericles would have written, from a study of his life and policy, no speech would fit that conception better than this speech so far as its style is concerned; *but*, Thucydides was writing for his history, not so Pericles. Thucydides had ample excuse for deviating from the beaten track of the Epitaphios, not so Pericles. No one of the epitaphioi is so far from the set type of the Epitaphios in point of view of subject-matter as the funeral speech of Thucydides. And this in my judgment is conclusive as against the verbatim theory. Again, Thucydides' business was not to imitate the style of the masters of oratory for the benefit of posterity. His work was not that of a rhetorician, but he was to represent the motives which were at work in shaping the destinies of Greece and the policies of her foremost statesmen. The fact that Thucydides was in very close personal contact with affairs in Athens at that time argues well for his historical accuracy in his facts and philosophy, but, if it proves anything beyond this, it proves what is too much—that he gives the actual words of Pericles, for it is untenable that Thucydides would stop to imitate Pericles when he could get his very words. The verbatim theory is, as I have said, refuted by the extreme lack of harmony of the subject-matter of the speech with the unyielding requirements of the type. I find after having written the above that to Dahlman belongs the credit of the argument from non-conformity, though I have the consolation of knowing that I am not alone in my opinion. It is futile to urge that the absence of mythical embellishment is rather a proof of the fidelity with which Thucydides has reported a speaker who regardless of the vulgar taste was resolved to treat a well-worn theme in a new and higher strain. Such, however, is the insinuating statement of Jebb. But this will not hold, because: (1) It is not characteristic of Thucydides to report a speaker with such fidelity, as he, Thucydides, himself admits. (2) The taste which made the Epitaphios what it was, is not vulgar when looked at from a historical point of view. (3) The theme was not at that time so well-worn as to have become tiresome, since this is the first extant speech of the kind that has come down to us. The type had not been set long enough to have become tiresome. And (4) so great departures from the norm were not made in later times when the theme was worn even to being threadbare.

To these considerations are to be added (1) the testimony of Plutarch (Per. 8) to the effect that Pericles left no written speech. (2) Quintilian declared those extracts in his time to be spurious. (Cicero in Brut. 7., evidently refers to those speeches imputed to Pericles by Thucydides with no thought of expressing an opinion as to their genuineness.) (3) The picture of Athens as painted in the epitaphios forms a fitting comparison to that of Sparta presented in the speech of Archidamus in 1-80ff., in which we trace again the mind of Thucydides, so visible throughout his entire history in the pairing of speeches, and which contributes so much to its dramatic effect. In fact the speech seems rather to have been written for the student of history than for an audience of mourners. (4) It is observable that Thucydides, in ushering in his speakers, studiously employs terms implying that he is only reproducing the substance of what they said or might have said, e. g.: ἔλεγε τάδε or ταῦτα. (5) Thucydides used his speeches as his special means of tracing back the visible facts to the internal moving causes and in no speech is this principle more clearly seen than in this funeral oration. For there can be no doubt that the speeches attributed to Pericles and this one in particular, do accurately represent the characteristic features of Pericles' policy. No dramatist ever better understood the art of thinking and feeling everyone of his characters than Thucydides. From an Athenian he can become Archidamus, or Hermocrates. He can lose his individuality as the historian and don'or doff any make-up at pleasure. As an artist he plays each role with a view to the unity of the whole and here his individuality never forsakes him. He recognizes in Pericles the foremost statesman of his time; he represents him as a believer in Athens for Athenians, as an advocate of peace, arbitration, reciprocity,—the Henry of Navarre of Greece. He thinks Pericles, he feels Pericles, he writes Pericles—not Pericles the orator, but Pericles the man, the statesman, the policy, and in painting Pericles, he paints the Periclean age.

Stylistic Comparison Based on Participle and Finite Verb.

It would be interesting to make a minute comparative study of the five different epitaphioi in their use of hiatus; of questions; of doublets; of vocabulary; the reflexive pronoun; the articular infinitive; the correlatives τε . . . καὶ and τε . . . τε; the prepositions; antithesis, hyperbola, methaphor, and simile; syntax and periodology; the period and the individuality; the rythmical law according to which Demosthenes avoided the accumulation of more than two short syllables; and so forth, but the limits of space would not

permit, though inclination is strong. I have, however, taken up what I regard as perhaps the most general characteristic of the epideictic style—the play of the participle and the finite verb. To this end I have prepared diagrams showing the curves of the participle and finite verb for each speech. The representation of



Participial and finite-verb curves of the Ἐπιτάφιοι.

statistics by curves is by no means new,* but its novelty is not entirely lost as applied to language. I am willing to admit that the results obtained are rather more interesting than instructive, or more instructive because of their interest, than interesting because

*An idea suggested to me by Prof. Lodge of Bryn Mawr.

of their instructiveness. Not new facts so much as new confirmation. The five prominent elements in Greek expression (any language for that matter) are the noun, finite verb, participle, infinitive, adjective. I have accordingly prepared three sets of diagrams: (a) A set showing the curves of the participle and the finite verb for each speech. (b) A set showing the curves of *all* the five elements just stated for *each* speech. (c) A set showing the curves of *each* of the five elements just stated for *all* the speeches. I regret that only the first set of diagrams can be reproduced in this article.

Without entering into a detailed statement of all the inferences that can be drawn from these diagrams, I will, for the sake of illustration, remark briefly only on the first diagram (Lys. Epit.). I have chosen this as being the longest speech and the most polymetochic. The ratio of participle to finite verb in the epitaphios of Lysias is 1.6: 1, there being 380 participles and 238 finite verbs. The number of participles to the page is 22¾; the number of finite verbs, $14\frac{3}{10}$. It is especially to be remarked with regard to diagram (1), that where the wave of the participle in the epitaphios reaches its highest point (33), there, that of the finite verb reaches its lowest (10). The line of finite verbs is rather flat-breasted, while the bosom of the participle seems to be well developed. It will also be seen that the highest point reached by the finite verb is not so high as the average hight of the participle, and that the lowest point reached by the participle is not so low as the average hight of the finite-verb. It is also to be noticed that where the wave of the finite verb reaches its highest points, there the participle reaches its lowest points. They seem to attract and repel each other alternately until they finally conclude to dwell together in unity. The highest number of participles on any full page is 33; the lowest 15. The highest number of finite-verbs on any full page is 19; the lowest 10.

We see from this that the epitaphios "swells with participles in the true epideictic style." A good example of the heaping-up of participles may be seen in sec. 27, another in sec. 31. In fact there are many places in the epitaphios, where to quote Professor Gildersleeve, "there are hardly enough finite verbs to hold the sentences down; where the finite verb has to be reached through a crowd of circumstances; the logical relations are not clearly expressed, and the play of color in which temporal causal conditional adversative rays mix and cross is maddening". (See Amer. Jour. Phil. vol. 9.)

Other notable points:

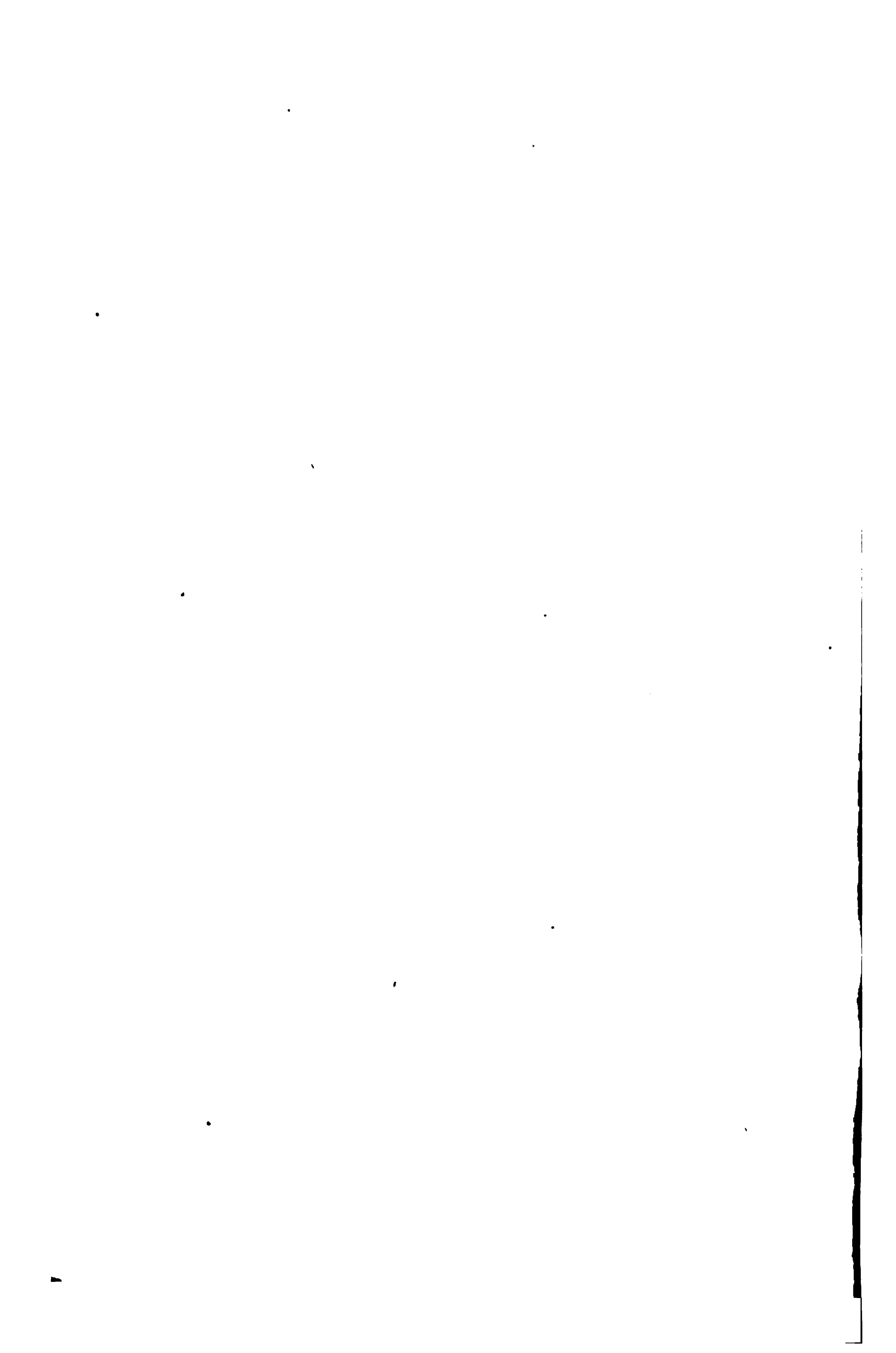
(1) Lysias', the only epitaphios in which the participial average is higher than the finite-verb average.

(2) Correspondence of curves of participle and finite verb in endings of speeches.

(3) High point of both participle and finite verb in the beginning of diagram (2). How accounted for—nouns.

(4) Peculiar ending of curves in Thucydides' epitaphios.

(5) High (comparative) point of adjective in Demosthenes.



A New Species of Dinictis from the White River Miocene of Wyoming.

BY ELMER S. RIGGS.

Among the specimens collected by the Kansas University Geological expedition of 1895 is one of a sabre-toothed cat having the general characteristics of *Dinictis*, but differing markedly from any described species. It comes from the Oreodon beds of eastern Wyoming. The distinctive characters are: The absence of the second lower molar, the slenderness of the base and the concave outer border of the upper sectorial as seen from above and the presence of but two incisors in the mandible. The name *D. paucidens*, sp. nov., is proposed for it.

The material consists of a skull, mandible, radius, ulnæ, humeri, the atlas, axis, and eighteen other vertebræ, together with numerous fragments. The skull is badly weathered in the frontal and premaxillary region, but the lower surface, the arches and the occipital portion are well preserved. The mandible is almost complete. The teeth are all present save the incisors and the upper tubercular molars. Compared with *D. felina* Leidy, the type of the genus, the skull is a little longer and broader across the arches, but narrower across the basi-occipital region. The capacity of the brain cavity is not above two-thirds as great, and the root of the zygomatic process does not project below the basi-cranial axis. The sagittal crest is thin and high, the arches slender, and the supraoccipital is moderately projecting. The otic bullæ are well inflated, and the paroccipital processes slender and directed backward. The infraorbital foramen is large, though not enough of its border remains to determine its shape. The palatine foramina lie opposite the anterior border of the third premolar. As in *D. cyclops* Cope, the sphenoorbital and rotund umforamina are conjoined and situated well forward. The latter receives a small ali-sphenoid canal connecting it with the foramen ovale. The last foramen corresponds in position with that of *D. felina* as described by Scott,* but is less prolonged transversely by the alisphenoid canal, which opens at its inner border. The foramen lacerum medius is large,

*Proceedings of the Academy of Natural Sciences of Philadelphia 1889.

oval in shape, and separated from the eustachian canal by a thin plate of bone. Just back of the otic bulla is the round and rather small jugular foramen. On the median side and lying near it is the small posterior opening of the carotid canal. The precondyloid foramen is located directly back of the latter and is similar in size and shape. The postglenoid foramen enters at the antero-superior angle of the external auditory meatus and forms a groove across the entire anterior wall. The mandible is broader and stronger than in *D. felina*, the symphysis and the flanges are deeper, and the coronoid process is shorter, narrower and less recurved. The flanges form a sharp projecting ridge at the angle with the anterior border. The anterior mental foramina are small and inconspicuous. The condyle is long and narrow, the masseteric fossa comparatively shallow and sloping at its borders.

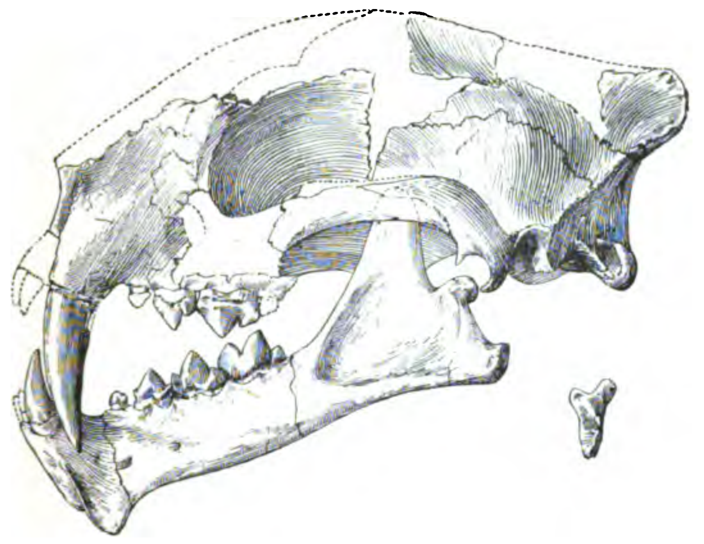
$$\text{Dentition I}\,\frac{?-?}{2-2,}\ \text{C}\frac{1-1}{1-1},\ \text{P M}\frac{3-3}{3-3}\ \text{M}\frac{1-1}{1-1}$$

The upper incisors together with the premaxillary border, are lost entirely. The upper canines are laterally compressed like those of *D. felina* but somewhat broader and longer than the estimates given of that type. They are two-edged, lenticular in cross section and minutely denticulate on both borders as in *D. cyclops*, although in this specimen the denticulation on the anterior border is almost worn away. The upper conical premolar is two-rooted, broader and more laterally compressed than in *D. felina*, but no trace of a secondary cusp remains. A short diastema separates it from the third premolar as well as from the canine. The anterior lobe of the third premolar is wanting, but the posterior one is prominent and is separated from the principal lobe by a transverse fissure similar to that of the carnassials. The upper carnassial, as before mentioned, is peculiar in its slender base as seen from above, its concave outer border, the prominence of its anterior basilar process, and the slenderness of the neck connecting with the internal conical cusp. The heel is low and retreating, the principal cusp short, and the transverse fissure open and marked. The crown of the tubercular molar is lost; the roots remaining indicate a rather strong tooth.

The crowns of all the lower incisors are broken away, but the position of the two roots remaining in one ramus and the one in the other, points to the conclusion that there were but two incisors on a side. The outer root is much stronger than the inner one. The lower canine is almost round at the base, slender, somewhat

recurved and slightly drawn to an angle on the posterior side throughout its entire length. The second premolar is separated from the canine by a descending diastema somewhat longer than in *D. felina*, and almost double that of *D. squalidens*. The tooth is a trifle smaller than the corresponding upper one and like it is separated from the third premolar by a short diastema. It is two-rooted and similarly compressed, but retains a slight posterior lobe. The third and fourth premolars are similar in size to those of *D. felina*, but the basilar lobes are larger and more angular. The lower sectorial has a more prominent heel, but bears only the slightest trace of a postero-internal cusp. No trace of a second molar can be found, although fractures in both of the mandibles just back of the carnassials would have brought it to light had even a rudiment existed.

The length and proportions of the bones of the front leg indicate



Skull of *Dinictis paucidens* Riggs.

a very slender animal. The humerus though slenderer is almost as long as that of the robust *D. bombifrons* described by Mr. Adams in the American Naturalist for June, 1895, while the radius is considerably longer and more slender. A series of measurements compared with those of *D. felina* will give an idea of the relative size. In Leidy's description of the generic type measurements of two different specimens are given. Those of the second and larger are used on account of their corresponding most nearly with the measurements of this specimen.

Measurements.	D. felina. M.	D. paucidens M.
Skull, length from condyle to incisors	.159	.165
" " " occiput " "		.176
" breadth across zygomata	.113*	.122
Mandible, length from angle to incisors	.123*	.129
" " of entire dental series	.071	.069
" " of molar series	.051	.049
" breadth of incisor series	.015	.014
Length of upper molar series	.045	.043
Upper canine, length	.042*	.046
" " longitudinal diameter	.015	.016
" " transverse "	.007	.007
Length of upper diastema	.007	.005
Lower canine, length	.016	.022
" " longitudinal diameter	.008	.008
" " transverse "	.007	.007
Length of lower diastema	.012	.014
Breadth of crown of upper second premolar	.005	.006
" " " " " third "	.016	.017
" " " " " sectorial "	.021	.019
" " " " lower second "	.003	.005
" " " " " sectorial "	.019	.018
Transverse diameter of brain case		.045
Depth of flange from base of canine	.031	.032
" " symphysis	.027	.031
" " mandible at front of sectorial	.018	.021
Breadth of front border of mandible	.023	.025
Distance from condyle to angle of mandible	.021*	030
" " " to summit of coronoid process	.032	.029
Height of occipital crest above occipital condyles		.055
Breadth of skull across mastoid processes	.070	.062
Distance from anterior margin of glenoid cavity to posterior maxillary border	.044	.046
Length of glenoid cavity	.023	.029
Length of bony palate	.072	.073*
Humerus, length of	.172	.189
" breadth of head and great tuberosity	.038	.040
" " of distal end	.042	.043
Ulna, length of	.172	.190
" olecranon to posterior lip of sigmoid cavity		.029
" " to coronoid process		.046
Radius, length of		.157
" diameter of head		.018
" " of distal end		.027

It will be seen that the specimen upon which this species rests is somewhat larger than *D. felina*, and considerably longer and slenderer of limb. The absence of the second lower molar might

*Estimated.

not be especially significant in itself, since that tooth is described as just disappearing in *D. bombifrons*. Also the denticulate superior canine is described in *D. cyclops*, the absence of the third incisor in *D. squalidens*, and the general size and proportions of the skull in *D. felina*, but in no existing type are all these characters combined. Moreover the peculiar shape of the upper carnassial, the slight development of the postero-internal lobe of the lower carnassial, the presence of but two incisors in the mandible and the proportionate length and slenderness of the forearm, appear to be constant characters. These, together with the absence of the second lower molar, seem sufficient grounds for establishing a new species, which from the small number of teeth may fittingly be termed *Dinictis paucidens*. The type is preserved in the University of Kansas Museum.

Continuous Groups of Projective Transformations Treated Synthetically.

BY H. B. NEWSON.

Part II.

In Vol. IV, No. 2 of this QUARTERLY the writer gave a tolerably complete synthetic theory of the Projective Groups in one dimension and promised a similar theory for the same groups in two dimensions. The present paper is a partial fulfillment of that promise.

§1. Construction of Projective Transformations.

Let π and π^1 be any two planes intersecting in the line l and making any convenient angle with one another. It is known from the theory of projection that when we have given any four elements, lines or points, of the plane π and the four corresponding elements of π^1, then the projection of π on π^1 is completely determined. These four elements must be taken in perfectly general positions; i. e. if four points be taken, no three can lie on a right line; if four lines be taken, no three can pass through a point.*

Let us suppose that we have given four lines α, β, γ, δ, in the plane π and the four corresponding lines α^1, β^1, γ^1, δ^1, in π^1. The four lines α, β, γ, δ, intersect in the six points P, Q, R, S, T, U; the corresponding points in π^1 are P^1, Q^1, R^1, S^1, T^1, U^1. Let us call the points in which α, β, γ, δ cut the line l, A, B, C, D. Then we have in the plane π on the lines α, β, γ, δ, the four ranges (PQRA), (TSPB), (QSUC), (RTUD). If we construct in the plane π^1 on the lines α^1, β^1, γ^1, δ^1 the points A^1, B^1, C^1, D^1, so that the anharmonic ratios $(P^1Q^1R^1A^1)=(PQRA)$, $(T^1S^1P^1B^1)=(TSPB)$, $(Q^1S^1U^1C^1)=(QSUC)$, $(R^1T^1U^1D^1)=(RTUD)$; then A^1, B^1, C^1, D^1 will lie on the line l^1, which in π^1 corresponds to l in π. Let the lines joining AA^1, BB^1, CC^1, DD^1 be a^1, b^1, c^1, d^1; the lines a^1, b^1, c^1, d^1, l, l^1 all lie in π^1 and touch a conic K^1, every tangent to which cuts l and l^1 in corresponding points.

Let us now consider the line l to belong to the plane π^1 and find the line l_1 in π which corresponds to l in π^1. This can be done

*See the following standard references: Reye's Geometrie der Lage, Band II, page 7. Clebsch-Lindemann's Vorlesungen ueber Geometrie, Band I, page 256. Lie's Continuierliche Gruppen, page 21.

exactly as before by considering ranges of four points each on α^1, β^1, γ^1, δ^1 and constructing the corresponding ranges on α, β, γ, δ. As before we shall have four points on l in π^1 and their four corresponding points on l_1 in π; the lines joining these corresponding points all lie in the plane π and together with l and l_1 determine a conic K in π, every tangent to which cuts l and l_1 in corresponding points. Let the points on l_1 corresponding to A, B, C, D on l be A_1, B_1, C_1, D_1; and let the lines joining AA_1, BB_1, CC_1, DD_1, be a, b, c, d. Now a and a^1 are corresponding lines of the two planes. For A and A_1 are two points of π whose corresponding points in π^1 are A and A^1; hence a^1, the join of A and A^1, corresponds to a, the join of A and A_1. For the same reason b and b^1, c and c^1, d and d^1, are corresponding lines of the two planes. Thus we have in general:

The tangents to the two conics K and K^1 from any point on l *are corresponding lines of the two planes.*

By the aid of the conics K and K^1 we can now construct the point P^1 in π^1 corresponding to any point P in π. Draw the tangents from P to K and let them cut l in Q and R; the lines in π^1 corresponding to these tangents from P are the tangents from Q and R to K^1; these meet in P^1, the point corresponding to P.

If we have given any line as g in π, we can construct the corresponding line g^1 in π^1 as follows: take two points P_1 and P_2 on g and in the manner just explained construct the corresponding points P_1^1 and P_2^1; the line g^1 joining P_1^1 and P_2^1 corresponds in π^1 to g in π.

Since the constructions in the two planes are exactly alike, these operations are strictly reversible. By means of the conics K and K^1, the configuration in π corresponding to any given configuration in π^1 can be constructed. And further, one of the planes as π^1 may be revolved about l until it coincides with π, and then the construction thought of as all in the same plane. We shall make constant use of this last conception.

The lines joining corresponding points of the planes π and π^1 form a linear congruence (Strahlen-Congruenz) of the third order and first class; (see Reye's Geometrie der Lage, II. Band, p. 94). By means of this congruence the projection of π on π^1 is completely determined. By a revolution about l this new set of points may be brought back to π and both sets of points thought of as existing in the same plane. This operation of projecting the points of π into a new system of points on π^1 and by a revolution about l bringing the new system back to π will be called a *Projective Transformation*

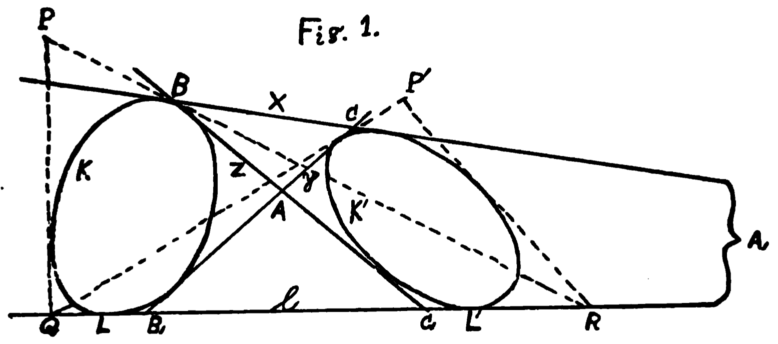
of the plane π. Such a projective transformation is determined and completely constructed by means of two conics K and K^1 in the pland π and touching the line l.

Theorem I. A projective transformation of the points of a plane is completely determined by means of two conics K and K^1 touching a fixed line l *of the plane.*

It is clear from this method of constructing a projective transformation that any point P outside the conic K; i. e. a point from which two real tangents can be drawn to K, is transformed into a point P^1 outside of K^1. Similarly a point within K is transformed into a point within K^1. For the tangents from P to K in this case are a pair of conjugate imaginary lines, and their corresponding lines are conjugate imaginary tangents to K^1, and hence intersect in a real point inside of K^1. And finally it may be seen that any point P on K has its corresponding point P^1 on K^1.

In order to construct P^1 on K^1 corresponding to any point P on K, draw the tangent to K at P; from the point where this tangent to K cuts l draw the tangent to K^1; its point of contact with K^1 will be the point P^1 desired. It follows from this last construction that the polar of any point P with respect to K corresponds to the polar of P^1 with respect to K^1. This fact suggests a method for constructing a point P^1 within K^1 which corresponds to a point P within K. Construct p the polar of P with respect to K; then construct p^1 the line corresponding to p; and finally construct the pole of p^1 with respect to K^1; this will be the point P^1 desired.

The transformation determined by K and K^1 transforms the line l into some line l^1. Since l is a tangent to K, l^1 must be a tangent to K^1. The line l^1 is easily seen to be the tangent to K^1 from the point of contact of K and l. The tangent to K from the point of contact of K^1 and l is transformed into l.



The chief results may be concisely stated as follows:

Theorem 2. The conic K divides the plane π into the regions O (outside) and I (inside); the conic K^1 divides the plane π^1 into two corresponding regions O^1 and I^1. The projective transformation determined by K and K^1 transforms the region O into O^1 and the region I into I^1; it transforms the boundary of O and I into the boundary of O^1 and I^1.

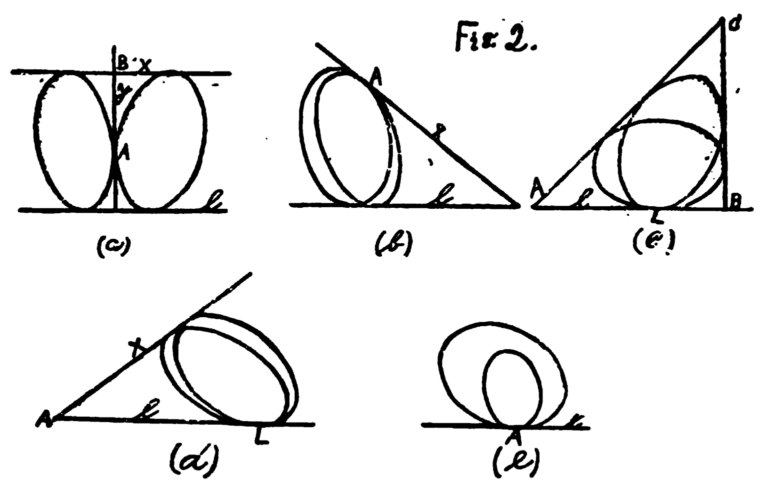
These two conics touching the line l have generally three other common tangents which will be designated by x, y, z, (Fig. 1). We shall next examine the relation of these three lines to the transformation determined by the two conics K and K^1. Let us take a point P on the line z and draw the two tangents from it to the conic K; one of these tangents is the line z which cuts l at C_1, the other cuts l at some point as Q. The corresponding point to P is found by drawing tangents to K^1 from C_1 and Q. One of these tangents is again the line z; the tangent to K^1 from Q intersects z in P^1 the corresponding point to p. In like manner every point on the line z is transformed into a point on the line z; in other words the line z is an invariant line of the transformation. In the same way it may be shown that the lines x and y are also invariant lines of the transformation. The fixed line l is not an invariant line, although a common tangent to the two conics.

The points A, B, C, which are the intersections of the invariant lines x, y, z, are invariant points of the transformation. This is evident from the fact that the two tangents from A to K are the lines y and z; the two tangents from C_1 and B_1 to K^1 are also y, and z, which intersect at A the startling point. Thus A is a self corresponding point, or an invariant point, of the transformation; the same is true of B and C.

It is easy to see from the construction that these three points and these three lines are the only ones left invariant by the transformation determined by K and K^1. In particular cases where the conics K and K^1 are specially related to one another, e. g. touch one another, the invariant figure may be different. These special cases will be determined later. When the conics K and K^1 do not intersect, the sides and vertices of the invariant triangle are all real; but if the conics intersect in two real and two imaginary points, they have only one real common tangent other than l, the other two being imaginary. In this case the invariant triangle has one real and two imaginary vertices, one real and two imaginary sides. If the two conics intersect in four real points, the invariant triangle is again real in all its parts. We shall call this most general transformation, Case I.

For special positions of the conics K and K^1 the invariant figure

may be different. Thus special cases arise when the conics touch one another, or touch the lines l at the same point, or have contact of the second or third order, etc. These special cases we now proceed to examine.



Let us first consider the case of a projective transformation where the two conics touch one another. The invariant figure in this case consists of y, the common tangent to the two conics at A their point of contact, and another common tangent x intersecting y at B, (Fig, 2 (a)). It does not matter whether the two conics intersect in two other real or imaginary points, the invariant figure is the same in either case. This kind of a projective transformation may be considered as a special case of the last when two sides of the invariant triangle coincide. We shall call this Case II.

Instead of simple contact as in last case the two conics K and K^1 may have contact of the second order at a point A. When the conics have a contact of the second order they can and must intersect in one other point. In this case the common tangent to the two conics at A is the only invariant line of the transformation and the point A is the only invariant point on the invariant line, (Fig. 2. (b)). We shall call this Case III.

Again the two conics may both touch the line l at the same point, the contact being of the first order. In this case the two conics K and K^1 have two other common tangents x and y which intersect at some point C. (Fig. 2 (c)). It is at once evident from the figure that the transformation determined by K and K^1 leaves the lines x and y and the point C invariant. A little further consideration of the construction shows that the points A, B, and L on the line l are

invariant points. But if a projective transformation leaves more than two points of a line invariant, it leaves all points on the line invariant, (Cont. Gruppen, page 117). Therefore every point on the line l is an invariant point of the transformation. Any line g drawn throngh C intersects l in some point as G. Therefore the line g having two points G and C invariant is an invariant line. Thus we see that every line through C is an invariant line. Hence we conclude that the transformation determined by the two conics K and K^1 touching the line l at the same point leavesthe point C, all points of the line l, and all lines through C invariant. The two remaining points of intersection of the two conics may be either real or imaginary, the result is the same in either case. If these remaining points of intersection are coincident, i. e. if the two conics have double contact, the resulting transformation is still of the same character and the invariant figure the same. This constitutes Case IV.

When the two conics have a contact of the second order at the point L on the line l, the invariant figure takes still another form. In this case only one other common tangent x can be drawn to the two conics. This common tangent intersects l at A, (Fig. 2 (d)). The transformation determined by the two conics in this position leaves invariant all points of the line l and all lines through A.

If the two conics have contact of the third order at L, then l is the only common tangent they have, (Fig. 2 (e)). Such a transformation leaves invariant every point of the line l and every line through L. The invariant figure is therefore the same as before. This constitutes Case V.

We have now enumerated all the special positions which the conics K and K^1 can take and there are no more cases to be considered.

Theorem 3. Every projective transformation of the plane determined by two conics K and K^1 touching a fixed line l *of the plane leaves a certain plane figure invariant. There are five cases to be considered:*

Case I. When the two conics are not in contact, the invariant figure is a triangle.

Case II. When the two conics have contact of the first order not on l, *the invariant figure consists of two invariant lines, their point of intersection, and an invariant point on one of the invariant lines.*

Case III. When the two conics have contact of the second order not on l, *the invariant figure consists of one invariant line and one invariant point on that line.*

Case IV. When the two conics have a contact of the first order on l, *the invariant figure consists of all points on the line* l, *a single point* A *not on* l, *and the pencil of lines through* A.

Case V. When the conics have contact of the second or third order on l, *the invariant figure consists of all points on the line* l *and all lines through a point* A *on* l.

We need a convenient and expressive notation for these five kinds of projective transformations in the plane. We shall designate a transformation of the first kind by the single letter T. When we wish to designate the conics which determine the transformation, we shall write it T (KK^1). When we wish to call attention to the triangle which remains invariant we shall use the notation T(ABC). Transformations belonging to cases II and III will be designated by T^1 and T^{11} respectively. Transformations belonging to cases IV and V will be designated by S and S^1 respectively.

(To be continued.)



Editorial Notes.

The department of Palæontology has for sale or exchange a magnificent slab, of about twenty square feet in area, of Uintacrinus Socialis from the Kansas Cretaceous. It has numerous complete heads in excellent preservation. Address Prof. S. W. Williston, Lawrence, Kansas.

Mr. Vernon L. Kellogg, formerly managing editor of the QUARTERLY and an instructor in the University of Kansas, has recently been made Professor of Entomology in Stanford University where he has been associate professor for two years past. Professor Kellogg's Monograph on New Mallophaga, published by the Stanford University, has attracted wide and favorable attention.

In a recent paper *by Professor Cragin "On the Stratigraphy of the Platte Series, or Upper Cretaceous of the Plains" the author proposes a number of new terms for various divisions of the Cretaceous of Kansas, with many of which the

*Colorado College Studies, vi, 53.

writer can not agree. In the first place, I doubt the necessity or even desirability of introducing a term that is perfectly synonymous with "Upper Cretaceous," already well established, to include the Dakota, Benton, Colorado, Montana and Laramie groups. These rocks, presenting as they do marine, brackish water and fresh water deposits, certainly have nothing in common that renders any other term than that in use desirable.

Osborne Limestone. The writer has already restricted the name Ft. Hays† Beds to this group of limestones. It is true that Mudge‡ used this term in a wider

†Trans. Kans. Acad. Science, xiii, 108.
‡Bull. U. S. Geo. Surv. Hayden, No. 3, 218.

sense. "The massive stratum of limestone above described, together with all the deposits above the sandstone of the Dakota, I shall call the Fort Hays division." To use Professor Cragin's own words, "if a geological subdivision must be given a confessedly new name whenever one chooses to pare it off or add to it a little, or has doubt about the original disposal of some small fraction of it, confusion worse confounded will increasingly result and finally reign supreme in the science of stratigraphic geology." The benefits that Mudge conferred upon Kansas geology are certainly worthy of this concession. In any event the writer has thus restricted the name and Professor Cragin's name is a pure synonym.

Smoky Hill Chalk. Professor Marsh has already named these beds the Pteranodon Beds, of which fact Professor Cragin seems to be ignorant. Professor Cragin subdivides these beds into two zones, the "Norton Zone" and the "Trego Zone." Aside from the fact that Norton Zone is an unfortunate term, inasmuch as the deposits in Norton county are very sparse and uncharacteristic, the author gives no characters by which the two zones can be distinguished. The present writer had fondly hoped that he exploded the error made by Mudge and followed by others that the "blue shale" was characteristic of the lower horizons. There is no marl anywhere in the Niobrara deposits and absolutely no lithological differences between the upper and lower beds, as the writer has convinced himself by microscopic examinations.

The sole difference discoverable is the relative greater abundance in some places among the upper rocks of iron, giving, when acted upon chemically, a more buff or yellow color to the chalk and a deeper color to the "blue shales." There *is* a paleontological distinction between the upper and lower beds, as the writer has already pointed out,.(op. cit.), and if one needs characteristic terms for the two horizons, the names Hesperonnis and Rudistes beds would be far preferable. Nor is there any geological horizon of jasper, and I am surprised that the author should have so intimated. As is well known, the Nebraska Tertiary beds lie *unconformably* upon the Niobrara, and the silicification of the uppermost part of the chalk from the percolating waters could not possibly produce a geological horizon.

The resent writer was the first to call attention to the presence in Kansas of the Ft. Pierre deposits. Since that time he has had the opportunity of studying the same group of rocks in various other regions to the northwest. There is absolutely no difference, yet discovered, either lithological or palæontological to distinguish the outcrops in Wallace county from those of the typical beds in Dakota and elsewhere. Professor Cragin proposes to call these beds in Kansas the "Lisbon Shales," but he gives nothing whatever to distinguish them from the beds above them. Why then should they have a distinct name? Nor does he give any reasons for naming the Arickaree shales of the Fox Hills group in Cheyenne county. One is hardly justified in naming geological horizons without attempting to define them. There has already been too much of such aimless nomenclature. The author seem to have been unaware of the publication of the paper by the undersigned in the Trans. of the Kansas Academy.

S. W. WILLISTON.

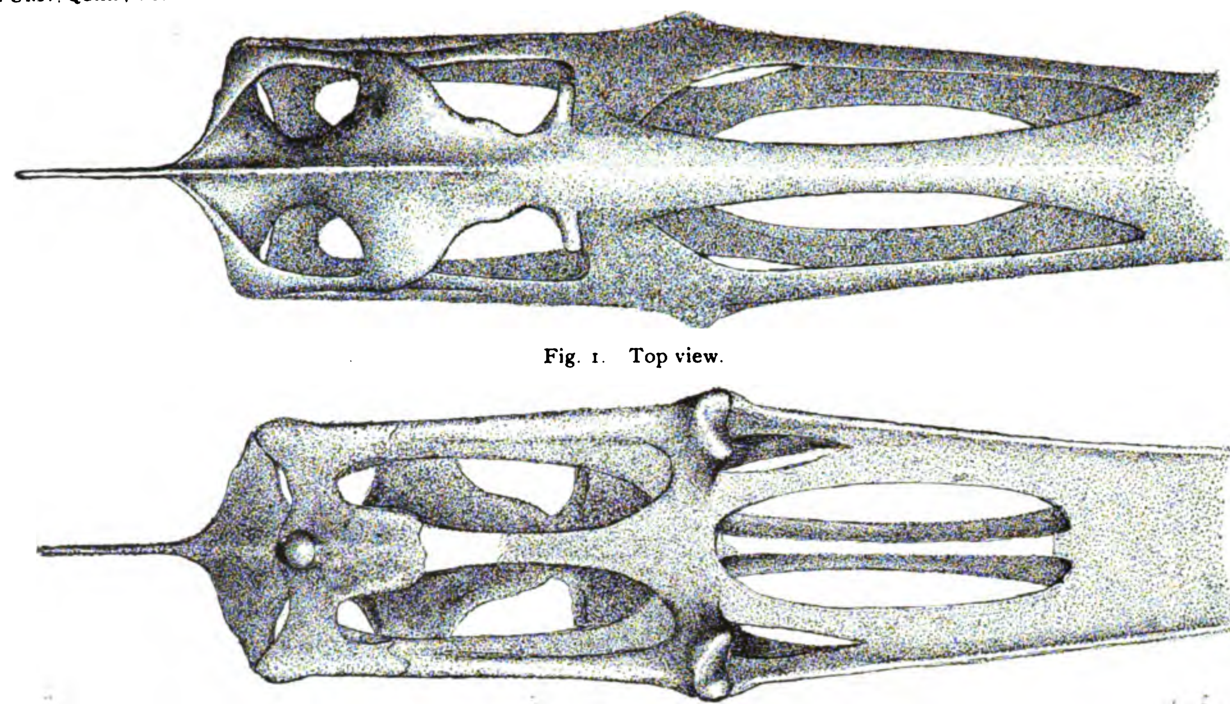
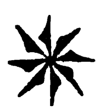


Fig. 1. Top view.

Fig. 2. Bottom view.

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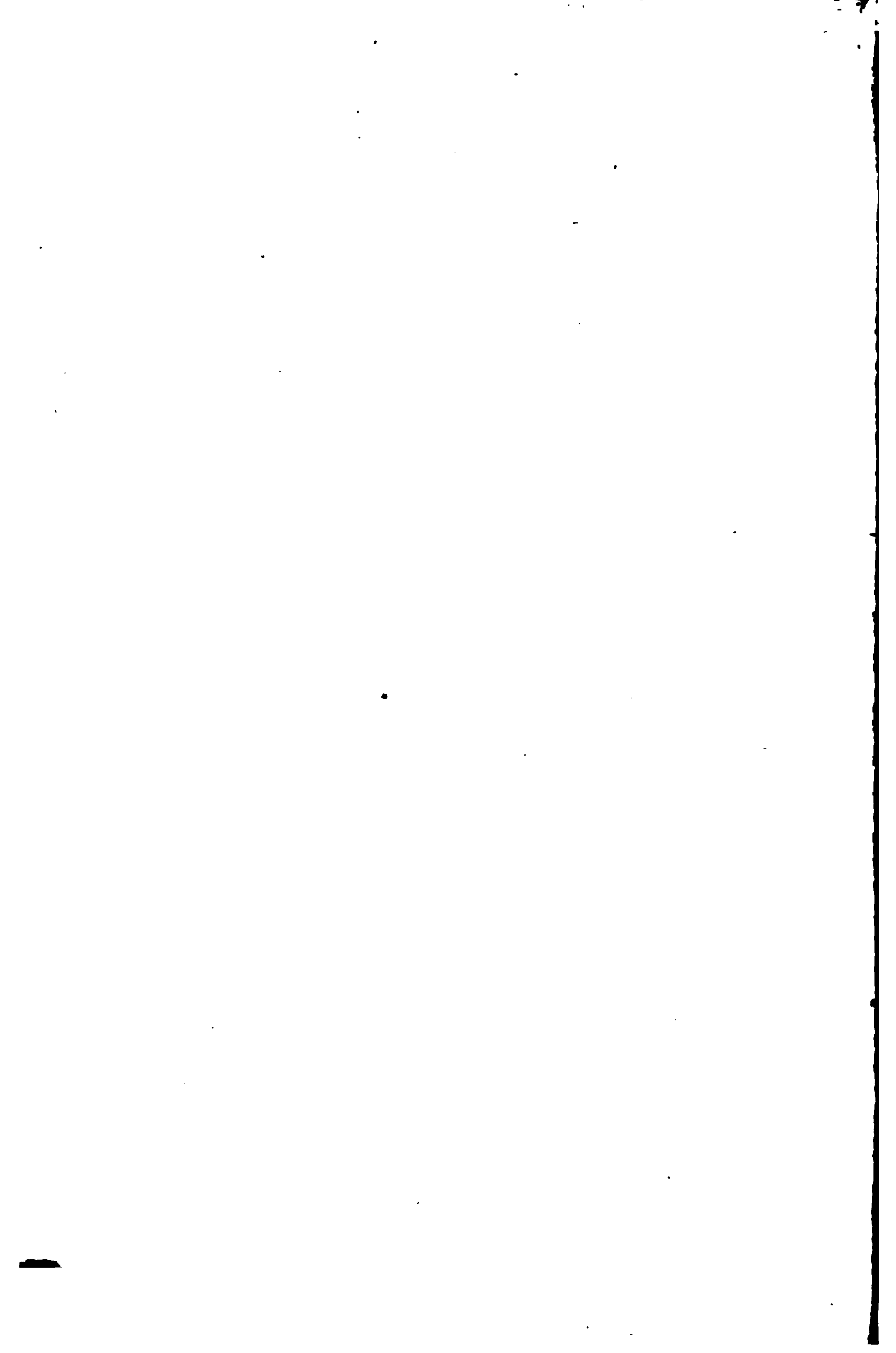
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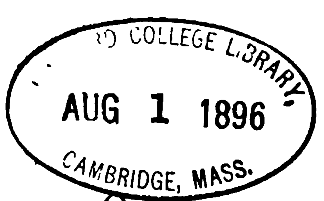
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KANSAS UNIVERSITY QUARTERLY

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Projective Groups of Perspective Collineations in the Plane, Treated Synthetically.

[A Dissertation presented to the Faculty of the University of Kansas to attain the degree of Doctor of Philosophy.]

BY ARNOLD EMCH.

Preliminary.

In presenting this paper the author has attempted to consider the subject of Perspective Collineations from the modern point of view of the theory of groups. Hitherto collineation has always been treated either by means of descriptive geometry, or analytic geometry, and without reference to group properties.

Sophus Lie's "Vorlesungen über Continuierliche Gruppen," which will be chiefly referred to, treats of the projective transformations of the plane of which collineation is only a special case. There are five types of projective transformations in the plane under which all the groups can be subordinated, and it has been the aim of Professor Newson to enumerate the projective groups of the plane by means of the synthetic method, and to classify them according to those five types.

This paper treats in the same manner the two types of transformations that are known as perspective collineations.

For suggestions in the treatment of the subject the author is thankfully indebted to his collaborator, Prof. H. B. Newson.

§1. Representation of Perspective Collineation.*

If the corresponding lines α, α^1; β, β^1; γ, γ^1; δ, δ^1; intersect each other on the line of intersection of the two planes π and π^1, and fulfill the further condition that all the connection-lines of

*Perspective Collineation has the same meaning as the German *Centrische Collineation* (Fledler), or the French *homologie*.

corresponding points pass through one and the same point, the projectivity, thus produced, is perspective collineation.

In the general case to the line l, the line of intersection of the planes π and π^1, belonging to the plane π, corresponds a line p^1 in the plane π^1, and to the line l, belonging to the plane π^1, corresponds a line p in the plane π. Connecting the corresponding points of l and p^1, and of l and p two conics K^1 and K, in π^1 and π respectively, are produced which determine the projective transformation. In the case of a perspective collineation, however, these two conics are indeterminate; since the lines p and p^1 coincide with l. We can, therefore, choose any four points A, B, C, D, on l, in π, and connect them with their corresponding points A^1, B^1, C^1, D^1, in π^1, which coincide with the former, i. e., we can draw any four lines a^1, b^1, c^1, d^1, in π, through A, B, C, D, respectively, which with l determine the conic K^1. The conic K is determined by those lines a, b, c, d, in π, which correspond to the lines a^1, b^1, c^1, d^1, according to the original conditions.

Thus, the two conics K and K^1 are collinear and fully determine the collineation. Since K^1 touches the line l, K touches l at the same point. As it will be seen from this, the two conics K and K^1 characterizing the general projective transformation exist also in perspective collineation; but there is a multiplicity of two conics tangent to each other and tangent to the line l at the same point. As there are ∞^4 conics tangent to l one and the same perspective collineation can always be represented by ∞^4 combinations of such two conics. The line l is the axis of collineation, and the centre C of collineation is obtained by the intersection-point of the two other common tangents of the conics K and K^1.

We are now ready to make the following statement:

Theorem 1. Each two conics tangent to each other determine a perspective collineation with the common tangent at their point of tangency as the axis and the intersection-point of their two other common tangents as the centre of collineation.

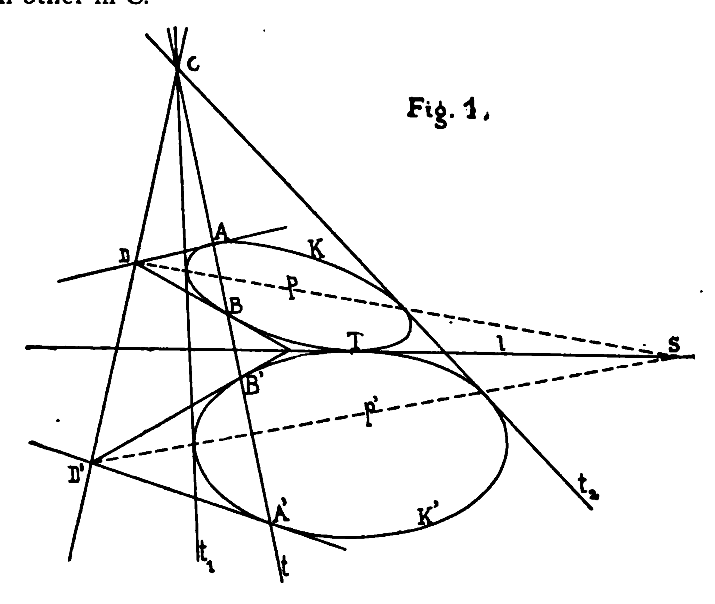
The general theorem concerning the construction of collineation by means of two conics tangent to the same line, as well as this special theorem, are obtained in a natural way by studying the congruence of right lines (3. 1) formed by all the right lines connecting corresponding points of two collinear planes in space. The focal surface of the congruence is a developable surface of the 3. class, and its edge of regression a curve in space of the 3. order. Any two osculating planes of this curve intersect the surface in two conics which are tangent to their line of intersection. Desig-

nating the points of intersection of any ray of the congruence with the two osculating planes as corresponding points, the two planes are collinear and we have immediately our general theorem if we revolve one of the osculating planes into the other about their line of intersection. In the case of a perspective collineation the congruence is of the order 1 and the class 0. The developable surface is not determinate, so that we may choose a cone of the 2. class which with any two planes determines a perspective collineation.

As in the general case of projectivity, to each point P the corresponding P^1 is obtained by drawing two tangents from P to the conic K which will intersect the line l in two points; from these two points draw the tangents to the conic K^1. Their point of intersection gives the required point P^1.

That this construction gives perspective collineation we can also prove without referring to the general case of projectivity.

Assume the two conics K and K^1 in the required position (Fig. 1) and draw the common tangents t_1 and t_2 which intersect each other in C.



K and K^1 now belong to a system of conics tangent to t_1 and t_2 and to l at a fixed point T. The polars p, p^1....of C in regard to the conics K, K^1,....of the system, therefore, intersect each other in one and the same point S on l, and if we draw any other line t through C which intersects K in A and B, and K^1 in A^1 and

B^1, the tangents in A and B to the conic K intersect in a point D of the polar p and those in A^1 and B^1 to the conic K^1 in a point D^1 of the polar p^1. As is well known from synthetic geometry the points D and D^1 lie on a ray through C. Moreover, the tangents in A and A^1, and in B and B^1 meet in the line l, hence, the points D and D^1 are obtained by our construction of the collineation. To every point corresponds one and only one point and both lie in a ray passing through the centre C. Two corresponding straight lines always meet in a point of the line l. These two conditions, however, constitute perspective collineation and hold for any point, or line of the plane and their corresponding elements.

In the next chapter we shall make those constructions which will be necessary in the study of group-properties of perspective collineations.

§2. Classification of Perspective Collineations.

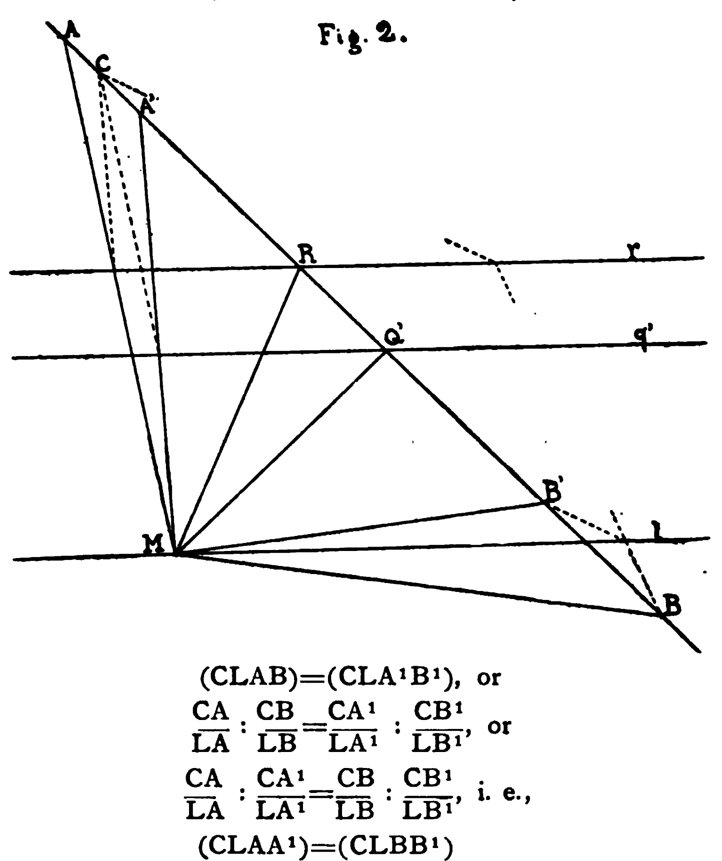
The two conics, K and K^1, determining the perspective collineation, being given we can ask for the line q^1 which corresponds to the infinitely distant straight line q. Drawing the two parallel tangents to the conic K from each point at infinity, and from their intersection-points with the axis l of perspective collineation tangents to the conic K^1, the points of q^1 are obtained by the intersection of each such pair of tangents to the conic K^1. Conversely, there exists a straight line r whose corresponding line r^1, or what is the same, q^1, is at infinity. The lines q^1 and r may be called counter-axes (German "Gegenaxen") of perspective collineation, and are parallel to the axis of collineation.

In central projection and perspective the constructions are usually made by aid of the centre and axis and the counter-axes of perspective collineation. A perspective collineation is determined by centre and axis and any one of the counter-axes, and, as immediately follows by construction, also by centre and axis and two corresponding points of the collineation.

It is now of great importance to state the connection between these two determinations.

Each perspective collineation transforms a ray through the centre into itself and also each point of the axis into itself. In other words, it leaves the points of the axis and the rays through the centre invariant. Each ray through the centre represents two coincident projective point-ranges, and each pencil of rays through a point of the axis two coincident projective pencils of rays.

Taking a ray s through the centre C, its corresponding ray s^1 is coincident with s and intersects the counter-axes q^1 and r in the two counter-points Q^1 and R of the ray (German "Gegenpunkte").* Since C and L, L being the intersection-point of s with l, correspond to themselves the following relation between these points and two pairs of corresponding points A, A^1, and B, B^1, on the ray s exists: (See Fig. 2.)



$$(CLAB)=(CLA^1B^1), \text{ or}$$

$$\frac{CA}{LA}:\frac{CB}{LB}=\frac{CA^1}{LA^1}:\frac{CB^1}{LB^1}, \text{ or}$$

$$\frac{CA}{LA}:\frac{CA^1}{LA^1}=\frac{CB}{LB}:\frac{CB^1}{LB^1}, \text{ i. e.,}$$

$$(CLAA^1)=(CLBB^1)$$

Substituting for the pair B, B^1 the pair Q, Q^1, or R, R^1, this last projectivity becomes:

$$(CLAA^1)=(CLQQ^1)=(CLRR^1), \text{ or}$$

$$(CLAA^1)=(CL\infty Q^1)=(CLR\infty), \text{ or}$$

$$\frac{CA}{LA}:\frac{CA^1}{LA^1}=\frac{CQ^1}{LQ^1}:\frac{CR}{LR}=\text{const.}$$

*We avail ourselves of the designation of Fiedler in his "Darstellende Geometrie," I. Band, and for the following classification especially refer to §22, page 95, of this book.

Thus, any pair of corresponding points in the perspective collineation has a constant relation to the counter-points, and we have the well known

Theorem 2. Each pair of corresponding points on a ray through the centre forms a constant anharmonic ratio with the intersection-point of the ray with the axis of perspective collineation.

Any point M on the axis l may be connected with the points C, L, Q, R, ∞, A, A^1, B, B^1, and designating these rays by small letters, there is obviously

$$(claa^1)=(clbb^1)=\text{const.}$$

This fact can be stated as the dualistic of the above theorem, viz:

Theorem 3. Each pair of corresponding rays through a point on the axis forms a constant anharmonic ratio with the ray through the centre and the axis of perspective collineation.

We call this constant the characteristic anharmonic ratio of the perspective collineation and designate it by k.* By aid of it a classification of the perspective collineation can easily be made, and so far as it will be of avail for our further consideration we will discuss the different cases of perspective collineation from this point of view. Among all the ∞^1 values of k the special case $k=-1$ deserves the greatest attention, and it shall be considered first, because it enables us at once to draw important conclusions from its combination with particular positions of the center, the axis, and the counter-axes of the perspective collineation.

From the assumption $k=-1$ follows:

$$\frac{CQ^1}{LQ^1}=\frac{CR}{LR}=-1, \text{ or}$$

$$CQ^1=-LQ^1, \text{ and } CR=-LR; \text{ i. e.,}$$

the counter-points and therefore also the counter-axes are midway between C and L, and, therefore, coincide. For every pair of corresponding points the relation exists:

$$(CLAA^1)=-1=\frac{CA}{LA}:\frac{CA^1}{LA^1}=\frac{CA^1}{LA^1}=\frac{CA}{LA}$$

From this follows

$$(CLAA^1)=-1=(CLA^1A) \text{ and}$$

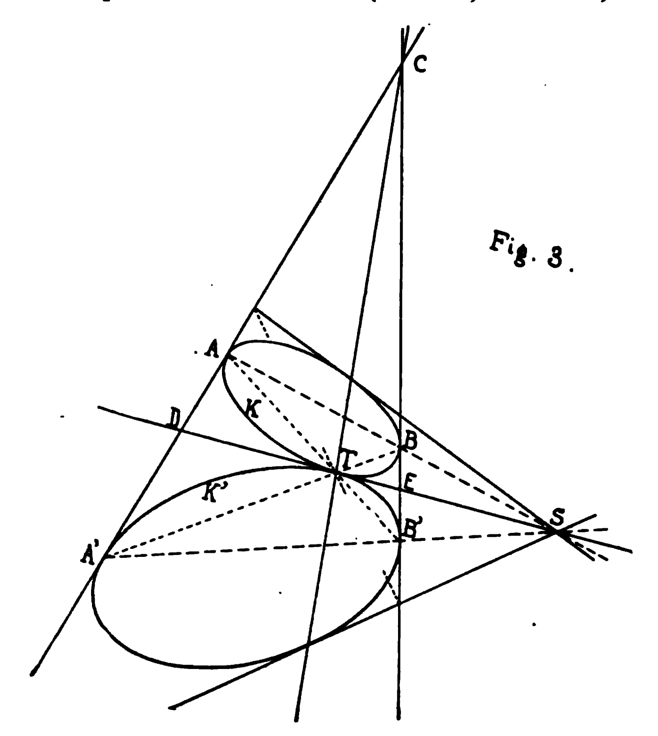
in a similar way:

$$(claa^1)=-1=(cla^1a), \text{ i. e.,}$$

in this collineation the points and rays of each pair are interchangeable. The collineation whose characteristic anharmonic ratio is $k=-1$ is therefore involutoric.

*This constant was first introduced into Geometry by Fiedler.

The same result is also obtained by starting from the two conics K and K^1 which determine the collineation. Fig. 3 represents the involutoric position of the conics $(CADA^1)=CBEB^1)=-1$.



The polars of C in regard to the conics K and K^1 intersect each other in S on l. The polars of S in regard to K and K^1 pass therefore through T, the common point of tangency of the conics with l, and through C, i. e., they are identical. Hence the point T is the intersection-point of $\overline{AB}^1$ and $A^1\overline{B}$; $(SETD)=-1$. Thus, designating the points of tangency of the common tangents to the conics by A, A^1 and B, B^1, the conics are in involutoric position, if their common point of tangency with l, T, coincides with the intersection-point of $\overline{AB}^1$ and $A^1\overline{B}$.

Besides involution we have to consider those collineations which result from special positions of the centre and axis of perspective collineation, or what is the same, from special positions of the conics K and K^1.

In the following development it would not be necessary to take conics K and K^1 into consideration. We shall do it here in order to show how the representation of special collineations

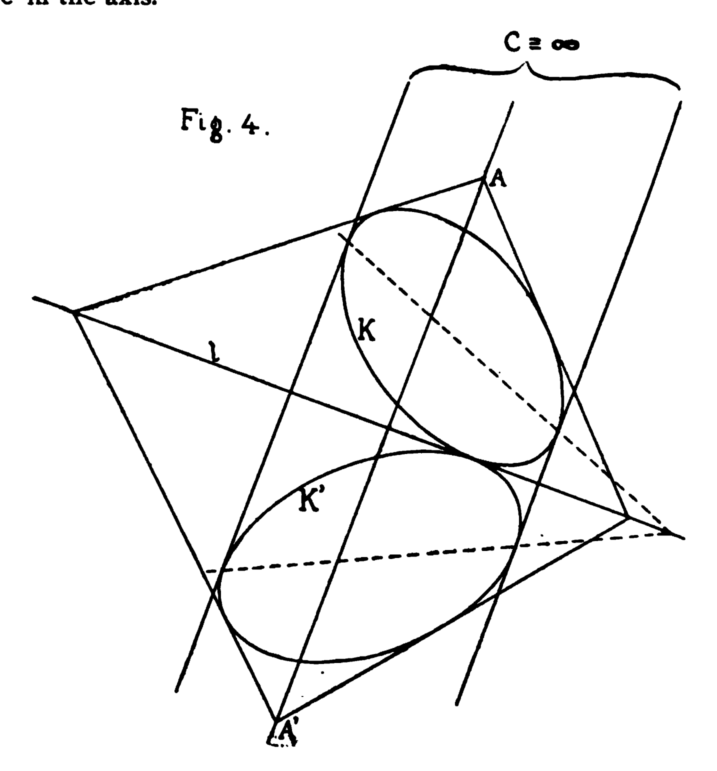
is made by means of those conics. For the arrangement we refer to the book of Fiedler already mentioned.

(*a*). The conics K and K^1 have two parallel common tangents, such that the centre C is at infinity (Fig. 4). There is

$$k=(\infty LAA^1)=(claa^1), \text{ or}$$

$$k=\frac{LA^1}{LA};$$

the corresponding point ranges are similar with the point of similitude in the axis.



For the counter-points there is

$$k=(\infty L \infty Q^1)=(\infty LR \infty),$$

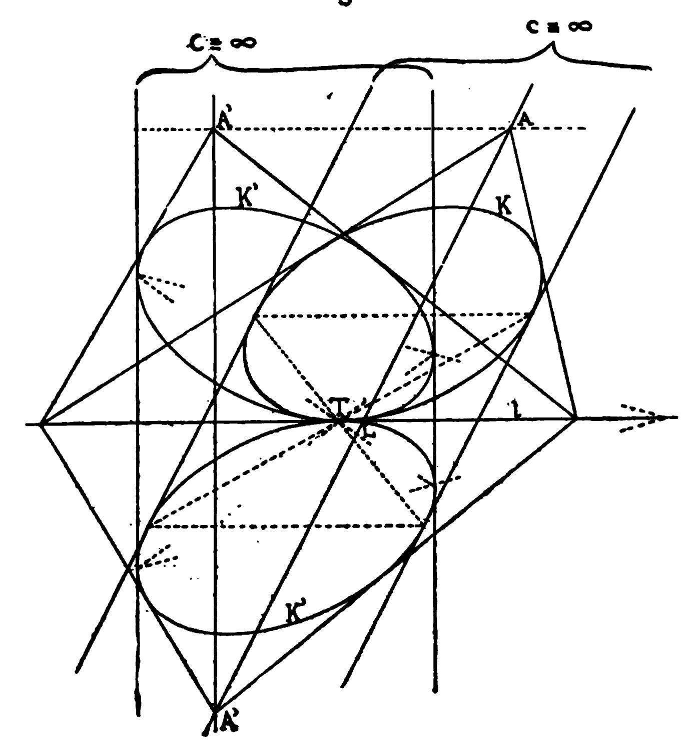
i. e., Q^1 and R are at infinity. The counter-axes q^1 and r form two coincident point-ranges of which the centre and the point at infinity of the axis are the double-points. Parallel straight lines have parallel corresponding lines. Such a collineation is called affinity, or dilation.*

*The word affinity is used in Mobius' "Barycentrischer Calcul."

(*b*). If K and K^1 have two parallel common tangents, and $k=-1$, i. e., if the collineation is dilation and involution (Fig. 5), there is:

$$(\infty LAA^1)=-1, \text{ or}$$
$$LA^1=-LA; \ (claa^1)=-1.$$

Fig. 5.



Each pair of corresponding points lies in a fixed direction and is equidistant from the axis. The ranges in corresponding straight lines are symmetric with the centre of symmetry in the axis. Corresponding triangles have the same area. This collineation is called oblique, or orthogonal symmetry in regard to the axis, according as the direction of the centre is oblique, or orthogonal to the axis.

(*c*). K and K^1 have two parallel common tangents and $k=+1$. In this case one of the conics of involution is revolved about the axis through $180°$, (Fig. 5). The centre is at infinity and its direction is parallel to the axis. In such a collineation corresponding figures have equal areas and it is therefore called the affinity of figures of equal areas.

It is also obtained by revolving one half-plane of oblique symmetry into the other. Corresponding points lie always on the same side of the axis and are, as in the previous case, equidistant from the axis.

(*d*). As the previous cases, (*a*), (*b*), (*c*), were characterized by the assumption of the centre C being at affinity, there remains to consider the collineation with an infinitely distant axis. Obviously the conics K and K^1 become coaxial parabolas which intersect each other either in two finite real, or two imaginary points (Fig. 6). There is

$$k=(C \infty AA^1)=CA : CA^1=(c \infty aa^1);$$

the distances of corresponding points from the centre have a constant ratio and form similar ranges.

Corresponding straight lines are parallel, and corresponding ranges similar, so that their constant ratio is k.

Fig. 6

The collineation thus characterized is termed similarity of systems in similar position. According as k is positive or negative, real or imaginary intersection-point of the parobola, the similarity is said to be direct or inverse similarity.

(*e*). If to the former case the further condition $k=-1$ is added, the relation becomes

$$k=(C \infty AA^1)=(c \infty aa^1)=-1, \text{ or}$$
$$CA=CA^1.$$

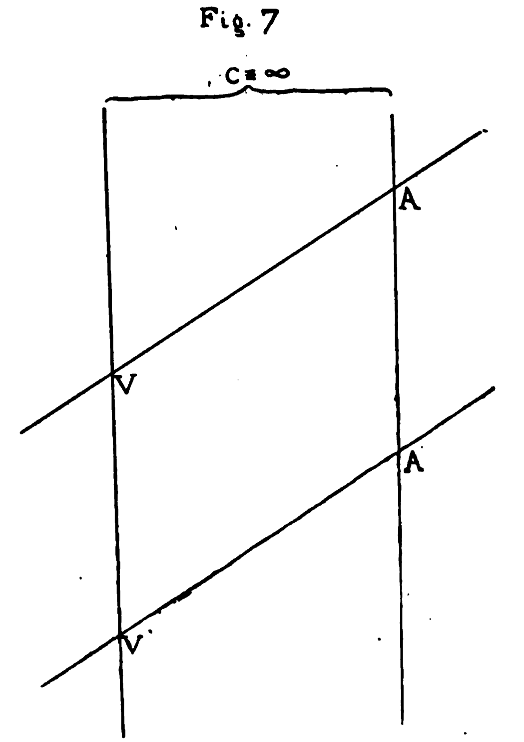
The conics K and K^1 have two imaginary intersection-points and are equal (K and K^1 in Fig. 6.). Corresponding points are in opposite directions and at equal distances from the centre.

Systems related in such a manner are said to be in central symmetry.

The value $k=+1$ together with an infinitely distant axis gives no collineation in the proper sense of the word. In this case the two conics K and K^1 coincide and determine what is called an identical collinear transformation. We shall meet this conception in one yet of the following chapters.

Assuming the conics K and K^1 as coaxial parabolas, and tangent at their vertices, two collineations arise according as the line at infinity or the finite common tangent is taken as the axis of collineation. In the first case the finite point of tangency of the parabolas is the centre of collineation. As is easily seen the relation of corresponding points becomes that of similar systems in similar position. In the second case the centre of collineation is the infinitely distant point of the common axis of the parabolas,

Fig. 7

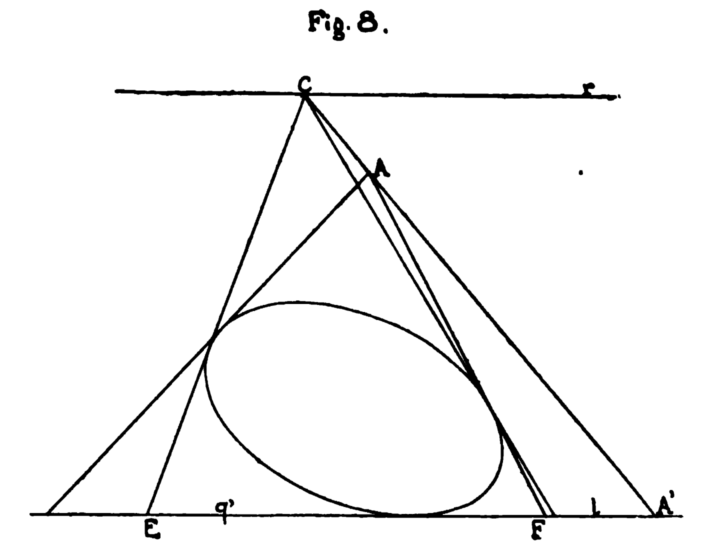


and its direction is orthogonal to the axis of collineation (the common finite tangent of the parabola). This, however, is orthogonal affinity. Adding the condition $k=-1$ the two cases represent central and orthogonal symmetry.

(*f*). The two conics K and K^1 may be represented by degenerate parabolas, i. e., by straight lines which coincide. Centre and axis of collineation are at infinity, and as they are coincident it follows from $k=(\infty\ \infty\ AA^1)$ $k=+1$. The two systems are therefore similar and in similar position; in the relation of dilation and of corresponding equal areas. Hence they are congruent. How the construction of corresponding points is made is seen from Fig. 7. Let $V\infty$ and $V^1\infty$ represent the two coaxial parabolas. To a point A the corresponding A^1 is found by drawing AV and $A\overset{\infty}{C}$ parallel to VV^1 (the two tangents to the conic VV^1) and intersecting AC by the parallel to AV through V^1. (AV and AC intersect the axis of collineation, or the line at infinity in two points. The tangents from this point to the conic $V^1\infty$ are A^1V^1 and A^1C). We add this construction here in order to show that also in the case of singularities the construction is applicable.

We have now seen that all the common cases of perspective collineation are expressed by the characteristic constant k together with the positions of the centre and axis of collineation, or result from certain positions and relations of the two conics. What remains yet to consider are the so-called *pseudo perspective collineations.**

Fig. 8.



Here to one point may correspond a whole system of points and vice-versa. These singularities can be classified according to

*Prof. Newson introduced the term *pseudo transformation* into geometry. Singular perspective collineations can therefore also be called *pseudo perspective collineations.*

$$k=0,\ k+\infty,\ k=\frac{0}{0}$$

(*g*). For $k=0$ take for K any conic tangent to l, and for K^1 a degenerated ellipse EF in l and tangent to K^1. Obviously q^1 lies in l and C in V (Fig. 8), hence $k=0$. As the construction shows, to each point A of the plane corresponds a point A^1 of the axis.

Conversely, to each point A^1, except the points of l, corresponds the centre C, i. e., the whole plane of the other system corresponds to the centre C. To each ray g through C corresponds its point of intersection with l.

(*h*). For $k=\infty$ results an analogous case in which r coincides with l and C with q^1.

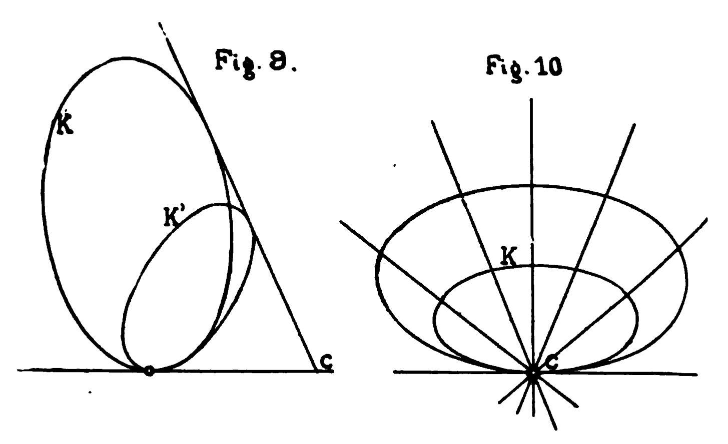
(*i*). For $k=\frac{0}{0}$, centre, axis, and counter-axis of collineation are coincident and the conics K and K^1 indeterminate. The points of each system correspond to the centre C. To each straight line in either system corresponds the axis l and to each ray through the centre in one system all the rays through the centre in the other system.

(*k*) *The last important case which is to be considered here is characterized by $k=+1$ and the centre C in the finite part of the axis l. The counter-axes q^1 and r are opposite and equidistant from l. These conditions are realized by two conics K and K^1 related by involution, but of which one conic is revolved about the axis of collineation into the half plane of the other. After having revolved one half-plane into the other, one or two common tangents of K and K^1 coincide with l. The center C becomes accordingly the intersection-point of the other common tangent with l, or the common point of tangency of K and K^1. In the first case the two conics have a contact of the second, in the other a contact of the third order. If no special assumption about the position of the centre and the conics is made the two conics will be in double contact, if, however, the connection line of the centre and the point of tangency of the conics in involution is perpendicular to the axis l the case of a triple contact arises. Figs. 9 and 10 illustrate these two cases.

What has been found here by logical deduction from the laws of collineation can also be proved by assuming one conic tangent to l, the center on l, the counter-axes q^1 and r opposite and equidistant from l and by constructing the corresponding conic. As the

*This is what Sophus Lie in his "Vorlesungen ueber continuierliche Gruppen" calls *elation*. In accordance with Lie we put this case at the end of the classification.

above cases are of great importance in the theory of groups, we state them again in a theorem:



*Theorem 4. Two conics with a contact of the second or third order determine a perspective collineation with the common tangent at the point of contact as its axis and a finite point in the axis as its centre.**

The centre is, of course, in both cases determined by the two conics and is found as indicated above. In fact the two cases are alike and thus it was not necessary to distinguish them in the above theorem.

§3. Number and Invariant Properties of Perspective Collineation.

From the preceding development it is known that each two conics tangent to each other determine a perspective collineation with the common tangent at their point of tangency as the axis and the intersection-point of their two other common tangents as the centre of collineation. As there are ∞^3 conics tangent to a straight line at a certain point, $\frac{1}{2}\infty^3(\infty^3-1)=\infty^6$ pairs of conics tangent to each other and to the line at that point can be formed, hence, just as many collineations. But there are discrete groups among these pairs which represent the same collineation. For, as a perspective collineation is also determined by the centre, the axis, and two corresponding points, it may be represented ∞^3 times by pairs out of those ∞^6. Hence, there are $\infty^6 : \infty^3=\infty^3$ pairs representing different perspective collineations. Taking all the points of the axis as points of tangency for pairs of conics determining a per-

*Our theorem involves the fifth type of Lie's table, page 66, "Continuierliche Gruppen."

spective collineation, we obtain $\infty^6 \cdot \infty^1 = \infty^7$ such pairs, or perspective collineations; but we have seen above, that among these are ∞^4 that represent the same perspective collineation. Hence, there are only $\infty^7 : \infty^4 = \infty^3$ different collineations left having a certain straight line of the plane for their axis. From this follows that to each pair of tangent-conics of a system confined to a certain point of the axis corresponds a pair of tangent-conics of a system confined to any other point of the axis. As to the other perspective collineation of the plane it is sufficient to say that there are $\infty^3 \times \infty^2 = \infty^5$ different perspective collineations in a plane; each straight line of the plane giving ∞^3 such collineations. But it is obvious that each of those ∞^2 systems has the same properties as any of the rest; each and any configuration of the one system can be made coincident with a certain configuration of any of the other systems. It is of no importance for our purpose to study up relations between two different systems in a general position. However, we shall have to consider two different systems with a common centre.

This relation occurs in the study concerning the invariant properties of perspective collineations. Here we have to consider the collineations in regard to those elements which by all collineations do not change their position, or even remain invariable in their intermediate parts and as a whole. Such elements are said to be invariant in the collineation. We found that there are ∞^3 perspective collineations belonging to a straight line as their axis. Hence the

Theorem 5. There are ∞^3 perspective collineations leaving the points of a straight line invariant.

Dualistically a point and the invariant rays through it, or the centre of collineation, can be taken as the invariant element. The ∞^2 straight lines of the plane and the characteristic anharmonic ratio combined with the centre give also ∞^3 different perspective collineations with the same centre, and we have therefore,

Theorem 6. There are ∞^3 perspective collineations leaving the rays through a point invariant.

This is the relation between two collineations with a common centre to which we drew attention while considering all the perspective collineations of the plane.

The next invariant element to be considered is the line-element,* or a straight line and a point on it.

*See Lie's definition of it in his "Vorlesungen," page 202.

To this line-element all the perspective collineations can be constructed which contain it as an invariant element. This can be done in two ways, first by taking the point as the centre and the line through it as an invariant ray of the perspective collineation, second by assuming the point as a point of the axis and the ray through it as a ray through the indeterminate centre of the collineation. By the same reasoning as before we find that in each case ∞^3 perspective collineations have a line-element in common. The two cases are in a dualistic relation and may be expressed in the

Theorem 7. There are ∞^3 perspective collineations leaving a line-element invariant.

Combining a line-element with either the points of a straight line or the rays through a point as the invariant figure, two other cases are obtained. If the points of a straight line and another straight line through one of these points are invariant, the centre of collineation may be any point on that other invariant line. This gives ∞^1 centres, and one on the same line as the axis of collineation. Adding to each centre two corresponding points, or what is the same, a characteristic anharmonic ratio, the number of perspective collineations is multiplied by ∞^1, so that ∞^2 perspective collineations satisfy the given conditions. Hence the

Theorem 8. There are ∞^2 perspective collineations leaving the points of a straight line and another straight line invariant.

In the dualistic case, where the rays through a point and another point on a ray through the first point are invariant, the axis of collineation may be any straight line through that other invariant point. This gives ∞^1 axes and one and the same point as the centre of collineation.

Adding to each axis two corresponding lines, or, what is the same, a characteristic anharmonic ratio, the number of collineations is multiplied by ∞^1, so that ∞^2 collineations satisfy the given conditions. Hence the

Theorem 9. There are ∞^2 perspective collineations leaving the rays through a point and another point on one of the rays through the first point invariant.

Finally there exists a system of collineations in which all points of a straight line and all rays through a point are invariant. Here, a special collineation is simply characterized by two corresponding points, or the characteristic anharmonic ratio. There are just ∞^1 such collineations. This result may be stated in the

Theorem 10. There are ∞^1 perspective collineations leaving the points of a straight line and the rays through a point invariant.

If the straight line is a priori supposed to be the axis, the statement: there are perspective collineations leaving the points of a straight line invariant is not entirely logical. Making this assumption it is self-evident that the points of the axis are invariant. The same may be said in regard to the centre of perspective collineations. The reason for putting the above theorem into this form is to have conformity with the statements in the cases of general collineation. See for this remark Lie's "Vorlesungen," table of groups, pages 288-291.

After having found the number and invariant properties of the general perspective collineation the special perspective collineations as classified in the previous chapter can be subjected to the same investigation. As before, only one system out of the ∞^2 of the plane shall be taken into consideration. Following the division given in the classification, §2, we have first the involution. It is determined by the centre and axis, since $k=-1$ and, hence, there are just as many involutions belonging to a straight line as an axis, as there are points (centres) in the plane, i. e., ∞^2. Hence there are ∞^2 involutions leaving the points of a straight line invariant and dualistically ∞^2 involutions leaving the rays through a point invariant. The same numerical result is found in regard to a line-element. But there is only one involution leaving the points of a straight line and the rays through a point invariant. What has been said about the involution holds in general for a whole system of collineations with a constant characteristic anharmonic ratio.

Dilation is characterized by an infinitely distant centre and a characteristic k varying from $-\infty$ to $+\infty$, and includes oblique and orthogonal symmetry ($k=-1$). The centre may be one of the ∞^1 points of the line at infinity, and since k can assume ∞^1 values, there are ∞^2 dilations which leave the points of a straight line invariant. The dualistic interpretation of dilations leaving a point at infinity invariant, respectively a pencil of parallel rays, does not lead to the same numerical result.

A certain direction, i. e., the centre at infinity, being given, the ∞^2 straight lines and ∞^1 values of k may be combined to form dilations. Hence, there are ∞^3 dilations leaving a point at infinity invariant. Taking a point of the axis and a ray through it as a line-element, it follows that there are ∞^2 dilations leaving a line-element invariant.

By the same reasoning as in the general case we find that there are ∞^1 dilations leaving the points of a straight line and another

straight line invariant. There are, however, ∞^2 dilations in which the rays through the centre and a finite point in one of these rays are invariant.

If k=—1 dilation becomes oblique and orthogonal symmetry. Obviously there are ∞^1 such symmetries leaving the points of a straight line invariant. On the other hand there are ∞^2 (straight lines of the plane) symmetries leaving one and the same point at infinity invariant. There is only one symmetry belonging to a line-element.

Revolving one system of axial symmetry through 180°, k becomes +1, and the centres move to infinity in the direction of the axis. The relation is that of figures of equal areas, and the number of such collineations is ∞^1. For, taking a line parallel to the axis, or, connecting two corresponding points A and A^1, one and the same collineation is determined by any two points A and A^1 which include the same length $\overline{AA^1}$. This gives ∞^1 different collineations leaving the points of a straight line invariant.

In the case of similarity the axis is at infinity, the centre finite, and k ranges from $-\infty$ to $+\infty$, thus including central symmetry. The ∞^2 positions of the centre together with ∞^1 values of k give ∞^3 similarities leaving the points of the line at infinity invariant.

The centre of similarity may lie on a finite straight line and since each finite point of the plane represents ∞^1 similarities, there are ∞^2 similarities leaving the points of the line at infinity and another straight line invariant (line-element with its point at infinity). Taking the point of the line-element in the finite part of the plane, there are just ∞^1 similarities leaving this line-element, or also a point invariant.

All similarities with k=—1 are, as it is known, central symmetries. As k=—1 their number is ∞^2.

Assuming the centre of similarity at infinity, k becomes +1 and the collineations are congruences. A congruence is determined by the direction of its centre and two corresponding points which subtend the same length. As there are ∞^1 directions and ∞^1 sects in the plane, the number of congruence in the plane is ∞^2. All of them leave the line at infinity invariant. The number of perspective collineations belonging to a certain direction, or leaving the centre at affinity invariant, is ∞^1.

The pseudo-collineations which are characterized by k=0, ∞, $\frac{0}{0}$, have as their numbers ∞^2, ∞^2, ∞^1, respectively. In all three cases the axis is the invariant element.

In the case of the relation of K and K^1 being in a contact of the second or third order, each point of the axis as a centre gives ∞^1 perspective collineations (determined by the centre and the ∞^1 corresponding points subtending different sects). Hence, the points of a straight line are invariant for ∞^2 such collineations.

Dualistically, the rays through a point are invariant for ∞^2 such collineations. In other words there are ∞^2 of those collineations leaving a line-element invariant and again, there are ∞^1 such collineations leaving the points of a straight and the rays through a point on this line invariant.

We have now investigated all the properties relating to number and invariants of the general and special perspective collineations which are essential from the standpoint of the theory of groups.

In the next chapter we will consider some of the infinitesimal properties of perspective collineations.

§4. Identical and Infinitesimal Transformation and W-Curves of Perspective Collineation.

The axis and centre of perspective collineation being given, ∞^1 perspective collineations can be determined which leave these elements invariant. Each of these collineations is determined by the characteristic anharmonic ratio, or a pair of conics K and K^1 touching the axis at the same point and having for the other two common tangents two rays through the centre. As is known from §3 there are ∞^1 such pairs or different collineations. If especially the two conics K and K^1 coincide, the collineation is an identical one. All the conics tangent to the axis at a certain point and tangent to two fixed rays through the centre may be taken for the representatives of the identical collineation. An infinitesimal collineation differs from an identical collineation by an infinitesimal amount, i. e., a point and its corresponding one have an infinitesimal distance and two corresponding lines include an infinitesimal angle. Thus, in order to obtain the infinitesimal from the identical collineation we have to choose for K^1 the conic infinitely close to K in the same system. This conic shall be designated by δK. According to this proposition a point A is transformed into $A+\delta A$, and a line a into $a+\delta a$.

A and $A+\delta A$ lie on a ray through the centre, and a $+\delta a$ intersect each other in a point of the axis. Applying a whole system of infinitesimal perspective collineations to each other and in a certain succession, an integral, or finite, perspective collineation is obtained. In this operation the corresponding point A^1 to A starts from A

having the direction to the centres and describes a certain curve which passes through the centre. Any point A^1, or $\int A+\delta A$, as corresponding to A in regard to the conics K and $\int \delta K$, lies with A on a ray through the centre. The curve which A^1 describes is therefore a straight line through the centre and is invariant for all perspective collineations of the system.

Since any conic tangent to the axis may serve to determine a perspective collineation with a given centre, axis, and characteristic k, it is obvious that there is only one infinitesimal perspective collineation belonging to a centre and an axis, or leaving these two elements invariant. By integration all the ∞^1 finite perspective collineations may be obtained which leave the points of a straight line and the rays through a point invariant.

From the fact that the centre and axis determine an infinitesimal perspective collineation, it follows that there are ∞^2 infinitesimal perspective collineations either leaving the points of a straight line, or the rays through a point invariant. The integration gives in both cases the ∞^3 finite perspective collineations leaving the same elements invariant. Moreover, it follows that the plane has ∞^4 infinitesimal perspective collineations which by integration give the ∞^5 finite perspective collineations of the plane.

It is not necessary to enter into a study of the infinitesimal perspective collineation of the special cases, because the result is essentially the same. It is sufficient to mention that the collineation having the centre in its axis and the characteristic $k=+1$ has simply ∞^1 infinitesimal collineations leaving the points of the axis invariant. By integration the ∞^2 finite collineations of this kind are obtained.

In this last case as well as in the general case the whole of W-curves* consists of the pencil of rays through the centre. This pencil becomes a pencil of parallel rays if the centre is at infinity, as it occurs in some of the special cases of perspective collineation. We have here found the same result as Lie in his "Vorlesungen" on pages 69 and 70.

§5. Groups of Perspective Collineations.

Suppose a system of perspective collineations which is restricted by certain conditions, for instance, such as to leave given elements or combinations of elements invariant, or to be characterized by a

*"W-Curven," or "selbstprojective Curven" in Lie's Vorlesungen, page 68.

special value of the characteristic anharmonic ratio. By any of the perspective collineations belonging to such a system a point A is transformed into A^1. Taking another collineation of the same system and A^1 as an original point in it, the corresponding point will be A''. Whenever now A and A'' are related in such a manner as to be a collineation of the given system, i. e., subject to the same conditions, and, inversely, if each point A^1 can be transformed back into its corresponding A by a collineation of the system, such a system of collineations is said to be a continuous group of collineations, or simply a group of collineations. By this statement it is easy to enumerate the groups which may occur in the general and special cases of perspective collineations. We may, however, occasionally avail ourselves of a theorem of Lie concerning a criterion of groups by means of invariant properties of transformations. The theorem is:

"The system of all projective transformations of the plane leaving a certain figure invariant has the property of a group. The transformations of the system are inverse by pairs."*

In enumerating the groups we follow the order of chapters 2 and 3. Thus, we have first to consider the general perspective collineations. In Fig. 11 we assume l as the axis common to ∞^3 perspective collineations of the plane and two collineations of this system determined by ($CLAA^1$) and ($C_1L_1A'A''$), or ($claa^1$) and ($c_1la'a''$) respectively, where C and C_1 are the centres of the two perspective collineations. The transformed point to A in the first collineation is A' and the transformed point to A' in the second collineation A''. Taking an other point B on a, the points B' and B'' can be constructed, or in general, to each point on a there are coresponding ones on a' and a''.

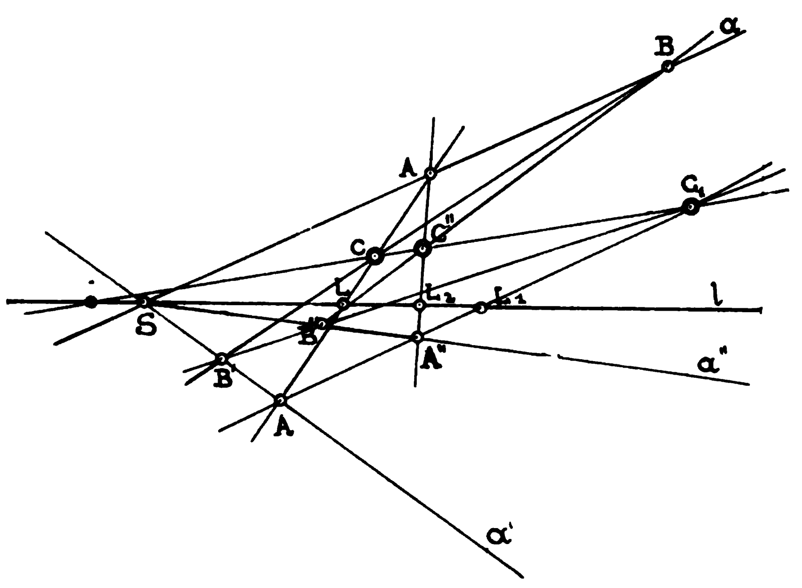
Now the point-range AB....is perspective to the point-range A'B'....with C as the centre and, again, A'B'....perspective to the point-range A''B''....with C_1 as centre. The three ranges have a self-corresponding point, S, on l. Hence, the point-ranges AB....and A''B''....are also perspective, i. e., AA'', BB''.... pass through one and the same point C'' which obviously may be taken as a new centre of collineation with l as an axis, and A, A''; B, B'';....as coresponding points. As is immediately seen, to each transformation in a perspective collineation may be found its inverse belonging to the same system. We have therefore the

Theorem 11. The system of all perspective collineations leaving the points of a straight line invariant forms a three-termed group.

*Lie's "Vorlesungen," page 113.

A collineation resulting from two other collineations is related to these in such a manner that the three centres lie in a straight line. For, considering the triangles ABC and $A''B''C_1$, it is seen that

Fig. 11.



their corresponding sides intersect each other in three points S, B', A', of a', or that the triangles are homologous. Hence $\overline{AA}''$, $\overline{BB}''$, $\overline{CC}_1$, intersect each other in a point, in the centre C'' of the third collineation. Hence C, C_1, C'', lie in a straight line.

This fact leads us at once to a sub-group of perspective collineation. By choosing the centres of perspective collineation constantly on a straight line the centre of a collineation resulting from two of these collineations is a point of the same line. To each perspective collineation may also be found its inverse belonging to the same system. From §3 is known that there are ∞^2 such collineations and we have therefore:

Theorem 12. The system of perspective collineations leaving the points of a straight line invariant forms a two-termed group.

Without needing a direct proof the following dualistic statements of the two preceding theorems can be made:

Theorem 13. The system of all perspective collineations having the rays through a point invariant forms a three-termed group.

And

Theorem 14. The system of all perspective collineations leaving the rays through a point and another point in one of these rays invariant forms a two-termed group.

For a proof of these theorems we refer either to Lie's theorem at the beginning of this chapter, or to the first proof of theorem 11. In the dualistic case the reasoning is exactly the same.

There is another three-termed group of perspective collineations, which is obtained by a special interpretation of the groups which leave the points of a straight line or the rays through a point invariant. The line-element, the point in which is taken as the centre, as an invariant figure, is equivalent with the rays through the centre. The system of collineations is therefore in both cases the same. On the other hand the point of the line-element may be a point of the axis. The invariant configuration is therefore that of the points of a straight line and another straight line. But the axis may be any of the rays passing through the point of the line-element, so that the number of perspective collineations is ∞^3 as before. Thus, the

Theorem 15. All the perspective collineations leaving a line-element invariant form a three-termed group.

The next and last sub-group of perspective collineation concerns the points of a straight line and the rays through a point as the invariant configuration. As usual let the point (centre) be C and the line (axis) l and two collineations represented by (CLAA') and $(CL_1A_1A_1')$. Applying the second collineation to A', the corresponding point A'' of A' is obtained which lies upon the same ray through C, as A and A'. The new collineation is therefore represented by (CLAA''), i. e., by a collineation of the same system. Since each of these collineations and its inverse belong to the system the following statement may be made:

Theorem 16. All the perspective collineations leaving the points of a straight line and the rays through a point invariant form a one-termed group.

The groups of perspective collineation are also easily obtained by the configurations in space.

Assume the two planes π and π' intersecting each other in l, and a centre (C) without these planes as a perspective collineation in space. Drawing the bisecting plane π_1 of π and π' and a perpendicular from (C) to the bisecting plane, intersecting π and π' in C and C', respectively, these two points will coincide when one of the planes π, or π' is revolved into the other about l as an axis.

From this we see that the line l is invariant for every perspective collineation having any point of space as its centre. The number of such collineations is ∞^3. A point C and a line l are invariant for ∞^1 such collineations, for there are ∞^1 centres, (C), on the perpendicular to the plane π_1 in C, and ∞^1 planes through the perpendicular. A plane through such a perpendicular intersects π and π' in two lines, which after revolving one plane into the other become an invariant line. As there are ∞^2 points (centres) in the plane perpendicular to π_1, the points of l and another straight line will be left invariant by ∞^2 perspective collineations.

In this manner we can successively deduce all the groups from intuition in space. We shall not carry on the enumeration of the other groups by this method; it is sufficient to have shown the possibility of this method, which in fact is identical with the other.

In the general case we had considered the ∞^3 perspective collineations leaving the points of a straight line, and, dualistically, the rays through a point invariant. Each collineation of the system is characterized by a certain anharmonic ratio. Two perspective collineations with the characteristics k and k_1 applied one to the other determine a new perspective collineation of the same group, whose characteristic may be designated by k. Now it is known that k'' is an algebraic relation between k and k_1, say:

$$k''=f(k, k_1).$$

Suppose now that two perspective collineations of the same characteristic applied one to the other produce a perspective collineation with the same characteristic, such that

$$k''=f(k'', k'').$$

If this relation shall hold for all values of k'', it must be an identical one; i. e., k''=k'', as it occurs in the identical perspective collineation. From this we conclude that in general a system of perspective collineations with a constant characteristic does not form a group. From the above relation we can find the special values for which

$$k''=f(k'', k'')$$

by resolving the equation

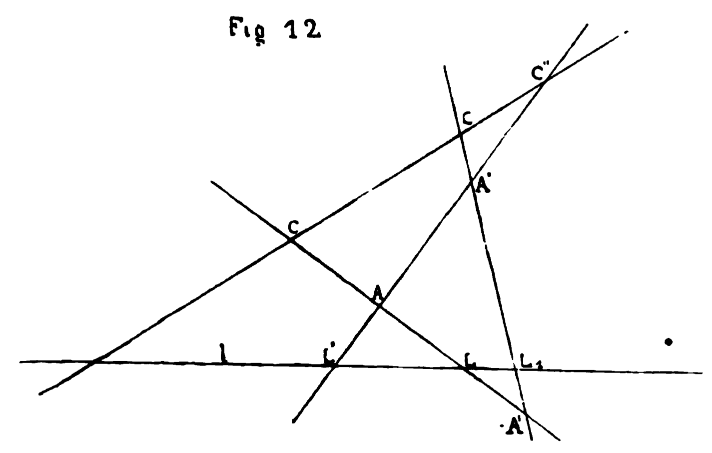
$$k''-f(k'', k'')=o.$$

It is therefore necessary to know the form of $f(k, k_1)$.

For this purpose we consider the three perspective collineations (CLAA'), (C_1L_1A'A''), (C''L''AA''), of which the last results from the two others as described before.

The sides of the triangle AA'A'' are intersected by two transversals l and the line joining the centres C, C_1, C''. Thus,

according to the theorem of Menelaos we have the relations:



(1). $LA \cdot L_1A' \cdot L''A'' = LA' \cdot L_1A'' \cdot L''A$, or

(2). $\frac{LA}{LA'} \cdot \frac{L_1A'}{L_1A''} = \frac{L''A}{L''A''}$, and in the same way:

(3). $CA \cdot C_1A' \cdot C''A'' = CA' \cdot C_1A'' \cdot C''A$, or

(4). $\frac{CA}{CA'} \cdot \frac{C_1A'}{C_1A''} = \frac{C''A}{C''A''}$

Dividing (4) by (2) we have

$$\frac{\frac{CA}{CA'} \cdot \frac{C_1A'}{C_1A''}}{\frac{LA}{LA'} \cdot \frac{L_1A'}{L_1A''}} = \cdot \frac{\frac{C''A}{C''A''}}{\frac{L''A}{L''A''}}, \text{ or}$$

$$(CLAA')\,(C_1L_1A'A'') = (C''L''AA'')$$

Designating the characteristics respectively by k, k_1, k'', we find the required fundamental relation in the form:

$$k'' = k.\ k_1$$

If these three characteristics are equal, say equal to k, the relation becomes

$$k = k^2, \text{ or}$$

$$k^2 - k = 0\ ;$$

Whence either $k=0$, or $k=+1$. Excluding the singular case $k=0$ we can therefore say, that only those perspective collineations with a constant characteristic are liable to form a group, for which $k=+1$. If a perspective collineation $(CLAA')=k$ is given we find its inverse $(CLA'A)=\frac{1}{k}$, which is a number of the same kind. Thus, to each perspective collineation we can

find its inverse. In the construction this fact is self-evident. If the characteristics of two perspective collineations are of the same sign the sign of the resulting third perspective collineation is always positive; if they are of a different sign, the sign of the third is always negative. From this we conclude, as we know already, that the system of involutions does not form a group:

$$-1 \times -1 = +1.$$

Among the general cases of perspective collineations we have to study the system of dilations in the first place. The centres are all at infinity. Hence we have only the relation

$$\frac{LA}{LA'} \cdot \frac{L_1A'}{L_1A''} = \frac{L''A}{L''A''}, \text{ while}$$

$$\frac{CA}{CA'} \cdot \frac{C_1A'}{C_1A''} = \frac{C''A}{C''A''} = 1.$$

The characteristic k'' of a dilation resulting from two other dilations of the same system is therefore expressed as before

$$k'' = k.\ k_1.$$

To each dilation can also be found its inverse, so that the respective characteristics are k and $\frac{1}{k}$. To sum up we can say:

Theorem 17. The system of dilations leaving the points of a straight line invariant forms a two-termed group.

If the centre at infinity and the axis are kept fixed the dilations differ according to their characteristics. A, A'; A', A'' being two pairs of corresponding points on a ray through the centre at infinity, A, A'', will be the corresponding pair of the resulting dilation, and there is evidently

$$\frac{LA}{LA'} \cdot \frac{LA'}{LA''} = \frac{LA}{LA''}, \text{ or}$$

$$k \cdot k_1 = k''; \text{ i. e.,}$$

the third dilation belongs to the same system. As there are ∞^1 such dilations we have

Theorem 18. The system of dilations leaving the points of a straight line and a point at infinity (centre of dilation) invariant forms a one-termed group.

The same is true for the system of dilations leaving the points of a straight line and another straight line invariant.

The dualistic interpretation, however, does not lead to the same result. From the general case it is known that all the collineations leaving the rays through a point invariant form a three-termed group. If this point is at infinity the collineations are dilations. Hence the

Theorem 19. The system of dilations leaving the rays of a parallel pencil of rays invariant forms a three-termed group.

Theorem 20. The system of dilations leaving the centre of dilation and another finite point invariant forms a two-termed group.

The one-termed dualistic group of dilation is the same as the original one; this group is self-dualistic.

In the oblique and orthogonal symmetry, which is dilation with $k=-1$, there is no group, for two symmetries applied one to the other give a collineation with the characteristic $k=+1$, $(-1\times -1=+1)$.

In the case of dilation with $k=+1$ the centre is at infinity in the direction of the axis. As is known from §3, this is the relation of corresponding equal areas. Two points, A, A', on a ray parallel to the axis determine the relation

$$k=\frac{\overline{\infty A}}{\overline{\infty A'}}=+1.$$

Taking A', A'' as a corresponding pair in another collineation of this kind,

$$k_1=\frac{\overline{\infty A'}}{\overline{\infty A''}}=+1, \text{ we obtain}$$

$$\frac{\overline{\infty A}}{\overline{\infty A'}}\cdot\frac{\overline{\infty A'}}{\overline{\infty A''}}=\frac{\overline{\infty A}}{\overline{\infty A''}}=+1, \text{ or}$$

$$(+1)\times(+1)=+1.$$

Thus the

Theorem 21. The system of perspective collineations characterized by corresponding equal areas and leaving the points of a straight line invariant forms a one-termed group.

In discussing the relation $k''=k.\ k$, it was pointed out that there are groups with the characteristic $k=+1$. This assertion is now proved; but we yet shall find other groups with the characteric $k=+1$.

Instead of the line joining the centres we can suppose the line l of a system of perspective collineations to be at infinity. In this case we have similarity which is expressed by

$$\frac{CA}{CA'}\cdot\frac{C_1A'}{C_1A''}=\frac{CA}{CA''}, \text{ or again}$$
$$k''=k.\ k_1.$$

Just as in the case of dilations, to each similarity can be found its inverse belonging to the same system. There are ∞^3 such similarities, therefore

Theorem 22. The system of similarities leaving the points of the line at infinity invariant forms a three-termed group.

If the centres C, C_1, C'', are confined to a straight line the theorem follows:

Theorem 23. The system of similarities leaving the points of the line at infinity and on another straight line invariant forms a two-termed group.

If the axis at infinity and the centre are kept fixed, the similarities differ according to their characteristics. A, A'; A', A'' being two pairs of corresponding points on a ray through the centre, A, A'' will be the corresponding pair of the resulting similarity, and there is evidently:

$$\frac{CA}{CA'} \cdot \frac{CA'}{CA''} = \frac{CA}{CA''}, \text{ or}$$

$$k \cdot k_1 = k'', \text{ i. e.,}$$

the third similarity belongs to the same system. As there are ∞^1 such similarities we have

Theorem 24. The system of similarities leaving the line at infinity (axis of similarity) and the rays through a point invariant forms a one-termed group.

The same is true for the system of similarities leaving the rays through a point and another point (on the line at infinity) invariant.

In central symmetry to the similarity is added the condition $k = -1$. Central symmetry as well as oblique and orthogonal symmetry does not form a group.

If the centre of similarity is at infinity we have congruence in which $k = +1$. Taking A, A' and A', A'' as corresponding pairs in two congruences, A, A'' will be a corresponding pair in a third perspective collineation, resulting from the first two. There is

$$\frac{\overline{\infty A}}{\overline{\infty A'}} \cdot \frac{\overline{\infty A}}{\overline{\infty A''}} = \frac{\overline{\infty A}}{\overline{\infty A''}} = +1$$

As there are ∞^2 such collineations we have the

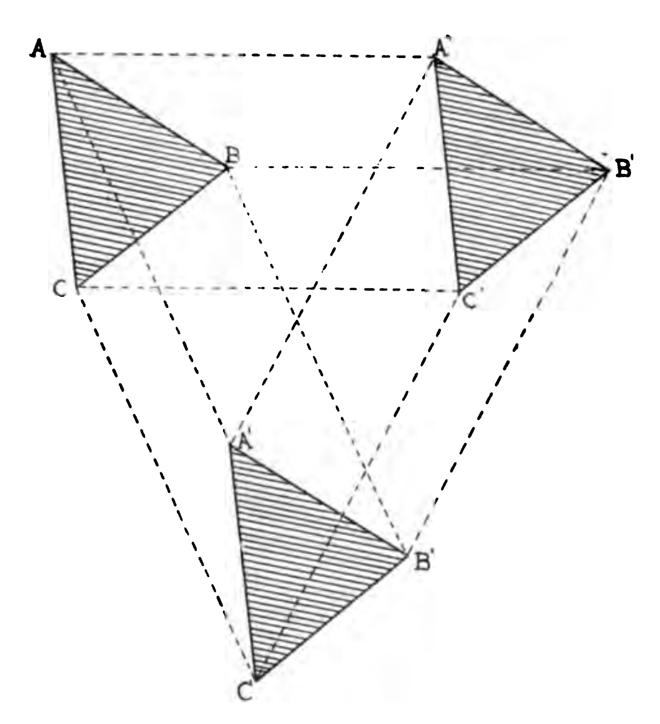
Theorem 25. The system of congruences leaving the points of the line at infinity invariant forms a two-termed group.

Supposing A, A', A'' in a straight line, the above relation still holds. The system of congruences has in this case a fixed centre at infinity and consists of ∞^1 congruences. Hence

Theorem 26. The system of congruences leaving the points of the line at infinity and the rays of a pencil of parallel rays invariant forms a one-termed group.

On account of the conspicuity in which the character of a group appears in the groups of congruences, we give an illustration in Fig. 13.

Fig. 13.



The pseudo-perspective collineations which have the characteristic $k=o, =\infty, =\frac{o}{o}$ may conveniently be represented in space. The cases arise when the centre of collineation is in either one of the planes π and π', If the centre (C) is in π then (CLAA')= (CLR∞)=(CLC∞)=o. Every point of π' is projected into C. Taking C as a point of π' (by revolving the plane π about the axis l into π') and performing the projection for another centre, say C_1 in π, the point C is projected into C_1. But the points in π' and C may be taken as a new pseudo-collineation of the system. The same can be said if the centre (C') is in π', or if (C'LAA')= (C'L∞C')=$\pm\infty$. The inverse of a collineation for both cases $k=o$ and $k=\infty$ is indefinite, but as the other conditions for a group are satisfied, we may say that the pseudo-perspective collineations with either the characteristic o or ∞ form a two-termed group.

In the pseudo-perspective collineation with the centre in the axis and k indefinite there is no group at all.

With this we close the study of groups of perspective collineations, perspective collineations taken in its common meaning, and proceed to that particular kind of perspective collineations in which centre and axis coincide and $k=+1$. This kind of collineations might be considered as the supplement of involution. For, the involution being obtained by revolving one plane, say π, into the other π' through an angle of α^0, the collineation in question is obtained by revolving π into π' through $180-\alpha$. According to Lie (page 202 of his "Vorlesungen") we may call this collineation an elation.

But here the word does not mean entirely the same as in Lie's definition. We use it here only as a convenient word to express those collineations.

In Fig. 12 C and L, C_1 and C_2; C'' and L'' coincide, so that the characteristic of an elation is $+1$,

The fundamental relation is

$$(+1)\times(+1)=+1$$

and since there are ∞^2 elations restricted to a line we have

Theorem 27. The system of elations leaving the points of a straight line invariant forms a two-termed group.

and dualistically:

Theorem 28. The system of elations leaving the rays through a point invariant forms a two-termed group.

Taking A, A', A'' on the same ray throngh the centre, there is

$$\frac{CA}{CA'}\cdot\frac{CA'}{CA''}=\frac{CA}{CA''},$$

and the same is true for LA, LA', and so on, since L coincides with C.

Therefore the relation as above:

$$(+1)\times(+1)=+1.$$

Theorem 29. The system of elations leaving the points of a straight line and the rays through a point invariant forms a one-termed group.

The centre of elation can also be considered as the point of a line-element. From this point of view we have another theorem.

Theorem 30. The system of elations leaving a line-element invariant forms a two-termed group.

Summing up, the following groups of perspective collineations are possible. (The Roman numerals denote the types of transformations as given in Lie's "Vorlesungen über Continuierliche

Gruppen," page 510 to 512, to which the groups belong, and the Arabic numerals in brackets denote the numerals of the groups in Lie's table of projective groups of the plane (pages 288 to 291).

A. THREE-TERMED.

(1). Invariant line-element, III, (14).
(2). Invariant points of a straight line, III, (21).
(3). Invariant rays of a pencil, III, (22).

B. TWO-TERMED.

(4). Invariant line-element, V, (24).
(5). Invariant points of a straight line and another invariant straight line, III, (32).
(6). Invariant rays of a pencil and another invariant point, III, (33).
(7 and 8). See two-termed groups of type V.

C. ONE-TERMED.

(9). Invariant points of a straight line and invariant rays of a pencil being not on the straight line, III, (38).
(10). Invariant points of a straight line and invariant rays of a pencil on the straight line, V, (39).

As groups of the special cases of collineations we have within the type III, the following:

A. DILATION.

(*a*). THREE-TERMED.—Invariant rays of a pencil of parallel rays.
(*b*). TWO-TERMED.—(1). Invariant rays of a pencil of parallel rays and another invariant point.
(2). Invariant point of a straight line.
(*c*). ONE-TERMED.—Invariant points of a straight line and invariant rays of a pencil of parallel rays.

B. CORRESPONDING EQUAL AREAS.

(*a*). ONE-TERMED.—Invariant points of a straight line.

C. SIMILARITY.

(*a*). THREE-TERMED.—Invariant points of the line at infinity.
(*b*). TWO-TERMED.—Invariant points of the line at infinity and another invariant straight line.
(*c*). ONE-TERMED.—Invariant points of the line at infinity and invariant rays of a pencil.

D. CONGRUENCE.

(*a*). TWO-TERMED.—Invariant points of the line at infinity.

(*b*). ONE-TERMED.—Invariant points of the line at infinity and invariant rays of a pencil of parallel rays.

E. PSEUDO-PERSPECTIVE COLLINEATIONS.

(1). Two-termed pseudo-group.

(2). One-termed pseudo-group.

In type V we have elations and under these the following groups:

A. TWO-TERMED.

(1). Invariant line-element, (24).

(2). Invariant points of a straight line, (29).

(3). Invariant points of a pencil, (30).

B. ONE-TERMED.

Invariant points of a straight line and invariant rays of a pencil on the straight line.

Finally we will add a formal representation of groups of perspective collineations in the plane.

The whole plane contains ∞^5 perspective collineations (combinations of the ∞^2 straight line and the ∞^2 points of the plane $= \infty^4$, and ∞^1 values of the characteristic anharmonic ratio. Designating a general perspective collineation by the index c, an n-termed group by G_n and its dualistic by Γ_n, a self-dualistic group by H_n, dilation by the upper index d, corresponding equal areas by a, similarity by s, congruence or translation by t, elation by e, and the dualistic interpretation of the group $G_2{}^d$ by Γ_2^δ, the following symbolic equations between the different groups and sub-groups of collineations exist:

$$\infty^5 C = \infty^2 G_3^c + \infty^2 \Gamma_3^c + \infty^3 H_3^{c}{}^* + 1 G_3^s + \infty^1 \Gamma_3^d + 1 H_2^t$$

$$G_3^c = \infty^1 G_2^c + 1 G_2^d + 1 G_2^e + 1 G_1^a$$

$$\Gamma_3^c = \infty^1 \Gamma_2^c + \infty^1 \Gamma_2^\delta + 1 \Gamma_2^e$$

$$H_3^c = \infty^1 G_2^c + \infty^1 \Gamma_2^c + 1 H_2^e$$

$$G_2^c = \infty^1 H_1^c + 1 G_1^d + 1 H_1^e$$

$$\Gamma_2^c = \infty^1 H_1^c + 1 H_1^e$$

$$G_3^s = \infty^1 G_2^s + 1 H_2^t$$

$$G_2^s = \infty^1 G_1^s + 1 H_1^t$$

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* These ∞^3 groups are all equivalent. especially ∞^1 groups can always be made identical by a simple translation.

$$\Gamma_3^d = \infty^1 \Gamma_2^d + 1 H_1^t + 1 \Gamma_2^\delta$$
$$\Gamma_2^d = \infty^1 \Gamma_1^d + 1 H_1^t$$
$$\Gamma_2^\delta = \infty^1 H_1^c + 1 \Gamma_1^s + 1 H_1^e$$

$$H_2^t = \infty^1 H_1^t$$
$$H_2^e = \infty^1 H_1^e$$

§6. Historical Sketch.

To show what position the subject of this dissertation occupies in geometry it will be necessary to give first a brief account of the development of geometrical methods which gradually lead to the modern standpoint. Projective or synthetic geometry is essentially a product of the 19th century, though it is well known that Pappus and Menelaus found some very important theorems concerning projective properties many hundred years ago and that Desargues discovered the fundamental theorems of perspective and involution in the 18th century.

The origin of projective geometry must be sought in the methods of descriptive geometry, which, by the achievements of Lambert and Monge, became at once very valuable in geometrical investigation. The first classic work on projective geometry was Poncelet's "Traité des propriétés projectives des figures," which appeared in 1822. In this great treatise the properties of figures are investigated which are unaltered by projection, or which are invariant. Poncelet introduced the so-called central-projection with a perspective-centre and a perspective-axis into the consideration of plane figures. While in France the "new geometry" was chiefly promoted by Gergonne and Chasles, in Germany its fruitfulness was shown to the scientific world by the three great investigators, Möbius, Plücker, and Steiner. The classical works of this period are:

Möbius, Barycentrischer Calcul, 1827.

Plücker, Analytisch-geometrische Untersuchungen, 1828.

Steiner, Systematische Entwickelung der Abhängigkeit geometrischer Gestalten von einander, 1832.

Chasles, Aperçu historique sur l' origine et le développement des méthodes en géométrie, 1838.

From these times also dates the separation of the mathematicians into two schools. One of them, the synthetical school, was represented by Steiner, Möbius, v. Staudt, Schröter, and has as its present principal leaders: Durège, Reye, Sturm, and Fiedler.

The other, the analytical school, has as its representatives: Plücker, Hesse, Aronhold, Gordan, Cayley, and many others.

Meanwhile the brilliant results of modern synthetic and analytic geometry have had a great influence upon pure analysis. The modern theory of functions was created, and by the investigations of Jacobi, Abel, Cauchy, Riemann, Hermite, and Weierstrass, it has reached a dominant position in almost all branches of mathematics. It became more and more a prevailing opinion that in fact the synthetic and analytic methods in geometry are identical and it is now generally acknowledged that the two methods differ only in their formal representation. Fiedler in 1874 defined the homogeneous co-ordinates as anharmonic ratios which lead at once from synthetic to analytic, and from analytic to synthetic geometry. The greatest step in overcoming the difficulties between synthetic and analytic methods was however taken by Klein and Lie about 1871. Klein in his "Erlanger Programm" clearly outlined the standpoint from which the problems of modern mathematics have to be considered. The fundamental idea of higher geometry is to find all the "groups" and to investigate their properties, i. e., to find geometrical truth. As to Lie, it is well known that the achievements of this great mathematician concerning the theory of groups, since about 1874, influenced and still influence many of the most important fields of mathematics.

The old conception of invariants is abandoned and its place has been taken by the conception of groups.

In this paper it has been attempted to make a little contribution to geometry by applying the theory of groups to the well-known subject of perspective. In works on groups, which hitherto has been published, the treatment is almost exclusively analytical and it may be pointed out that our paper is the first bearing on groups in which the synthetical method is used.

Lie divided all projective transformations into five types. Before the investigations of Lie were known, only two of these types have usually been treated, the general projective transformation (collineation), and the perspective collineation. Our perspective collineations make up two of those five types: perspective, and elation. As has been said already in the preface, the other three types are being investigated in the same way by Prof. Newson.

For references we give the following list of books:

Sophus Lie, Vorlesungen über continuierliche Gruppen, Leipzig, Teubner, 1894.

Sophus Lie, Theorie der Transformationsgruppen, 3 Vols., Leipzig, Teubner.

Sophus Lie, Vorlesungen über Differential Gleichungen, Leipzig, Teubner, 1891.

Lindemann-Clebsch, Vorlesungen über Geometrie, Vols. 1 and 2, Leipzig, Teubner, 1888-93.

Klein, Vorlesungen über höhere Geometrie, 2 Vols., MS. Notes, Göttingen, 1893.

V. Staudt, Geometrie der Lage, Nürnberg, 1846.

Fiedler, Darstellende Geometrie, 3 Vols., Leipzig, Teubner, 1883-85.

Reye, Geometrie der Lage, Baumgärtner, Leipzig, 1886.

Cremona, Elements of projective geometry, Oxford, Clarendon Press, 1885.

Klein, Vorlesungen über das Ikosaeder, Leipzig, Teubner, 1885.

Mathematische Annalen, vols. 28 and 29.

Hoplophoneus occidentalis.

BY E. S. RIGGS.

(With Plate I.)

[Submitted to the Faculty of Kansas University as a thesis for the degree of A. M.]

The species *Hoplophoneus occidentalis* Leidy is based upon a fragment of a mandible from the White River Beds, described in Leidy's Extinct Mammalian Fauna of Dakota and Nebraska. It shows the lower molar dentition to be P. M.$_{\overline{3}}$, M.$_{\overline{1}}$; but of these the crowns of the third premolar and the sectorial are lost, as are both extremities of the mandible, so that even the size of the animal could only be approximated. Nothing more was ascribed to this species until 1894, when Osborn and Wortman referred two specimens to it*. In 1895 Dr. Williston published in this QUARTERLY† a preliminary description of two specimens of a large sabre-toothed cat obtained during the previous summer in the White River Beds of South Dakota by the University Geological Expedition, to which he gave the name of *Dinotomius atrox*. In January, 1896,‡ Mr. Geo. I. Adams determined a mandible (No. 1407 Amer. Mus.) as *Hoplophoneus occidentalis* and described the smaller form of Osborn and Wortman as a distinct species, suggesting also that *D. atrox* was a synonym of *H. occidentalis*, since it agreed with the American Museum specimen.

In his preliminary description of *D. atrox*, Dr. Williston, describing a complete tooth, characterized the inferior sectorial as follows: "Molar much as in the cat, save there is a well-developed internal posterior tubercle and the heel is rudimentary." An accompanying plate, although concealing the anterior portion of this tooth, represented it as described. Nevertheless, Mr. Adams in reproducing Dr. Williston's drawing for comparison with the American Museum specimen, has taken the liberty to reconstruct this tooth so as to show a prominent heel and no postero-internal cusp, and then states that the two specimens agree. Again, in the *American Journal of Science* for June, 1896, Mr. Adams reproduces

*Bulletin of the American Museum of Natural History.
†Kansas University Quarterly, Vol. III, No. 3.
‡The American Naturalist, January, 1896.

the same drawing, and, after stating that the species is best known from the Kansas University specimen, in the face of Dr. Williston's description and figure, states as a characteristic of the species, that "the postero-internal cusp is wanting." Now since this cusp is as strongly developed in the Kansas University specimen as in *H. primævus* or *H. robustus*, as specimens before me show, if Mr. Adams is correct in his statement that *H. occidentalis* does not have the cusp, then we have to do with two distinct species.

In Leidy's type there is little more than the number and size of the teeth in the lower molar series, the form of the fourth premolar, and the general size and shape of the body of the ramus, upon which comparison can be based. Below is given a series of measurements in which the Kansas University specimens are compared with Leidy's figure and with the above-mentioned American Museum specimen by means of data kindly furnished me by Dr. Osborn. All measurements are given in millimeters.

	Leidy's Type	Am. Mus.	Kans. Univ. Smaller.	Kans. Univ. Larger.
Length from condyle to alveolar border...		170	168	
Molar series, length......................	47	75	46	
Breadth of base of third premolar........	09		09	
Breadth of base of fourth premolar........	15		16	
Breadth of base of sectorial..............	21		20	
Depth of symphysis.....................		65 (?)	63	72
Depth of flange from base of canine.......		63.5	67	78
Depth of mandible at base of fourth premolar		30	30	
Condyle to angle......		25	27	
Length of diastema......................		37	41	48

It will be observed that, so far as can be determined by measurements, the Kansas University specimen agrees very closely with Leidy's type. The fourth premolar is similar in size and shape to the one remaining tooth, and, like it, is directed somewhat backward. On the other hand, the mandible of the American Museum specimen is slightly larger, has a more retreating coronoid process, a shallower flange, a shorter diastema, and is longer from the sectorial to the condyle. However, these differences are no greater than those between the two Kansas specimens, and unless the absence of the postero-internal cusp be found constant, the differences would not seem to be specific. Then, so far as can be determined by comparison, *D. atrox* and *H. occidentalis* agree as closely as individuals of the same species may be expected to agree, and, as Mr. Adams suggests, may be regarded as identical. Moreover, the

dental, cranial, and skeletal characters will later be shown to be consistent with those already described in *Hoplophoneus*.

The distinctive characters of *H. occidentalis* as shown by the Kansas University specimens are: Its size, which exceeds that of any other member of the genus by one-fourth; its markedly concave sagittal crest; its strongly recurved canine, trenchant and denticulate at the point, but rounded at the margins throughout three-fourths of its length; the third lower premolar much reduced, and lower sectorial with postero-internal cusp, but heel rudimentary.

The material upon which this restoration and description is based is composed of parts of two skeletons found almost together and in exactly the same horizon, just below the *bullatus* layer of the Oreodon beds. They differ somewhat in size, but no more than might be due to age or sex. The smaller of the two shows by the less completely ossified epiphyses and the slightly worn teeth, that it was a younger animal; while the firmly ossified sacrum and the well worn teeth of the larger one indicate an older animal. Between the two specimens the skeleton is anatomically complete, save the lower incisors and canines, some whole vertebræ and many of the pophyses, the sternum and most of the ribs, half of the scapula and radius, the shaft of the fibula, half the bones of the hind feet, and nearly all those of the front feet. In the restoration I have had for comparison the skulls (more or less complete) of *Dinictis felina*, *D. paucidens*, *Hoplophoneus robustus*,* and *H. primævus*, a mandible of *Pogonodon sp.*, together with various bones of the skeleton of *D. paucidens*, and *H. robustus*. Also a skeleton of *Felis leo*, *Felis concolor*, *Felis domesticus*, and *Lynx rufus*. Where parts were wanting, they have been supplied by comparison with other members of the genus so far as the parts were present, but frequently I have had to rely upon the African Lion. In the description I have used *H. robustus* along with the lion for comparison.

The material above mentioned forms part of the paleontological collection of the University of Kansas. For the privilege of its use and for his careful direction and criticism in the preparation of this description and restoration I am indebted to Dr. Williston.

DESCRIPTION OF THE SKELETON.

The skull is complete in the smaller specimen, save the upper canines and the crowns of the lower canines and incisors. An almost complete canine from the larger one, however. shows the

*Adams. *American Naturalist*, January, 1896.

characters of this tooth. The skull is deep but narrow, the zygomatic arches, though somewhat crushed, evidently did not stand out so prominently as in *H. robustus.* The sagittal crest is concave, rising into a prominence at the occiput, which is strongly overhanging. The post-orbital processes are projecting and curve slightly forward; the supraorbital margin is less prominent than in *H. robustus.* The zygomatic processes project well below the basicranial axis as is common in species of this genus. The mastoid process is strong and much roughened for muscular attachments. The posterior nares open on a line with the posterior border of the sectorial. A median ridge extends the entire length of the bony palate. A groove leads backward from the posterior nares as far as the anterior portion of the basi-occipital, where it divides and the branches lead respectively to the precondylar foramina.

The mandible is marked by a deep descending flange, second only to that of *Eusmilus* in prominence, its long diastema, and its deep masseteric fossa sloping away on its superior border to the short, stout coronoid process. The condyle is proportionally longer and more slender than in *H. robustus,* and the angle is more projecting. The two specimens differ quite markedly in the anterior portion of the mandible. In the larger one, in which only that portion in front of the third premolar remains, the flange is eleven millimeters deeper, the chin seven millimeters broader, and does not show the constriction below the base of the lower canine which is present in the smaller specimen. The diastema is seven millimeters longer, and the rami are much thicker and stouter at the superior border. There are three mental foramina in the smaller specimen and two in the larger.

The infraorbital foramina, as in *H. robustus,* are proportionally smaller and more triangular than in the lion. The post-glenoid foramina are present; but small and directed far inward. The lachrymal foramen lies well within the orbit, is small and nearer the infraorbital foramen than in *H. robustus,* and is directed more downward. The spheno-palatine foramen lies just on the median side of, and near the posterior palatine foramen, as in the last-named species, and is only a trifle larger. The optic foramen is small, laterally compressed, and is situated directly above the sphenoidal fissure. Above and in front of the optic foramen, situated about midway of the spheno-frontal suture, is the well-developed foramen spinosum. The spheno-orbital foramen, the rotund foramen and the anterior opening of the ali-sphenoid canal appear at the

surface as a common, large, anteriorly directed opening. Just within their opening, however, the three diverge, forming distinct canals. The posterior opening of the ali-sphenoid canal is in front of, and near the oval foramen, but not included in a common fissure with it as in *H. robustus*. The carotid canal is represented by a groove alongside of the basi-occipital, within the optic bulla. It terminates just without the posterior end of a ridge bounding the median groove of the basi-occipital. Midway between this and the anterior border of the occipital condyle is the expanded opening of the re-condylar foramen. The dental foramen opens just back of the anterior margin of the coronoid process, and forms a groove to the base of the condyle.

Dentition I. $\frac{3}{3}$ C. $\frac{1}{1}$ P. $\frac{2}{2}$ M. $\frac{1}{1}$. **The dentition** is complete except the crown of the lower incisors and canines. The upper incisors are proportionally shorter and stouter than those of *H. robustus.* The first and second are similar in size and shape; the third is only a trifle longer, but much stouter. The canine is strong and decidedly recurved; its margins are well rounded throughout the greater part of its length, but near the point they become thinner and trenchant. The posterior edge is minutely denticulate, but the condition of the specimen does not show whether or not this was true of the anterior edge. The third premolar is removed from the canine by a wide distance. It agrees very closely, both in size and shape with the corresponding tooth of *H. robustus.* The superior sectorial has a shorter and blunter median lobe, a lower heel, and a more prominent anterior secondary lobe. The tubercular molar is two-rooted and very similar to that of *H. robustus.* The lower incisors show quite a variation in size. The first is small and compressed; the second considerably larger; the third is almost as stout as the lower canine. The third lower premolar is no larger than that of *H. robustus.* Its posterior lobe is less prominent, the anterior one has disappeared entirely. The fourth premolar is similar in every respect to Leidy's figure of the type. The median lobe is shorter, and the secondary lobe less prominent than in *H. robustus.* The lower sectorial has only a very slight heel, but the postero-internal cusp is distinctly present.

MEASUREMENTS OF SKULL.

	Smaller Specimen. mm.	Larger Specimen. mm.
Condyles to premaxillary border	240	
Occiput to premaxillary border	265	
Breadth across post-orbital processes	86	

	Smaller Specimen. mm.	Larger Specimen. mm.
Breadth across post-orbital constriction	38	
Premaxillary border to line of superior canines	23	
Premaxillary border to line of front of orbits	101	
Height of occiput above base of condyles	85	80
Occiput to line of post-orbital process	127	
Breadth of zygomata	150*	
Breadth of occiput at constriction	45	60
Breadth ocross occipital condyles	52	
Breadth of foramen magnum	23	
Height of foramen magnum	17	
Length from condyles to anterior border of posterior nares	133	
Breadth across posterior margin of upper sectorials	92	
Posterior maxillary border to anterior margin of glenoid cavity	45	
Greatest diameter of orbit	45*	
Transverse diameter of nares	26	
Height of nares	35	
Greatest diameter of infraorbital foramen	15	
Breadth of external auditory meatus	4	5
Length of superior dental series, including canines	92	
Length of superior canine		
Longitudinal diameter of base of canine		30
Transverse diameter of canine	14	
Breadth of incisor series	39	
Breadth of third premolar	14	
Breadth of sectorial premolar	25	
Breadth of tubercular molar	13	
Length of crown of upper first incisor	11	
Transverse diameter at base	6	
Length of crown of third incisor	14	
Longitudinal diameter of base	10	
Mandible, length from condyle to incisor	167	
" depth of flange from base of canine	67	78
" depth of symphysis	63	72
" greatest breadth of chin	42	45
" depth of ramus at base of third premolar	32	
" depth of ramus at coronoid process	50	
Lower canine, longitudinal diameter	10	
Diastema, length of	41	48

*Approximated.

	Smaller Specimen. mm.	Larger Specimen. mm.
Third premolar, breadth of crown	9	
Fourth premolar, breadth of crown	16	
Sectorial molar, breadth of crown	21	
Height of condyle above angle	27	
Length of condyle	39	

VERTEBRAL COLUMN.

The cervical vertebræ are represented by the atlas, axis, and the third, in the larger animal, and the seventh in the smaller one. The atlas has a strong, rounded neural arch, but the ventral arch is comparatively narrow and light. The rudimentary spine is bifurcate. The transverse processes are too badly broken to be determined. Their base is perforated posteriorly by the vertebrarterial canal, which opens on the inferior surface further back than in the lion. Here the vertebral artery ran for a short distance in a deep grove and again passed under an osseous bridge forming an atlantar foramen, as in *Dinictis* and the *Viverridæ.* On the upper surface it is again open for a short distance before passing under the anterior root of the neural arch. The internal openings are much further back from the anterior margin than in the lion. The centrum of the axis is much compressed vertically. The inferior surface is divided by a sharp median ridge, flanked by concavities. The neural arch and spine are lost. The third cervical has no spine, but a neural prominence, which is bifurcate posteriorly. The vertebrarterial foramen is very small, the anterior zygapophyses depressed, the parapophyses directed more backward than in the lion. The seventh has well marked rib-facets, and a slender spine. The transverse processes are broken.

Of the **dorsal vertebræ,** nine are preserved in the smaller animal, many of which have lost their spines and transverse processes. The centra are proportionally broader than in the lion, and are produced into rounded lateral ridges which extend backward from the base of the transverse processes, and end in the capitular facets. These facets are plainly marked, and in most instances distinct from the intervertebral surface. In the first the transverse process is proportionally longer than in the lion, the tubercular facet is concave and looks downward; in the seventh the facet is also concave, but directed more outward; in the eleventh it is concave, elongate and directed forward as well as outward, and there is a deep fossa just back of it. The spinous processes are long and slender, and instead of the sharp anterior borders found

in the lion, they have a median grove at the base and are rounded near the extremity.

Seven fairly complete **lumbar vertebræ** are preserved in the two specimens, two of which are duplicates. The processes diverge less from the median line than in the lion, the postzygapophyses are short and stout, and lie close together with only a narrow notch between them. Their articular facets are directed more outward than downward. The anterior zygapophyses become somewhat longer toward the caudal end of the series. The articulating facets are nearly opposed to each other and are deeply concave vertically. The anterior margins of the lamina are less deeply concave than in the lion. The neural spines are broad, rising far back between the posterior zygapophyses and extending forward to the anterior border of the arch. The metapophyses are fairly well developed, and the anapophyses are as prominent as in the lion.

The sacrum is composed of three vertebræ. It is fourteen millimeters longer than that of the lion, but narrower at the anterior end, and the transverse processes are shorter and stouter. The centra are so completely ossified that all traces of their union have disappeared. The anterior zygapophyses are long and stout with their opposed faces concave. Those of the second and third vertebræ are also prominent. The first spine is twice as strong as that in the lion, and is directed backward.

The caudals are not only much longer and stronger than those of the lion, but the processes are better developed and a larger number have a complete neural arch. The anterior eleven are preserved in the small specimen. The first caudal has a neural spine as strong as the first sacral in the lion, and the third has a distinct rudiment. The zygapophyses are articulated as far back as between the eighth and ninth. The neural canal is present in the tenth. The transverse processes are strong in the first and second, become changed into a broad flat expansion in the sixth, which in turn gives place to an anterior and a posterior lateral expansion in the tenth. In the fifth and following vertebræ the posterior intervertebral notch is less deep, and, a short distance in front of the margin, there appears on each side a small foramen perforating the pedicle. Doubtless this foramen was for the passage of the nerve and vessels which, from the flexibility of the tail might otherwise have been subject to compression.

In the restoration of the remainder of the tail, I have figured the same number of vertebræ as in the lion, giving them as nearly as could be determined, proportions corresponding to the anterior

ones. From the fact that the arch extends further back in this animal than in the lion, it would seem certain that there could not be a less number of vertebræ, and it is very probable that there were more.

MEASUREMENTS OF VERTEBRÆ.

Atlas, breadth across the anterior articulating surfaces	55
" antero-posterior breadth of arch	26
" height of neural opening	31
Axis, greatest length	64
" breadth across anterior articulating surface	49
Seventh cervical, width of posterior and of centrum	38
" " length of centrum	30
" " height of neural opening	12
" " width of neural opening	24
Seventh dorsal, expansion of transverse processes	62
" " width of neural opening	18
" " height of neural opening	9
" " length of centrum	28
Second lumbar, length of centrum	41
" " width of centrum	29
" " width of neural opening	19
" " height of neural opening	9
Sacrum, length	104
" width of anterior end	63
" least diameter of first transverse process	35
First caudal, length of centrum	32
" " width	28
Entire length of first eleven caudal vertebræ	440
Width of eleventh caudal	28
Length of same	43

Pectoral girdle. Only the lower half of the scapula is present, and one of the sternal bones. The glenoid surface of the scapula is rounder in outline than in *H. robustus* and much more so than in *Felis concolor*. The anterior part of the ventral surface is more deeply concave, and the coracoid process, like that of *H. robustus*, curves inwardly, more strongly than in the lion. The neck is less constricted and the spine is much nearer the axillary border. At the origin of the teres minor muscle the border is thickened and massive, presenting a posterior face which stands at right angles to the subscapular surface. The same is true in *H. robustus*, but in a less marked degree.

Fore Leg. The **humerus** is described from two bones, one of which lacks the head, the other the distal third. The length is

determined by comparison. The great tuberosity is partly broken away, but evidently projected somewhat beyond the head. The bicipital groove is deep and narrow; the inner surface of the shaft is concave as far as to the lower extremity of the deltoid ridge. The lesser tuberosity is separated from the articulating surface by a deep groove, continuous with the concavity on the posterior surface of the shaft reaching almost to its middle. The anterior surface of the shaft is laterally compressed with the roughened, deltoid ridge unusually prominent. The supinator ridge rises above the middle of the shaft, curves outward and forward, forming a marked concavity on the antero-external surface of the shaft, and giving an unusual breadth to the distal end of the bone. The supracondyloid foramen is well rounded. The inner condyle is prominent and roughened. Back of the inner condyle and near the trochlear surface is a broad groove, a character which seems to be common in the species of *Hoplophoneus*, but which is lacking in *Dinictis*. There is only a trace of it in the species of *Felis* examined and in *Machærodus crassidens* Cragin (Williston).* The olecranal fossa is broad transversely, but shallow. The coronoid fossa extends as far outward as the exterior border of the capitellum, much as in *M. crassidens*. It is quite as deep as the olecranal fossa.

The ulna is a strong bone, rounded and convex on the posterior surface, slightly concave anteriorly and grooved on the external surface throughout the greater part of its length. The olecranon is stout, bent inward, and expanded at its roughened extremity. The great sigmoid cavity is narrowed antero-posteriorly, but broad from side to side; the beak is thin, but prominent on the outer border. The exterior border of the lesser sigmoid cavity, is not so prominent as in recent forms. On the interior border of the anterior surface, just in front of the great sigmoid cavity, is a roughened surface for muscular attachment, common to *Dinictis* but not found in the recent cats. The styloid process is short and stout, and separated from the round articulating facet for the radius by a shallow notch.

The radius, represented by the proximal half of one bone. The head is quite concave, as in *H. robustus*, and bears on its anterior margin a prominent protuberance exterior to which there is a notch separating it from the anterior prominence of the articulating surface for the ulna.

*Kansas University Quarterly, Vol. III, No. 3.

Of the **front foot**, only the unciform and the proximal half of the fifth metacarpal, and the distal end of the second are present. The unciform, as seen from the dorsal side, is roughly a triangle in which the anterior border is slightly concave and the posterior angle rounded. The surface for the cuneiform is extended downward posteriorly, and is not bounded below by a continuous groove as in the lion. The surface for the scapho-lunar is strongly convex throughout, curving around the posterior end. The surface for articulation with the os magnum extends downward to the lower border of the anterior face, and is continuous with the scapho-lunar surface posteriorly. The proximal end of the fifth metacarpal is less expanded than in the lion, its external tuberosity is less prominent, and is not separated from the articulating surface by a groove. Its posterior end is rounded, and articulates with about half of the anterior surface of the unciform. The distal end of the second metacarpal is proportionally broad and strong. A first phalanx is short, stout, and strongly curved; the protuberances of the proximal end are shorter and the inferior surface is less deeply notched than in the lion.

MEASUREMENTS OF THE FORE LEG.

	mm.
Scapula, length of glenoid cavity	39
" from base of spine to posterior border	13
" from base of spine to anterior border	29
" thickness of posterior border	14
Humerus, length	236*
" diameter of head and great trochanter	65*
" greatest diameter of distal end	74
Ulna, length	242
" end of olecranon to beak	53
" olecranon to coronoid process	79
Radius, approximate length	177
" diameter of head	32
Unciform, length	24
" breadth	22
Second metacarpal, breadth of distal end	19
Phalanx, length	35
" breadth of proximal end	20

*Approximate.

Pelvic girdle. Between the two specimens the pelvis is almost complete. Compared with that of the lion it is less constricted at the acetabula. The iliac rami of the ischia are straighter, and less divergent posteriorly. The ischiatic rami of the pubis are

stronger both relatively and actually. The anterior end of the ilium stands nearly vertical and the crest is curved strongly outward. The gluteal surface is divided longitudinally by a strong ridge, extending from the acetabulum to the crest. This character is even more prominent in *H. robustus* and is found also in *Dinictis*. Above it there is an elongated fossa, below a slightly concave surface. The muscular roughening for the *rectus femoris* is more prominent and extends further forward than in the lion. A sharp line separates the gluteal surfaces. The border below the acetabulum is thin and sharp. The ilec-pectineal eminence is scarcely noticeable. The iliac surface of the ilium is convex, the articulation with the sacrum close and admitting of little motion. The iliac ramus of the ischium is thicker, narrower and more rounded on the superior border than in the lion. The pubic ramus is thin and flat. The spine of the ischium is situated near the middle of the iliac ramus. The pubic symphysis is firmly co-ossified. The ischiatic ramus of the pubis is concave above, and is nearly as strong at posterior, as at the anterior end.

Hind Leg. **The femur** has more of the characters of the recent cats than has that of *H. robustus.* The shaft is straighter, the third trochanter less prominent, and not connected with the great trochanter by a sharp ridge. The head is directed less forward, and the patellar surface forms an anterior prominence. The great trochanter projects only slightly beyond the head. A marked groove extends downward from the pit for the ligamentum teres. On the outside of the shaft there is a prominent, roughened protuberance extending thirty-five millimeters above the condyle, which is not present in *H. robustus* or in the recent cats. The **patella** is rather long and narrow and is irregularly rounded on the anterior surface. The articulating face covers two-thirds of the posterior surface. It is concave vertically and convex from side to side.

The **tibia** is a strong bone, slightly curved forward, and laterally compressed. The anterior border is sharp and has a marked protuberance about midway of its length, where it makes a sharp curve inward. As seen from behind both borders are concave, that of the inner being slightly more marked. There is a distinct articulating facet for the proximal end of the fibula. The internal malleolus is strong and projects somewhat inward, ending in an angle toward the posterior border. The groove for the tendons of the *tibialis anticus* and the *flexor longus digitorum* is broad and shallow and is directed more obliquely forward than in the lion.

The astragular surface is, as in *H. robustus*, less deeply grooved and is placed more obliquely to the shaft than in recent cats. The antero-external border extends but little below the articular surface.

Of the **fibula** only the extremities remain. These indicate a stronger bone than that of *H. robustus*. The proximal end articulates with the tibia by a well-marked oval facet. The outer surface is roughened for ligamentary or muscular attachments. There is only a trace of a groove on the outer surface, instead of the deep concavity found in *H. robustus*. The distal end, as in the last-named species, is very unlike that of recent cats. It is narrower but thicker than the head and is roughly triangular in section. The internal surface bears at its lower anterior border a convex articulating facet for the astragalus which curves half way around the lower end. The posterior surface stands at a right angle with the last, and is almost as broad. There is a broad, shallow, peroneal groove at the inner side of the posterior tuberosity of the malleolus. The tendinous depression on the external surface is less marked than in the recent cats.

The foot is short and weak in the metatarsal region, as is true of all the early cats. The calcaneum is not more than two-thirds as large proportionally as that of the lion, and does not extend distally as far as the astragalus. The external process extends backward beyond the anterior margin of the superior articulating surface. Back of this and near the upper surface is a deep fossa. The sustentaculum is situated near the anterior border, opposite the external process and has a broad, shallow groove at its base. The anterior surface is quite concave; the articulating surface for the astragalus does not turn inward posteriorly, as in the recent cats.

The astragalus has a short, constricted neck, and a well-rounded head, but is markedly compressed vertically. The superior surface is but slightly concave laterally, corresponding to the slight convexity of the tibial surface, and does not extend to the posterior border as in recent cats. This does not permit of as great an angle between the foot and the tibia, and bears evidence of more plantigrade affinities. The head is less deflected from the antero-posterior axis than in the lion. There is no articulation with the cuboid, as is the case in *Dinictis*. The facet for articulation with the sustentaculum is long and deeply notched posteriorly. The posterior end is grooved for the tendon of the *flexor longus hallucis*. The groove between the inferior articulating surfaces is straight, and ends abruptly in a deep fossa.

The cuboid is narrower in proportion to its length than that of the lion. The posterior and anterior surfaces converge outwardly, making the external surface shorter than the internal. The posterior surface is convex; the groove for the tendon of the *peronæus longus* and the ligamentary prominence posterior to it extend across the entire inferior surface. The facet for the navicular is long and curved; that for the ecto-cuneiform is semilunar in outline. The anterior surface is concave to receive the fourth and fifth metatarsals.

The metatarsals are about one-half the length of those of the lion. Only the fourth, fifth, and half of the second are preserved. The fifth articulates with the cuboid by about half of its posterior end which is sloping and extends but little back of the facet. The tuberosity projects prominently outward and backward. The fourth overlaps the fifth and in turn is overlapped by the third much as in the recent cats. The shafts are sub-triangular in section and are strongly curved near the distal end. The second is about as strong as the fourth; its proximal end is laterally compressed. The superior surface is symmetrically rounded, instead of sloping outward as in recent forms and its inferior process does not project under the third. The exterior articulating surface and the ligamentary attachments indicate a fairly well-developed first toe. The proximal series of phlanges are short and stout. The second series are markedly concave above, indicating a perfectly retractile claw.

MEASUREMENTS OF PELVIS AND HIND LEG.

	Smaller. mm.	Larger. mm.
Pelvis, length	250	
" breadth between actetabula	66	
Ilium, breadth in front of actetabulum	37	36
" thickness above actetabulum	24	29
" greatest breadth	55	
" greatest diamemeter of actetabulum		42
Ischium, diameter back of actetabulum		27
Femur, length		285
" breadth of head and great trochanter	70	76
" diameter of head	34	35
" distance from head to lower margin of lesser trochanter	67	76
" breadth of condyles	59	61
Patella, length	47	
" width	31	

	Smaller. mm.	Larger. mm.
Tibia, length		234
" transverse diameter of proximal end	60	62
" transverse diameter of distal end	37	40
Fibula, approximate length	210	
" width of proximal end	28	
" width of distal end	27	
" thickness of distal end	16	
Calcaneum, length	68	
" width across processes	38	
Astragalus, length	46	48
" greatest width	33	37
Second metatarsal, length	60*	
" " width proximal end	10	
" " vertical diameter proximal end	19	
" " width of distal end	16	
Fourth metatarsal, length	61	
" " width of proximal end	17	
" " width of distal end	17	
Fifth metatarsal, length	55	
" " width of proximal end	18	
" " width of distal end	14	

SUMMARY.

In short, the characters of *Hoplophoneus occidentalis*, as shown from these specimens, are: Size similar to that of the *Felis onca* but stouter bodied and limbs shorter in proportion; skull large in proportion to body, deep but narrow, brain case small, sagittal crest concave, occiput strong and overhanging, zygomatic processes drooping, superior canine trenchant only at point, and inferior sectorial with a rudimentary heel; atlas with an atlantar foramen; zygopophysis firmly interlocked and but little diverging from the median line; sacrum long but narrow at the anterior end; caudal vertebræ stronger with processes better developed and neural canal extending to the eleventh; scapula with neck little constricted, glenoid cavity deep and rounder than in recent cats, the posterior border at the origin of the *teres minor* thickened and massive; humerus with deltoid and supinator ridges unusually developed, lesser tuberosity separated from the head by deep groove, the internal epicondyle unusually prominent and separated from the trochlea by a broad notch; ulna with the oberanon two-ninths the length of

*Estimated.

the entire bone and the great-sigmoid notch narrow antero-posteriorly; pelvis articulating closely with the sacrum, ilium with a strong median dividing the gluteal surface into a superior and an inferior concavity, and the pubic ramus of the ischium unusually strong; femur with shaft nearly straight, patellar surface forming an anterior prominence, and a well-marked tubercle above the external condyle as in Felidæ; fibula not grooved on the external surface of the head, thick and articulating loosely at the distal end and having a strong posterior tubercle; astragalus only slightly grooved for the tibial articulation and the tibial surface does not extend to the posterior border, an evidence of planitigrade affinities; calcaneum short and having the sustentaculum near the anterior end: metatarsals short and curved; claws distinctly retractile.

DINICTIS PAUCIDENS.

In a recent paper on the extinct Felidæ of North America* Dr. Adams states summarily in a note that *D. paucidens*† is probably a synonym of *D. fortis*.‡ Such a statement would indicate either that Dr. Adams has not gone far enough into the description of this form to recognize the characters upon which it is based, or that *D. fortis* is a sufficiently generalized type to include whatever it may be found convenient to place under it. In the latter case, *D. felina*, the type of the genus, would fall a much easier victim, since *D. fortis* in becoming synonymous with *D. bombifrons*‡, has so far lost its distinctive characters that its dentition is essentially the same as that of the generic type, leaving as the only specific character a difference in size. However, trusting that this is due merely to oversight, I repeat here that the distinctive characters of *D. paucidens* are: "The absence of a second lower molar, the slenderness of the base, and the concave outer border of the upper sectorial as seen from above, and the presence of but two incisors in the mandible." These, together with the very "slight development of the postero-internal cusp of the lower carnassial," described as well developed in *D. bombifrons* (syn. *D. fortis*) and the "proportionate length and slenderness of the fore-arm," are differences sufficient to satisfy the most exacting.

*American Journal of Science, June, 1896.
†Riggs, Kansas University Quarterly, April, 1896
‡Adams, American Naturalist, June, 1895.

On the Dermal Covering of Hesperornis.

BY S. W. WILLISTON.

(With Plate II.)

A specimen of *Hesperornis*, collected in western Kansas the past year by Mr. H. T. Martin and now in the University Museum, is of especial interest from the information it affords of the dermal covering of this Cretaceous toothed bird.

The specimen, which is in excellent preservation, lies upon a chalk slab, with the head doubled partly under the pelvis. Some six or eight vertebræ, together with the humeri and coracoids and many of the ribs are wanting; otherwise the specimen seems perfect. The size is distinctively less than that of *H. regalis*, and it does not seem to be due to immaturity. Possibly the species is identical with *H. gracilis*, which has been only imperfectly described.

The photographic illustration given in Plate II was taken from the fragment removed from the slab over the right tarso-metatarsal, the surface of the slab itself being less clearly, though more fully marked. I have sketched in the bone to show the relative size and position.

The podotheca is seen to be scutellate in front. The structure is shown so clearly in the photograph that I need not enter into a fuller description. The scutes are all smooth, not imbricated, and distinctly separated from each other. They are a little longer from side to side below, though not much. I count twenty-six on the slab, and to the back part of the bone, while impressions of the feathers will be seen on the opposite side.

These feathers were evidently long, reaching nearly to the phalangeal articulation, and are clearly semiplumulaceous in character, the pennaceous shaft of considerable size, the vanes long and wavy. The shaft of one feather is seen in the illustration lying close to the outline of the bone, and is of considerable size. I doubt not that the feathers throughout were of this character, or wholly plumulaceous. I find distinct impressions of the wavy vanes at the back of the head and elsewhere, but in no case is there the impression of a true feather, as I think would surely be the case had the bird possessed them.

This plumulaceous character of the plumage is not unexpected. Although Marsh nowhere mentions the plumage in his work, I know that he personally had the opinion that it was of a downy character. That the feathers of the tarsus should extend to the feet in a wading bird seems surprising, but there can be no other interpretation of the specimen.

The Duty of the Scholar in Politics.

BY FRANK HEYWOOD HODDER.

[Phi Beta Kappa Address, delivered at the University of Kansas, June 8, 1896.]

The duty of the scholar in politics has been the subject of so many addresses upon occasions of this character that it is difficult to say anything new respecting it. It is, however, suggested both by the occasion and by the direction of my own studies. Mr. Disraeli is reported to have once replied to an opponent in Parliament: "The honorable gentleman has said things both true and new but the things true are not new and the things new are not true." It is, after all, the things true which are not new that are important. Especially is this the case with respect to duty, whatever its direction. It rarely happens that we do not know our duty but often that, knowing it, we fail in the doing.

By the scholar, in this connection, I do not mean the specialist but rather the man of education and independence, the man who is well informed upon all important topics of current interest and who does his own thinking respecting them. This definition does not include all graduates of colleges and universities and it does include many who never had the advantage of college training. The duty in politics of the man of education and independence is then the subject. The greater the education, the greater the influence he may exert and the greater the obligation to exert it. Especially great is the obligation in the case of the young men and young women educated at the expense of the state. Upon them rests the duty of using their influence for its welfare.

But I do not intend to range at large over the whole subject. I propose instead to emphasize one particular duty—namely the duty of the scholar to use his influence for the maintenance of international peace. The discussion of this particular duty is especially appropriate to the occasion by reason of the fact that it is totally disconnected from all questions of party politics. It is a duty pre-eminently of the scholar as a man governed by reason,

rather than by passion and prejudice. Recent events seem to present certain dangers to our national peace, which I shall consider in order. They are:

1st, misconstruction of the Monroe doctrine;

2d, a rising war spirit among the people; and

3d, enormous expenditures for war purposes.

First, the Monroe doctrine. I venture the assertion that the recent unwarranted construction of that doctrine is contrary to the teaching of the founders of the republic, a perversion of the true meaning of the original declaration, an encroachment upon the rights of foreign states and a menace to our peace and safety.

It is contrary to the teaching of the founders which was non-interference with the affairs of foreign nations and peace and friendship with all mankind. Three men may be called pre-eminently the founders of the republic. They were Washington, Madison and Hamilton, to whom more than to any others were due respectively the success of the revolution, the framing of the constitution and the establishment of government. The combined wisdom of these men was embodied in the farewell address issued by Washington upon his retirement from the presidency, a worthy guide to the American people for all time. In that address we find this advice:*

"Observe good faith and justice toward all nations. Cultivate peace and harmony with all..... It will be worthy of a free, enlightened and, at no distant period, great nation, to give to mankind the magnanimous and too novel example of a people guided by an exalted justice and benevolence..... The experiment, at least, is recommended by every sentiment that ennobles human nature."

"The great rule of conduct for us in regard to foreign nations is to have with them as little political connection as possible..... Europe has a set of primary interests which to us have none or a very remote relation. Hence she must be engaged in frequent controversies, the causes of which are essentially foreign to our concerns.....Our detached and distant situation invites and enables us to pursue a different course..... Why forego the advantages of so peculiar a situation? Why quit one's own to stand on foreign ground? Why entangle our peace and prosperity in the toils of European ambition, rivalry, interest, humor, or caprice?"

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*See "Statesman's Manual" for quotations from Presidential messages and addresses. Richardson's "Messages and Papers of the Presidents," now publishing by the Government, will supersede the earlier collection.

†See Wharton's "Digest of International Law," Vol. 1, sects. 45 and 57, for opinions cited above.

All parties at that time agreed in counseling peace.† Jefferson, the father of democracy, expressed the same sentiment. In an official letter in 1793, while Secretary of State, he said:

"We love and value peace; we know its blessings from experience. We abhor the follies of war and are not untried in its distresses and calamities. Not meddling with the affairs of other nations, we hope that our distance will leave us free in the example and indulgence of peace with the world."

Again in writing Monroe in 1823, advising the issue of this very declaration, he said:

"I have ever deemed it fundamental for the United States never to take an active part in the quarrels of Europe. Their political interests are entirely distinct from ours. Their mutual jealousies, their balance of power, their complicated alliances, their forces and principles of government are all foreign to us. They are nations of eternal war. All their energies are expended in the destruction of the labor, property and lives of their people. On our part never had a people so favorable a chance of trying the opposite system, of peace and fraternity with all mankind and a direction of all our means and faculties to the purposes of improvement instead of destruction."

And Monroe in the very message, now made the excuse for so much warlike demonstration, took pains to repeat this doctrine of non-intervention:

"In the wars of European powers, in matters relating to themselves we have never taken part nor does it comport with our policy to do so....With the existing colonies or dependencies of any European power we have not interfered and shall not interfere.....Our policy with regard to Europe is not to interfere with the internal concerns of any of its powers."

Statements of this character were frequently repeated by later statesmen. Van Buren in official letters, while Secretary of State, within five years of the issue of the Monroe declaration, said:

"It is the ancient and well settled policy of this government not to interfere with the internal concerns of any foreign country."

"An invariable and strict neutrality and an entire abstinence from all interference with the concerns of other nations are cardinal traits of the foreign policy of this government. The obligatory character of this policy is regarded with a degree of reverence and submission but little if anything short of that which is entertained for the Constitution itself."

Mr. Seward in 1863, at the very time he was protesting against the French occupation of Mexico, the only violation of the true Monroe doctrine ever attempted, wrote Mr. Adams:

"In regard to our foreign relations, the conviction has universally obtained that our true national policy is one of self reliance and self conduct in our domestic affairs, with *absolute non-interference* with those of other countries."

Again in 1866 Mr. Seward* in advising against interference in behalf of Chili said:

"If there is any one characteristic of the United States which is more marked than any other, it is that they have from the time of Washington adhered to the principle of non-intervention and have perseveringly declined to seek or contract entangling alliances, *even with the most friendly states.*"

Quotations of this character might be multiplied indefinitely but enough have been given to prove that the teaching of the founders from Washington to Monroe and John Quincy Adams was non-intervention and peace. Their authority cannot rightfully be invoked in support of any other policy.

Recent construction of the Monroe doctrine is a perversion of the true meaning of the original declaration. I venture this assertion without fear of contradiction by any special student of international law or of our political history. The Monroe doctrine consists of two parts corresponding to the two causes which occasioned its issue. John Quincy Adams wrote the first part, Jefferson the second, and Monroe embodied both in his annual messages for 1823 and 24. Adams, Jefferson and Monroe may therefore properly be considered its joint authors.†

The first part respects colonization. America is not subject to future European colonization. In 1821 the Czar Alexander of Russia issued a proclamation claiming the western coast of North America as far south as the 51st parallel. That territory was then claimed both by Great Britain and the United States. The proclamation of the Czar was accepted by both as evidence of an intention to establish a Russian colony in America. It is difficult for us to-day to reproduce in imagination the situation of the United States at that time. Our territory then as now extended from the Atlantic to the Pacific but that portion between the Alleghanies and the Mississippi was still sparsely settled and the vast expanse between the Mississippi and the Pacific, with the exception of

*"Works." Vol. 5, pp. 444-5.

†It is well known that Madison was consulted and advised the issue of the declaration. He, however, merely seconded Jefferson's suggestions.

Louisiana, Arkansas and Missouri, was absolutely unoccupied and almost unexplored. The territory of Mexico subsequently acquired by us was in the same condition. It would not then have been difficult for Russia to have planted a colony either in or near this territory, upon the plea that it was unoccupied. To guard against this danger President Monroe, acting upon the advice of Adams, issued this declaration:

"The American continents, by the free and independent condition which they have assumed and maintain, are henceforth not to be considered as subjects for future colonization by any European powers.....With their existing colonies or dependencies we have not interfered and shall not interfere."

There was not the slightest intention of assuming a protectorate over other American states for the purpose of guarding their territory from European colonization. That such was the case is absolutely proved by the language used by Mr. Adams two years later in a special message to the Senate on the subject of a Congress of American states.

"An agreement," he said, "between the parties represented at the meeting that each will guard, by its own means, against the establishment of any future European colony within its borders, may be found advisable. This was announced to the world, more than two years ago, by my predecessor, as a principle resulting from the emancipation of both the American continents."

This statement Mr. Schouler* observes is remarkable as an exposition of the Monroe doctrine from the pen of the one most competent to make it, that is from the pen of the one who originally wrote it—in effect that European exclusion from this hemisphere was to be the work not of the United States, acting as the champion of the two Americas, but of each American republic as the protector of its own rights. Mr. Webster speaking at the same time expressed the same opinion.†

"It was highly desirable to us," he said, "that new states should settle it as a part of their policy not to allow colonization within their respective territories. We did not need their aid to assist us in maintaining such a course for ourselves, but we had an interest in their assertion and support of the principle as applied to their own territories."

The Russian claim was immediately abandoned in treaties with both Great Britain and the United States. Since that time there

*"History of the United States," Vol. 3, p. 362.

†"Works," Vol. 3, pp. 200-207.

has not been the faintest suggestion of an intention on the part of any European power to establish any new colony upon either of the American continents. The rapid growth of American populations has practically resulted in the actual occupation of every part of both continents. An occasion then for an application of this part of the Monroe doctrine has not presented itself and cannot present itself.

The second part of Monroe's declaration respects intervention. It consists of two distinct propositions. European interference with American states for the purpose of subverting their governments cannot be permitted and the extension to America of the European political system cannot be permitted. At the close of the Napoleonic wars in 1815 Russia, Austria and Prussia united in the so-called Holy Alliance. Their avowed object was the maintenance of the Christian religion. Their real purpose was the preservation of their political system of absolute monarchy, based upon the divine right of kings, by a pledge of mutual assistance in case of popular insurrection. The treaty between them was offered for signature to every power in Europe except the Sultan and the Pope. All acceded to it except Great Britain whose foreign minister replied that the principles of the Alliance were inconsistent with those of the British constitution. In 1821 the allies sent an Austrian army into Italy in order to prevent the adoption of a free constitution in Naples. And in 1823 they sent a French army into Spain to suppress popular insurrection there, and re-establish the despotism of Ferdinand VII. It was then proposed that the allies call a congress to arrange for the subjugation of Spain's revolted colonies in America and the re-establishment of Spanish authority over them. Information of this design reached the United States through Great Britain. In opposition to it Monroe, acting on the advice of Jefferson, issued the second part of his famous declaration:

"With the governments who have declared their independence we could not view any interposition by any European power in any other light than as the manifestation of an unfriendly disposition toward the United States.....The political system of the allied powers is essentially different from that of America.....We should consider any attempt on their part to extend their system to any portion of this hemisphere as dangerous to our peace and safety. .....It is impossible that the allies should extend their political system to any portion of either continent without endangering our peace and happiness ...It is equally impossible, therefore, that we

should behold such interposition in any form with indifference." In other words, European states could not be permitted to overthrow any American government for the purpose of establishing upon its ruins an absolute monarchy based upon the divine right of kings. There was not a word respecting intervention for any other purpose.

Monroe's warning was sufficient to induce the Holy Alliance to abandon their plan of interfering in American affairs. Since that time there has been but a single violation of this part of Monroe's declaration. During our civil war the unscrupulous government of Napoleon III invaded Mexico, overthrew her government and established in its place an Empire, sustained by French arms. Immediately upon the close of our war, Secretary Seward informed France that her troops must be withdrawn. They were withdrawn and the Empire fell. Since that time there has not been the faintest suggestion of an intention upon the part of any European power to interfere in the affairs of any American state for the purpose of overthrowing its government and establishing monarchy in its place. Constitutional government has been established in every European state except Russia and the European political system of which Monroe wrote has ceased to exist. An occasion, therefore, for a second application of this part of the Monroe doctrine has not presented itself.

Briefly stated the Monroe doctrine opposed new European colonies, subjugation of American states by European powers and the system of the Holy Alliance. New colonization has never been attempted, subjugation has been tried once and failed utterly, the system of the Holy Alliance has been dead for half a century. Any statement that goes beyond these three points is unwarranted by the original declaration. Monroe's declaration was a protest against new colonies. It is now applied to colonies that antedate our national existence. Monroe's declaration was a protest against intervention. It is now made the basis for intervention. Monroe's declaration was a protest against absolutism. It is now applied to a government which, despite monarchical forms, is more thoroughly democratic than our own. Such construction is a perversion of the true meaning of the original declaration.

Let us now inquire into the origin of this misconstruction of the Monroe doctrine. With the defeat of John Quincy Adams and the election of Andrew Jackson in 1828, the era of statesman presidents came to an end and an era of military favorites and politicians began. At the same time we abandoned the founders' policy of

peace and friendship with all mankind and assumed an attitude of defiance toward foreign nations. Slavery wanted more territory for its expansion and the South needed more slaves in order to keep abreast of the rapidly growing North. Longing eyes were turned toward Texas and its acquisition became the settled policy of the slave power. Jackson first tried to buy Texas but Mexico refused to sell. "To do so," Santa Anna replied, "would be to sign the death warrant of my country, for the United States would take one province after another until none remained." Jackson then sent Houston to Texas, at that time the territory of a friendly state with which we were at peace, with the understanding that he should colonize it with American citizens, foment revolution and, when a favorable opportunity presented itself, apply for admission to the United States. This conspiracy required time for its development but was carried out according to the program. The revolution came, Texas declared her independence of Mexico and applied for annexation to the United States. A treaty for the purpose failing of ratification in the Senate, President Tyler secured the passage of a joint resolution for the admission of Texas as a State in the Union.

Such was the situation when Polk became President of the United States on the 4th of March, 1845. In his inaugural address the new President said:

"I regard the question of annexation as belonging exclusively to the United States and Texas. Foreign powers do not seem to appreciate the true character of our government. Our union is a confederation of independent states, whose policy is peace with each other and all the world. To enlarge its limits, is to extend the dominion of peace over additional territories and increasing millions."

In his first annual message to Congress, again referring to Texas, he said:

"The United States cannot in silence permit any European interference on the North American continent; and should any such interference be attempted, will be ready to resist it at any and all hazards.....The nations of America are equally sovereign and independent with those of Europe. They possess the same rights, independent of all foreign interposition, to make war, to conclude peace and to regulate their internal affairs. The people of the United States cannot, therefore, view with indifference attempts of European powers to interfere with the independent action of nations on this continent.....We must ever maintain the

principle that the people of this continent alone have the right to decide their own destiny. Should any portion of them, constituting an independent state, propose to unite themselves with our confederacy, this will be a question for them and us to determine, without any foreign interposition."

This is the new version of Monroe's declaration. Monroe had protested against European interference for the purpose of destroying independent states and Polk extended the protest to any interference whatever.

Within the month the annexation of Texas was completed. But the South was not satisfied. She next coveted the rich soil of California. Again Mexico was asked to sell. Again she refused and Polk precipitated a war to compel her to do so. Mexico was prostrated and compelled to part with California for fifteen million dollars. This was Polk's way of extending the blessings of peace over additional territories and increasing millions.

Before peace with Mexico had been ratified, a peculiar situation presented itself in Yucatan. The white race in that peninsula were engaged in a protracted struggle with the Indians. As the price of assistance, they simultaneously offered the dominion and sovereignty of their country to Great Britain, Spain and the United States. In a special message, advising the occupation of Yucatan, President Polk said:

"We could not consent to a transfer of this 'dominion and sovereignty' to either Spain or Great Britain or any other European power. In the language of President Monroe.... 'the American continents, by the free and independent condition which they have assumed and maintain, are henceforth not to be considered as subjects for future colonization by any European power.' ....The present is deemed a proper occasion to reiterate and reaffirm the principle avowed by Mr. Monroe and to state my cordial concurrence in its wisdom and sound policy."

Here we have the new version of the first part of Monroe's declaration. The protest against new European colonies is construed to mean that no European power shall acquire territory upon this continent in any way whatever.

Polk's two statements were glaringly inconsistent. The first declared the right of the United States to acquire territory by the free gift of an independent state, the second denied the right of Europe to acquire territory in the same way. The first denied to Europe the right of interposition; the second asserted it for the United States. The first asserted that the nations of America were

sovereign and independent and alone had the right to decide their destiny; the second limited that right to a disposition conformable to our interests—in short, they might do as they pleased as long as they pleased to do as we pleased. In what mysterious way the sovereignty of the United States was suddenly extended over the entire continent was not explained. Nevertheless Polk's statement gave the Monroe doctrine its final form: Europe shall not interfere with American states and shall not acquire territory in America in any way. The United States may interfere and may acquire territory whenever her interests demand it. This, I take it, is the form in which the Monroe doctrine rests in the minds of the American people to-day.

Polk's misconstruction of the Monroe doctrine did not pass unchallenged. Mr. Calhoun was at that time the only surviving member of Monroe's cabinet. He was, therefore, of all men living the best acquainted with the circumstances and discussions attending the issue of the declaration. His pro-slavery sympathies and his own part in the annexation of Texas might have inclined him to accept Polk's construction. Instead he declared in the Senate that the case of Yucatan did not come within the Monroe declarations; that they did not furnish the slightest support for it.* It was not the extension of the European political system to this continent, for that system had already ceased to exist. It was not an interposition of an European power to oppress an American government, because that power would come, not to oppress, but to save. Even if England should assert her sovereignty over Yucatan, it would not bring the case within the Monroe doctrine because the tender of that sovereignty had voluntarily been made. It was not colonization. That word had a specific meaning. It meant the establishment by emigrants from a parent colony of a settlement in territory either uninhabited or from which the inhabitants had been partially or wholly expelled. The occupation of Yucatan could not be construed to be colonization by any forced interpretation. Yucatan might become a province or a possession of Great Britain but not a colony. In conclusion he said:

"What the President has asserted in this case is not a principle belonging to these declarations; it is a principle which, in his misconception, he endeavors to engraft upon them but which has an entirely different meaning and tendency....It goes infinitely and dangerously beyond Mr. Monroe's declaration. It puts it in the power of other countries on this continent to make us a party

*Calhoun's "Works," Vol. 4, pp. 454-66.

to all their wars.....If this broad interpretation be given to these declarations....our peace will ever be disturbed, the gates of our Janus will ever stand open, wars will never cease."

Who, then, was the author of this so-called Monroe doctrine? It was Polk, Polk the mendacious, as v. Holst has called him, the man who provoked a war of wanton conquest and based its declaration upon a lie. It is Polk's doctrine and not Monroe's. Not daring to sign his own name, he sought to give it authority by attaching that of one of the founders of the republic. When and why was it proclaimed? It was at the very time we were engaged in the annexation of Texas and the conquest of Mexico, the two acts in our national history of which we have least reason to be proud. Then it was that Polk twisted a declaration intended for the protection of free institutions into an excuse for the extension of human slavery. Its origin and purpose condemn it.

The policy which had succeded in Texas and Mexico, Polk next applied to Cuba. He first tried to buy Cuba but Spain replied that rather than sell she would see the island sunk in the ocean. Filibustering expeditions next tried to revolutionize Cuba, as Houston had revolutionized Texas, but failed. We next threatened Spain as Slidell had threatened Mexico. In the spirit of the Polk doctrine, our ministers to Great Britain, France and Spain, in the celebrated Ostend Manifesto* declared:

"After we have offered Spain a price for Cuba far beyond its present value and this shall have been refused, it will be time to consider the question 'does Cuba, in the possession of Spain, seriously endanger our internal peace and the existence of our cherished union.' Should this question be answered in the affirmative, then, by every law, human and divine, we shall be justified in wresting it from Spain if we possess the power.....We should be recreant to our duty, be unworthy of our gallant forefathers, and commit base treason against our posterity should we permit Cuba....seriously to endanger or actually to consume the fair fabric of our Union."

But anti-slavery opinion in the North was setting strongly against the slave power in its foreign as well as its domestic policy. The first republican platform in 1856 resolved that "the highwayman's plea that might makes right, embodied in the Ostend circular, was in every respect unworthy of American diplomacy and would bring shame and dishonor upon any government or people that gave it their sanction."

*House Ex. Docs., Vol. 10, No. 93; 2d Sess., 33 Cong., pp. 127-36.

The civil war destroyed the slave power and the desire to acquire territory for slave purposes. The doctrine devised by Polk in the interest of slavery seemed to be dead. But now after nearly half a century it is revived in the interest of foreign commerce. It suggests an old epigram:

"To kill twice dead a rattlesnake,
And off his scaly skin to take,
And through his head to drive a stake,
And every bone within him break,
And of his flesh mincemeat to make,
To burn, to sear, to boil and bake,
Then in a heap the whole to rake,
And over it the besom shake
And sink it fathoms in the lake—
Whence after all, quite wide awake,
Comes back that very same old snake."

The Polk doctrine is an encroachment upon the rights of foreign states. This fact is so clear that the wonder is that it does not appeal to every one the moment it is stated. The explanation perhaps is that frequent repetition secures its acceptance much as we incline to believe a false report that is often repeated. The first and most fundamental doctrine of international law asserts the sovereignty, independence and equality of states. They are sovereign in the regulation of their internal affairs, independent of interference in their relations with other states and equal in rights. This is precisely the doctrine stated by John Quincy Adams,* when urging the declaration in the cabinet meeting.

"Considering the South Americans as independent nations," he said, "they themselves and no other nations have the *right* to dispose of their condition. *We* have no right to dispose of them, either alone or in conjunction with other nations. Neither have any other nations the right of disposing of them without their consent."

From equality of rights results a corresponding equality of obligations. The same rights belong to all—the same duties rest upon all—the greatest as well as the smallest, the strongest as well as the weakest. Strength confers no privileges and weakness grants no exemptions. If the weak state injure the strong one, it must make reparation. It is the duty of the strong state to seek it peaceably, it is her right to secure it forcibly if necessary.

In 1854 the people of Greytown, Nicaragua, insulted the American minister and destroyed American property. The United States sent a war-ship there and, failing to secure an indemnity, bom-

*"Memoirs," Vol. 6, p. 168.

barded the town. Lord Palmerston, at that time prime minister of England, in referring to the incident in Parliament, said:

"We may think that the attack was not justified by the cause which was assigned. But we have no right to judge the motives which actuated other states in vindicating wrongs which they supposed they had sustained."*

In 1855 the United States became involved in a controversy with Paraguay, in which justice appears to have been largely upon the side of the weaker state. Reparation was demanded and refused. Thereupon President Buchanan sent a fleet of nineteen vessels, which forced an apology and the payment of an indemnity. In 1890 we threatened Venezuela with force in order to collect a private claim and in 1892 we threatened Chili with war to secure an apology for an injury. No European power interfered at any time to protect the weaker state.

In 1894 the authorities at Bluefields, Nicaragua, insulted the British consul there and a mob destroyed the consulate. Great Britain demanded an indemnity of the Nicaraguan government and proposed, in default of payment, to take possession of the port of Corinto and collect the duties there until the amount claimed was realized. Immediately the American press raised the cry of "Monroe Doctrine" and in effect denied the right of Great Britain to resort to the same measures of redress in her intercourse with independent states which we had many times employed in similar cases. We might have said as Lord Palmerston did of the Greytown bombardment that we did not think the punishment was justified by the cause assigned but we were bound to add as he did, that "we had no right to judge the motives which actuated other states in vindicating wrongs which they supposed they had sustained." To deny to foreign nations the same modes of redress that we employ ourselves is an encroachment upon their sovereignty, a violation of their independence and a denial of their equality.

In 1861 the United States was confronted with the most stupendous insurrection ever organized. The rebellion began in South Carolina in December of 1860. By the 8th of February, 1861, seven states had seceded and organized an independent government as complete in all respects as was the Union government. They were subsequently joined by four more states making eleven in all, exactly one-third of the total number at that time and including nearly a third of the area and population of the Union. For five months after the beginning of this rebellion no effort was made to

*Wharton's "Digest," Vol. 2, p. 596.

check or suppress it. It was for a time even doubtful whether such an attempt would be made at all. The first conflict of arms took place in April. The President of the United States immediately called for seventy-five thousand volunteers and declared a blockade of the seceded states. A war was immediately prepared, the most regularly equipped, the most regularly conducted and the greatest of modern times. In May and June European states issued proclamations of neutrality, recognizing the fact of war and the belligerency of the parties. We considered these proclamations an unjustifiable interference in our internal affairs and an evidence of great unfriendliness and made them for years the subject of a claim for damages against a foreign state.

In the neighboring colony of a friendly state there has raged for some time an irregular guerilla war. The government of the insurgents does not approach in completeness the government of the Confederate states. It has not a tenth part of the equipment, of the regularity, or of the prospect of success that the Confederates had. And yet it is seriously proposed that we recognize these insurgents as belligerents and advise Spain to grant them independence, on the ground that she can never conquer them. In what temper would the Union government have received such advice in 1861? Interference in the affairs of foreign states, which we resent when applied to ourselves, is an encroachment upon their sovereignty, a violation of their independence and a denial of their equality.

According to well settled rules of international law, interference in the affairs of independent states is justified in only two cases: first, when demanded by self preservation and second, when necessary to prevent the commission by a government upon its subjects of crimes repugnant to humanity. The protest of President Monroe came well within the first case. It is difficult for us now to realize the comparative weakness of the United States in 1823. We had at that time a population of less than ten million people sparsely settled over a wide area. Within ten years we had come out of a war with a single European power badly beaten and glad to make peace without mention of the causes of the contest. The establishment by powerful European states of new colonies upon our borders would have been a menace to our peace and safety. The subjugation of South American states by an European alliance acting in the interest of Spain would in principle have justified the conquest of the United States by a similar alliance acting in the interest of Great Britain. The circumstances justified the protest.

Very different are the recent cases. In no one of them is there any menace to our national existence. We have no right of interference, upon the same principle of law that an individual has no standing in a controversy in which his rights are not involved. The fact that states are located in the Western hemisphere gives us no protectorate over them. Much of Europe is actually nearer to us than many South American states and all of Europe is more easily accessible than any of them. International law knows no North, no South, no East, no West. The rights and duties of states are the same everywhere. The assertion by the President that an extension of the boundary of British Guiana is dangerous to our peace and safety is an absolute absurdity. And yet, so far as I am informed, only three newspapers in the United States had the courage to say so. The only other protest came from a few college professors, who in the popular view, by reason of the special study of particular questions, become thereby incapacitated for forming intelligent opinions respecting them. These few protests were met by crushing charges: their authors were dudes and Anglomaniacs and turned up their trousers when it rained in London. And now the government has come to the college professors because no one else can read the documents upon which rests the settlement of the questions involved. Two members of the Venezuelan commission are college presidents and former professors of history and the actual study of maps and manuscripts is being carried on by Mr. Winsor, the librarian of Harvard, Professor Burr of Cornell and Professor Jameson of Brown University. I am bound to say that the moderation of Great Britain in view of our repeated interference in her affairs is truly remarkable. I do not believe that the American people would for a moment brook a similar interference by any European state in matters that concern ourselves exclusively.

The case of Cuba affects us more nearly. We cannot but sympathize with the insurgents, struggling for liberty and independence, but we have no interest that justifies interference. The interest of Great Britain in our civil war was far greater, for the blockade closed her factories and caused widespread distress and actual starvation. It is reported that the contest in Cuba is waged with great cruelty, with the use of poisonous and explosive bullets, with summary trials and barbarous executions, storming of hospitals and massacre of non-combatants, but the evidence does not show that the cruelty is much greater on one side than on the other. "As for a state's having the vocation to go forth like Hercules,"

says President Woolsey,* "beating down wickedness, all over the world, it is enough to say that such a principle, if carried out, would destroy the independence of states, justify nations in taking sides in regard to all national acts and lead to universal war."

A doctrine which claims a right to interfere in controversies between other states or in their internal affairs, when our national existence is in no way imperiled or even remotely involved, is a violation of international law and an encroachment upon the rights of foreign nations.

The Polk doctrine is a menace to our peace and safety. A state that interferes in matters that do not concern her does so at her peril. Especially dangerous are alliances with states so unstable and changeable as those of Central and South America. Their internal affairs are in a state of confusion. Under the forms of republican institutions their governments are in fact a succession of military dictatorships—despotisms tempered by revolution. Within a period of forty years Mexico had nearly forty revolutions and more than seventy presidents. The history of the other states is very similar. So precarious are the lives of their statesmen that a right of asylum in foreign legations is admitted in all of them upon the ground that otherwise experienced men could not be induced to engage in affairs of government.† They are continually involved in wars with each other. Their wholesale repudiation of their debts continually embroils them with Europe. The government of to-day may be overthrown to-morrow. They ask our assistance only when involved in controversies with other states. At other times they reject our advice and repel our advances. Such protection is a thankless and fruitless task. Connection with them may at any time render us responsible for acts that we cannot control. Connection with one of them recently threatened a war in which we had no interest involved or principle at stake, a war with a state to which we are bound by ties of common blood, common language, common literature and common history, a war that would have caused incalculable loss and misery, a war that would have arrested the progress of the world for a decade and disgraced the closing years of the century. Let us take warning from experience and renounce a policy fraught with so much danger to our peace and safety.

The so-called Monroe doctrine is, therefore, contrary to the teaching of the founders of the republic, a perversion of the true

*"International Law," 6th ed., p. 19.
†Wharton's "Digest," Vol. 1, p. 693.

meaning of the original declaration, an encroachment upon the rights of foreign nations and a menace to the peace and safety of our own, and it is the duty of the scholar to impress these facts upon the people through the press, in the pulpit and on the platform.

I come now to the second danger that threatens our national peace—the existence of a rising war spirit among the people. I do not by any means believe that such a spirit has become general but it has infected considerable numbers and unless checked may at any time get the upper hand. I attribute this spirit in large part to the influence of the younger men who are rapidly gaining control of public and private affairs. The older men have retained control longer than usual by reason of the prominence and claims that service in the civil war gave them. They are now passing rapidly away and their places are being filled by the generation that has grown to manhood since the war. This change is accompanied by a rise of war spirit, much as the same spirit arose during the first half of the century at the passing of the men of revolutionary times.

One cause of this spirit is to be found in a desire to extend our territory. In Europe in recent times there has been a revival of activity in colonization, indicated by the occupation of the minor islands of the Pacific and the conquests of England and Germany, France and Italy in various parts of Africa. The principal motive of this movement has been a desire to find an outlet for surplus population without incurring the loss that emigration of that surplus to the United States involves. The American people have caught the infection without having the same reason for it. The result is a revival of the doctrine that it is the manifest destiny of the United States to acquire control of the whole continent. This doctrine is illustrated by an anecdote told of a dinner given by the Americans residing in Paris during the civil war. The first speaker proposed the toast: "The United States, bounded on the North by British America, on the South by the Gulf of Mexico, on the East by the Atlantic and on the West by the Pacific Ocean." "But," said the second speaker, "this is far too limited a view of the subject. Why not look to the great and glorious future which the manifest destiny of our race prescribes for us? Here's to the United States, bounded on the North by the North Pole and on the South by the South Pole, on the East by the rising and on the West by the setting sun." "If we are going," said the third speaker, "to leave the present and take our manifest destiny into

account, why restrict ourselves within the narrow limits that have just been assigned? I give you the United States, bounded on the North by the Aurora Borealis, on the South by the precession of the equinoxes, on the East by primeval chaos and on the West by the Day of Judgment."

The revival of this spirit is indicated by the frequent recurrence of articles in the magazines advocating the annexation of Canada, by a very general desire not long since for the acquisition of the Hawaiian Islands, by a strong feeling in some quarters at the present time for the occupation of Cuba and by the demand sometimes heard that we make the Isthmus canal our southern boundary. Such exuberance and enthusiasm are natural to youth. The fact seems scarcely to be considered that nearly every one of these measures involves war. I do not mean to disparage the importance of our vast extent of territory and of our boundless resources, a just source of pride to every patriotic American. The annexation of both Texas and California has been productive of incalculable good to us and to the territory involved but that does not justify the mode and motive of their acquisition. We ought not to acquire more territory by war and conquest. We ought not to annex islands so far removed from our present boundaries that a great and expensive navy would be necessary for their defense, costing more than the value of their total product. And we ought not to acquire territory of which the population is unfit to constitute a state in the Union. Quality is more important than quantity; domestic peace more valuable than foreign commerce.

A second cause of the war spirit is to be found in the existence of deep seated prejudices against particular nations, prejudices unreasoning and unreasonable. The strongest of these prejudices is directed against England. This is in part a survival of the passions of the revolution. Aversion to England and partiality to France were potent factors in our domestic politics from the revolution to the war of 1812. So strong indeed was their influence that a foreign observer was led to remark that "he found in the United States, many French and a few English but no Americans." Rightly understood the revolution furnished little reason either for hatred of England or gratitude to France. At least after the lapse of a century and especially as we were victorious, we can afford to be magnanimous. The English do not cherish the same resentment against us. An Englishman once said to me: "We don't bear you any grudge, you know, for beating us in the revolution. We are proud of you. It is just what we would have done in your place."

And I believe that this remark is characteristic of the feeling of the English people. Prejudice against England was revived by the events of our civil war. There was in truth far greater reason for hatred of France, whose government on the one hand continually urged Great Britain to interference and to a joint recognition of Southern independence and on the other tried to turn our distracted condition to her own advantage by establishing an empire in Mexico. The existence of what is called the Irish vote tends to perpetuate this prejudice and enables politicians to make capital by trading upon the passions of the people. Here again we cannot do better than turn to the advice of Washington's farewell address:

"Nothing is more essential than that permanent, inveterate antipathies against particular nations and passionate attachments for others should be excluded and that in place of them, just and amicable feelings toward all should be cultivated.....Antipathy in one nation against another disposes each more readily to offer insult and injury, to lay hold of slight causes of umbrage and to be haughty and intractable when accidental or trifling occasions of dispute occur....Hence frequent collisions and obstinate, envenomed and bloody contests. The nation, prompted by ill-will and resentment, sometimes impels the government to war contrary to the best calculations of policy. The government sometimes participates in the national propensity, and adopts through passion what reason would reject. At other times, it makes the animosity of the people subservient to projects of hostility, instigated by pride, ambition and other sinister and pernicious motives. The peace often, sometimes even the liberty of nations, has been the victim."

A third cause of the war spirit may be found in an extreme sensitiveness and a disposition to resent anything that looks like injury before the actual facts are known. The conduct of foreign relations is undoubtedly a weak point in republican institutions. Formerly they were considered the exclusive affair of government, diplomatic correspondence was secret and time was allowed for explanation or apology before definite action was threatened or taken. Now all public questions are discussed in the forum of the people and upon the first rumor of insult or injustice there arises a demand for instant apology and a threat of war. Governments like individuals dislike the appearance of yielding to pressure and a premature resort to it diminishes the chances of accommodation. The danger is that popular excitement may precipitate an unnec-

essary conflict. Fortunately the government has proved more moderate than the people and the danger so far has been avoided.

Nations have the rights of individuals and the same duties rest upon them—among others the duty of moderation.

"It not infrequently happens," says General Halleck,* "that what is, at first, looked upon as an injury or an insult is found, upon more deliberate examination, to be a mistake rather than an act of malice or one designed to give offense. Moreover the injury may result from the acts of inferior persons, which may not receive the approbation of their own governments. A little moderation and delay, in such cases, may bring to the offended party a just satisfaction whereas rash and precipitate measures may often lead to the shedding of innocent blood."

I would not abate one jot or tittle of our just rights but I would counsel moderation, a postponement of judgment until all the circumstances are known, an avoidance of irritating and insulting charges, a resort to peaceful measures of redress and above all no talk of war until it shall appear that war is necessary to save national honor. "He that is slow to anger is better than the mighty and he that ruleth his spirit than he that taketh a city."

The last and most important cause of the war spirit is to be found in the fact that the new generation have never known the horrors of war and are ignorant of its true character.

"Art and literature," says a recent writer on international law,† "combine to help on the work of slaughter. Poets and painters celebrate the 'pomp and circumstance of glorious war' till people come seriously to regard it as a thing of bands and banners, of glittering uniforms and burnished steel, of deeds of heroic daring and examples of lofty self-sacrifice. They forget the stern realities of cold and hunger, wounds and death, the shattered limbs, the fever thirst, the fiendish passions of cruelty and lust. They forget the demoralization it causes among both victors and vanquished and the widespread ruin that follows in its train. In the twenty-five years betwen 1855 and 1880 over two million men died in wars between civilized powers."

In our own civil war, upon the Union side alone, out of three hundred and fifty thousand dead, only sixty-seven thousand were killed in battle. Two hundred thousand died of disease, forty-three thousand died of wounds and forty thousand from accident, murder, execution, starvation or abuse. Thirty thousand one hun-

*"International Law," 3d ed., Vol. 1, p. 463.

†T. J. Lawrence, "Essays on Modern International Law," 2d ed., pp. 242-4.

dred and fifty-six Union soldiers died in Southern prisons and thirty thousand one hundred and fifty-two Confederate soldiers died in Northern prisons, within four of the same number on both sides.

"Who can calculate," says the same writer, "the awful mass of human misery that these figures represent?....Comparatively few of those that perish die upon the battle field. Thousands succumb from sheer exhaustion, having endured for weeks, perhaps months, the slow agony of failing strength, under the influence of privation and over-exertion. Thousands die of disease, many of them for want of the commonest comforts of the sick. Starvation demands one host of victims, fever another, neglected wounds a third. Vice of all kinds preys upon the soldiery and exacts its terrible toll of moral and physical ruin. Even well appointed and victorious armies melt away under the influence of sickness and fatigue unless constantly reinforced. What then must be the case with a broken or retreating army, an army separated from its supplies or cooped up in a beleaguered fortress? Let the three hundred thousand French soldiers, whose bones strewed the plains of Russia from Moscow to the Niemen provide the answer. Read in the history of a more recent period how a British army was destroyed by cold and privation in the trenches before Sebastopol, while transports rocked idly in the harbor of Balaclava, almost within sight of the starved men dying like flies for want of the comforts they contained. Consult English papers for the condition of the hospitals at Plevna, when the Russians entered the town and found the wounded with broken and unset limbs twisted out of all human recognition. In records such as these you will read the true history of war. No one acquainted with them can deny that much remains to be done to correct popular ideas and sentiments on the subject. There must be a great change in the ordinary modes of thinking and speaking of war before current opinion in regard to it conforms to the standard of Christianity."

It is not death alone that makes war terrible. Worse than dead are the wrecks of men, maimed in body and shattered in mind, who live afterward, a curse to themselves and a burden to their friends. No account has yet been taken of the suffering at home. Think of the three hundred and fifty thousand dead in our last war on the Northern side alone and then think of the thousands of mothers left childless, the thousands of wives left husbandless, the thousands of children left fatherless, the heart-burnings and heart-breakings it caused, and *then* talk lightly and wantonly of war.

"The real sorrows of war," says George Cary Eggleston,* in speaking of the South, "fall most heavily upon the women. They may not bear arms. They may not even share the triumphs which compensate their brothers for toil and suffering and danger. They must sit still and endure. The poverty which war brings to them wears no cheerful face but sits down with them to empty tables and pinches them sorely in solitude. After the victory the men who have won it throw up their hats in glad huzza, while their wives and daughters await in sorest agony of suspense the news which may bring hopeless desolation to their hearts."

I have heard men say that war would be a good thing, it would raise prices and make trade brisk. Truly when such remarks can be made, much remains to be done to correct popular ideas and sentiments upon the subject of war. The duty to do this rests upon those who know and feel the evil. It rests upon all alike, teachers in the schools and professors in the colleges, writers for the press and preachers in the churches, men of business on the street and statesmen in the halls of legislation. Lord Derby has said: "The greatest of England's interests is peace." Let us echo the sentiment: The greatest American interest is peace.

I come now to the third danger that threatens our national peace —enormous expenditure for war purposes. This expenditure, as Dunning said of the influence of the crown, "has increased, is increasing and ought to be diminished." The possession of great force is a standing temptation to use it.

It has been common for great men to give accounts of their early intellectual development and of books that have helped them. I see no reason why it may not also be permitted to small men to acknowledge their indebtedness to the influences that have moulded their opinions. In the library of the school where I received my training preparatory for college, there was a copy of the "Speeches and Addresses of Charles Sumner," which I often used to read when supposed by my instructors to be studying Latin or Algebra. The first speech in that collection made a powerful impression upon my mind. It was entitled "The True Grandeur of Nations," and defended the proposition that in our age there can be no peace that is not honorable, and no war that is not dishonorable. The oration was delivered on the fourth of July, 1845, before the city corporation of Boston. Mr. Sumner was himself a notable example of the scholar in politics—not always right, to be sure, but always honest and honorable. This speech was his first public appearance, the beginning of his public

*"A Rebel's Recollections," 3d ed., p. 58.

career. I desire to quote the passage,* which, according to the testimony of those present, made the strongest impression upon his hearers:

"Within cannon range of this city stands an institution of learning which was one of the earliest cares of our forefathers. Favored child in an age of trial and struggle—carefully nursed through a period of hardship and anxiety—sustained from its first foundation by the paternal arm of the commonwealth, by a constant succession of munificent bequests and by the prayers of good men—the University of Cambridge now invites our homage as the most ancient, most interesting and most important seat of learning in the land.....It appears from the last Report of the Treasurer, that the whole available property of the University, the various accumulations of more than two centuries of generosity, amounts to $703,000."

"Change the scene and cast your eyes upon another object. There now swings idly at her moorings in this harbor a ship of the line, the Ohio, carrying ninety guns, finished as late as 1836 at an expense of $835,000—more than $130,000 beyond all the available wealth of the richest and most ancient seat of learning in the land. Choose ye, my fellow citizens of a Christian state, between the two caskets,—that wherein is the loveliness of truth, or that which contains the carrion death."

"Pursue the comparison still further. The expenditure of the University during the last year amounted to $48,000. The cost of the Ohio for one year of service, in salaries, wages and provisions is $220,000, being $172,000 above the annual expenses of the University and more than four times as much as those expenditures. In other words, for the annual sum lavished upon a single ship of the line, four institutions like Harvard University might be supported."

A similar comparison between the cost of a modern warship and a modern University would be interesting, were the material at hand for making it. The average cost in recent years of a large man-of-war, without armament, has been over three million dollars. There have recently been added to our navy six battle ships—the Indiana, the Iowa, the Maine, the Massachusetts, the Oregon and the Texas, and two armored cruisers—the Brooklyn and the New York. Their total cost, making allowance for armament, is twenty-five million dollars. This amount exceeds by ten million dollars the total income of the four hundred and seventy-six colleges and universities in the United States to-day and at the present rate would defray the current

*Sumner's "Works," Vol. 1, pp. 80-2.

expenses of the University of Kansas for a period of two hundred and fifty years. And yet this twenty-five million is but a fraction of the total expenditure for war purposes which during the last five years has amounted to four hundred and twelve millions,* an average of over eighty-two millions a year—and the present Congress has surpassed all its predecessors in extravagance and voted the largest appropriations ever made and ordered the largest number of battle ships ever provided for at a single time—and all this in a period of peace abroad and commercial depression at home, with an enormous deficit in the national treasury and with widespread distress every winter in all our large cities, that has required for its relief an organization of charities hitherto unknown. Is it not time to call a halt in this enormous waste of wealth? Is there not some missionary work for educated men and women to do here at home in the way of arousing and civilizing public opinion upon this subject? "Let us," says General Walker,† "frown indignantly upon every proposed measure, upon every representative vote, upon every word of every man, whether in public or private speech, which assumes or gives countenance to the assumption that this people are to come under the curse of the war system or which threatens our friendly relations with any power on earth. Sixty-five millions, transcending in all the elements of industrial, of financial and, if you please, of military strength, the combined resources of any two of the greatest nations of the world, who shall molest us or make us afraid, who shall be so insane as to wantonly attack the greatest power on earth? Why then should we enter upon that career of competitive armament into which mutual jealousies and mutual fears have driven the nations of Europe—a career which once entered upon, has no logical stopping place short of complete exhaustion, impoverishment and financial bankruptcy and which in its turn finds that it has earned nothing but to be the object of universal dread and universal detestation? ....Let it then be our pride as it is our privilege to remain the great unarmed nation, as little fearing harm from any as desiring to wrong any. Let us follow the paths of peaceful, happy industry, developing the resources with which nature has so bounteously endowed us, reserving our giant strength for those competitions whose results are mutual benefits, and bestowing upon schools and colleges, libraries and museums, public parks and institutions of beneficence that wealth which others waste on frontier fortresses and floating castles."

*"Statistical Abstract of the United States," No. 18, p. 22.

†"The Growth of the Nation," an Address at Brown University, June 18th, 1889, printed in the Providence "Journal."

Editorial Notes.

The University of Pennsylvanvia sends out a handsome collection of the addresses at the opening of the recently purchased Bechstein Germanic Library. With this purchase—15000 volumes and 3000 pamphlets—the University of Pennsylvania at one step takes front rank among American universities for students of Germanic languages.

Dr. Geo. I. Adams, late assistant on the University Geological Survey, and a Fellow of Princeton College, has printed the substance of his Dissertation in the American Journal of Science, under the tittle "The Extinct Felidæ of North America." Some reference to the paper is made in an article in this number of the QUARTERLY.

THE UNIVERSITY GEOLOGICAL SURVEY OF KANSAS, Volume I, has been issued from the office of the state printer. The work is conducted by Prof. E. Haworth assisted by J. Bennett, G. D. Adams, M. Z. Kirk, E. B. Kneer and J. G Hall. The report consists of sections, mostly in the Eastern portion of the state, with reports on certain borings, and particular deposits, as coal and salt. A vast amount of useful information has been accumulated. It is illustrated by thirty-one plates, eleven figures, and occupies 310 pages. Volume II is in preparation.

The fifth volume of the Collections of the Kansas State Historical Society, which has just been published, contains nearly 700 pages, and is a well-printed book. It contains most of the addresses delivered before the society during the past six years, including the address of Rev. Doctor Cordley, on the Convention Epoch in Kansas History; that of Col. C. K. Holliday, on the Freemont Campaign of 1856; of Hon. James S. Emery, on History and Historical Composition; of Dr. Peter McVicar, on School Lands on the Osage Indian Reservation; of W. H. T. Wakefield, on Squatter Courts in Kansas; Mrs. Lois H. Walker's Reminiscences of Early Kansas Times; C. H. Dickson's Reminiscences of 1855; Hon. J. R. Mead's Trails in Southern Kansas; Hon. P. G. Lowe's account of Army Service on the Plains in 1852; memorial proceedings on Col. William A. Phillips; Hon Albert R. Greene's account of the Battle of Wilson Creek; Prof. O. E. Olin's Romance of Kansas History; Hon. John Speer's Incidents of Pioneer Days; Doctor Cordley's discourse on Judge S. O. Thacher; and Gov. Morrill's address at the annual meeting of the society, last January, on the Trials, Privations, Hardships and Sufferings of the Early Kansas Settlers. Besides, this volume contains a large fund of documentary historical materials pertaining to the troublesome times in early Kansas, including the official papers of the period of the administration of Governors Robert J. Walker, James W. Denver, and Samuel Medary, and of Acting Governors Frederick P. Stanton, Hugh S. Walsh, and Geo. M. Beebe. These papers for the most part have been lying hidden in the archives of the department of state, at Washington, during a period of over 36 years. At the personal request Hon. R. W. Blue. Secretary Olney directed a search to be made, which resulted in securing copies of these records. The documents complete the publication of the entire documentary history of the period of the Kansas territorial government from 1854 to 1861, the papers of former administrations having been published in the third and fourth volumes of the Historical Society's Collections.

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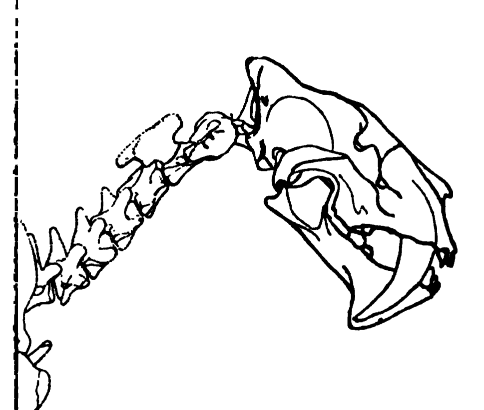
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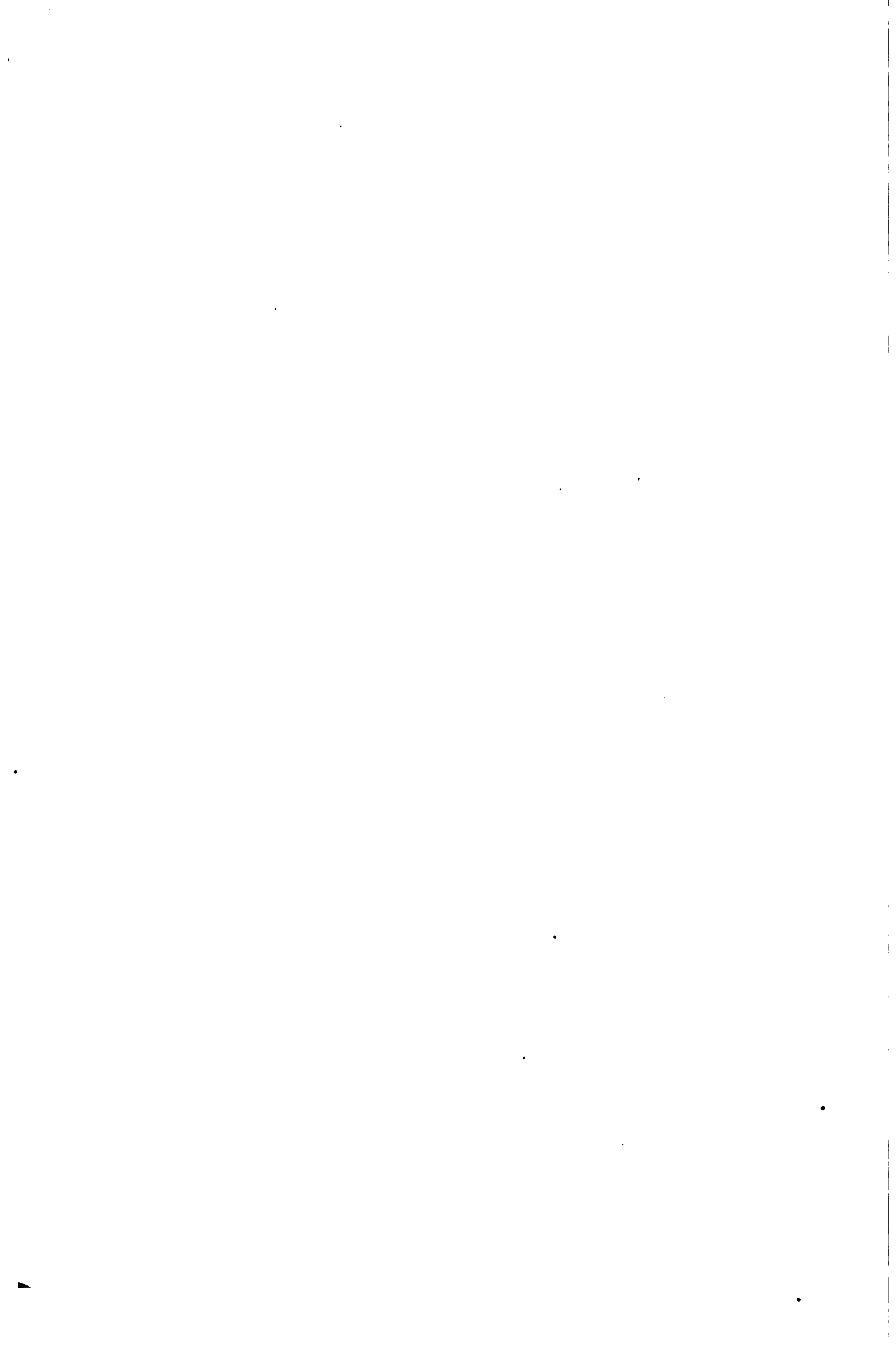
PLATE I.







DERMAL COVERING OF HESPERORNIS.
Enlarged about one-fourth.



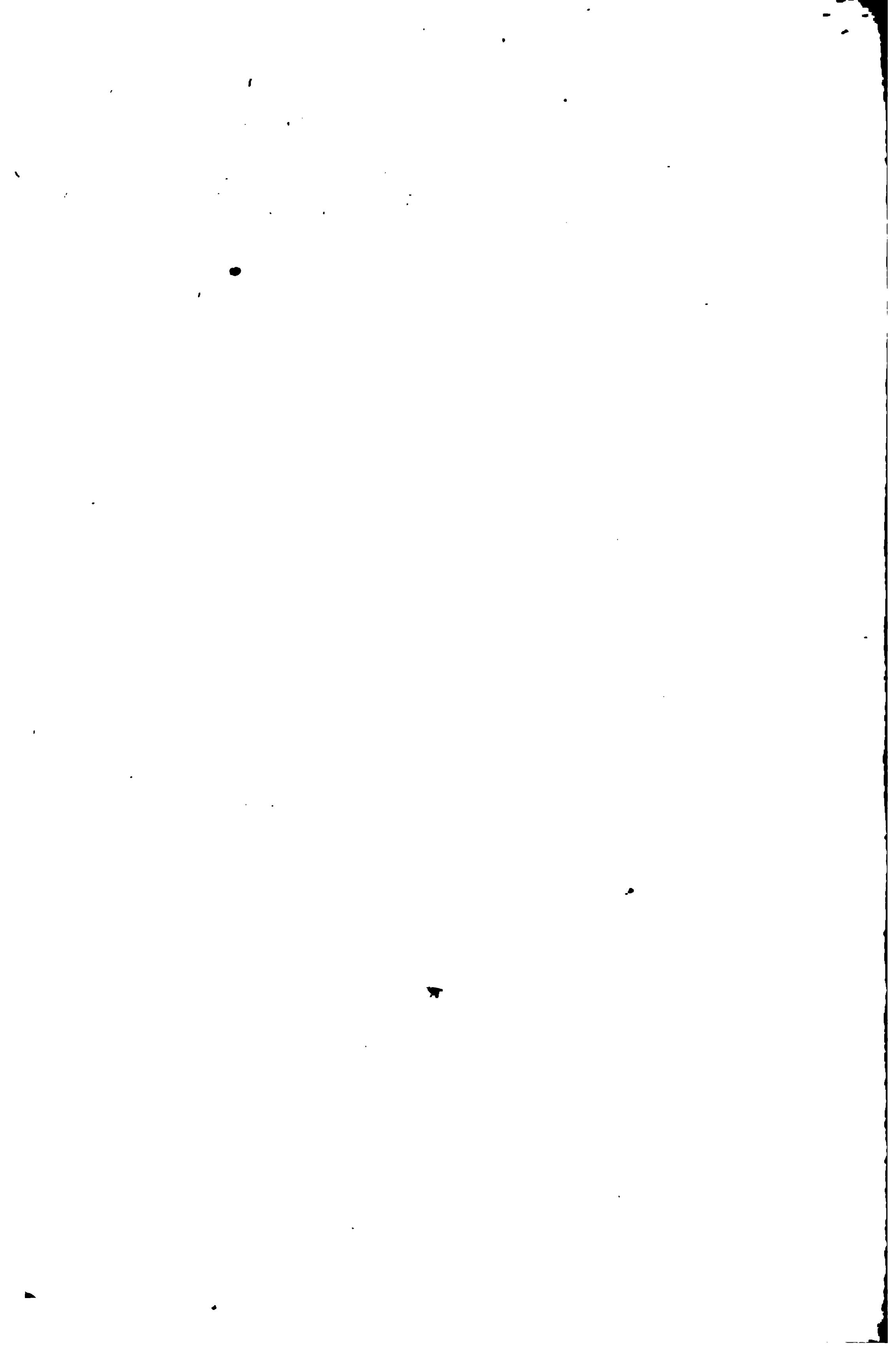
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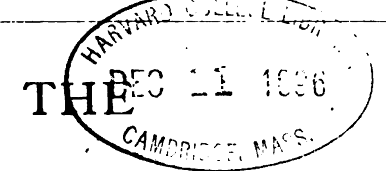
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Vol. V. OCTOBER, 1896. No. 2.



THE KANSAS UNIVERSITY QUARTERLY.

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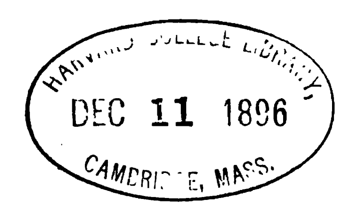
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KANSAS UNIVERSITY QUARTERLY

VOL. V. OCTOBER, 1896. NO. 2.

Continuous Groups of Projective Transformations Treated Synthetically.

BY H. B. NEWSON.

*Part II Continued.

§2 Groups in the Plane.

I have defined a projective transformation in a plane in the sense in which the the term will be used in this paper, and have given a simple method of constructing it. Having given four points A, B, C, D, no three of which are in the same straight line, we may choose as their corresponding points A′, B′, C′, D′; thereby a projective transformation T of the plane is completely determined such that any point P is transformed into a definite point P′. If now we choose four other points A″, B″, C″, D″, as the corresponding points to A′, B′, C′, D′, we would have obtained a projective transformation T_1 transforming P directly to P″. It is clear that two transformations T and T_1 together produce the same effect as T_2. Thus it may be shown in general that any two projective transformations of the plane are together equivalent to some third. Therefore all the projective transformations of the plane form a Continuous Group of Transformations.

The number of projective transformations in the plane is likewise determined from the same considerations. Having given four points A, B, C, D, a transformation is determined when their corresponding points are chosen; and there are as many transformations of the plane as there are sets of four points in a plane. Since the plane contains ∞^2 points, we easily see that there are ∞^8 such sets of points and hence there are ∞^8 projective transformations in the plane.

Another method of determining the number of projective transformations in the plane leads to the same result. From the method

of constructing a projective transformation referred to above, we see that any two conics touching a line l determine a projective transformation of the plane. Since the number of conics touching the line is ∞^4, the number of pairs of such conics is ∞^8 and hence there are ∞^8 projective transformation of the plane obtained by taking any line l as the fixed line of the construction. The line l was taken as the line of intersection of the two planes π and π', and in developing the construction of one projection of the plane upon the other the angle between the two planes was not considered. By making the planes π and π' intersect in some other line as l, we get another system of transformations which must be identical with the first system. If the angle between the two planes in the last position is not the same as in the first position, the transformations of the two systems will not be in the same order, but no new transformation will be introduced. We therefore infer that there are only ∞^8 projective transformations in the plane.

The group of the projective transformations of the plane will be called the General Projective Group and will be designated by the symbol G_8.

Theorem 4. There are ∞^8, projective transformations of the plane; these form the General Projective Group G_8 whose fundamental property is that any two transformations of the group are together equivalent to some third transformation belonging to the same group.

(For Lie's analytical proof see "Cont. Gruppen," Kapitel 2, §1.)

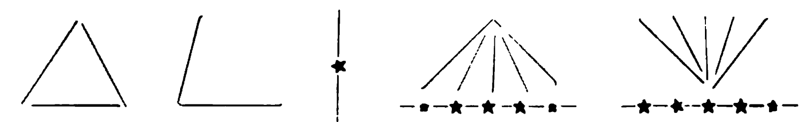
Every projective transformation of the plane leaves some line or lines and some point or points of the plane unaltered in position, or as we say, invariant. There are five types of these transformations, distinguished according to the kind of plane figure which is left invariant. (See Vol. IV, page 248 K. U. Q. and "Cont. Gruppen" page 35-6). If two transformations T and T_1 both leave any plane figure invariant, e. g. a line l, the transformation T_2 which is equivalent to the combination of T and T_1 must also necessarily leave l invariant. Thus considering the totality of transformations which leave l invariant, we see that the combination of any two transformations of the system are together equivalent to a third transformation of the same system. Hence the totality of transformations leaving a line invariant have the group property and form a sub-group of the general projective group. The same reasoning applies in general to the system of transformations leaving invariant any plane figure whatever.

Theorem 5. All projective transformations of the plane leaving a plane figure invariant have the group property and form a sub-

group of the general projective group. (See "Cont. Gruppen," page 113.)

By means of this theorem many of the sub-groups of the general projective group can be readily determined.

It will be convenient to have separate symbols to designate each of the five types of transformations referred to above. We shall represent the five types of transformations whose invariant figures are

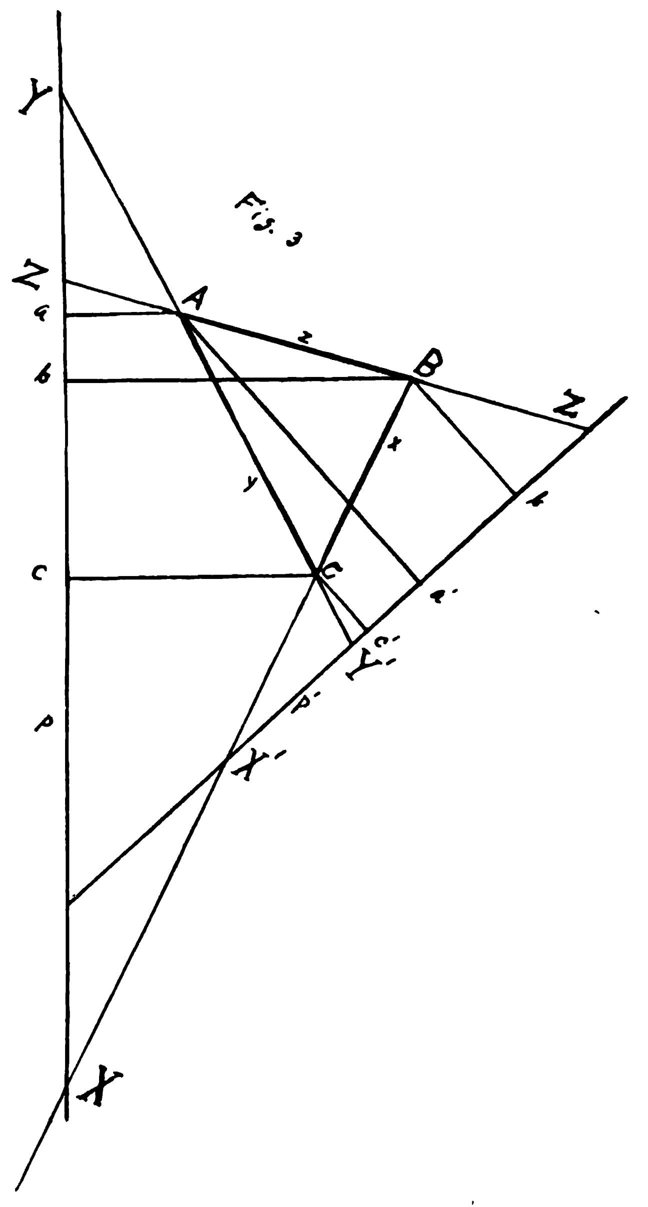


by T, T′, T″, S, S′, respectively.

We shall now consider more in detail these different types of transformations, beginning with the most general case (type 1) whose invariant figure is a triangle. Let the vertices of the triangle be represented by A, B, C; and the opposite sides by x, y, z, respectively. By means of a transformation T the line x is transformed into itself in such a way that the points B and C on it are invariant points of the transformation. Now we know that the one-dimensional transformation of the points on a line, which leaves two points of the line invariant, is characterized by the constant anharmonic ratio of the invariant points and any pair of corresponding points. (Kansas University Quarterly, Vol. IV., page 74.) Let k_x be the characteristic anharmonic ratio of the one-dimensional projective transformation along the line x. In like manner we have projective transformations of one dimension along each of the invariant lines y and z. We shall call their characteristic anharmonic ratios k_y and k_z respectively. In reckoning these anharmonic ratios the points will be taken always in the same order around the triangle. Thus we see that every projective transformation of the kind T in the plane determines three characteristic anharmonic ratios along the three invariant lines. It is also evident that the pencil of lines through the vertex A of the invariant triangle is transformed into itself in such a way that the rays AB and AC are invariant rays of the transformation. Also the anharmonic ratio of the invariant rays and any pair of corresponding rays of the pencil is constant for all pairs of corresponding rays; this anharmonic ratio is equal to k_x, the characteristic anharmonic ratio along the opposite side x. Similar considerations apply to the pencils of rays through the invariant points B and C.

We shall now proceed to show that these three anharmonic

ratios are not independent but are connected by a very simple relation. Let p and p′ (fig. 3) be a pair of corresponding lines in



the plane; and let p cut the lines x, y, z in the points X, Y, Z respectively; and let p′ cut the same lines in X′, Y′, Z′ respectively.

Since k_z=(ABZZ′); k_x=(BCXX′); and k_y=(CAYY′); we have

$$k_z=\frac{AZ}{BZ}:\frac{AZ'}{BZ'};\ k_x=\frac{BX}{CX}:\frac{BX'}{CX'};\ \text{and}\ k_y=\frac{CY}{AY}:\frac{CY'}{AY'}.$$

But by similar right triangles we have

$$\frac{AZ}{BZ}=\frac{Aa}{Bb};\ \frac{AZ'}{BZ'}=\frac{Aa'}{Bb'};\ \frac{BX}{CX}=\frac{Bb}{Cc};\ \frac{BX'}{CX'}=\frac{Bb'}{Cc'};\ \frac{CY}{AY}=\frac{Cc}{Aa};\ \frac{CY'}{AY'}=\frac{Cc'}{Aa'}.$$

Multiplying together and substituting we get

$$k_x\,k_y\,k_z=\frac{AZ}{BZ}.\frac{BZ'}{AZ'}.\frac{BX}{CX}.\frac{CX'}{BX'}.\frac{CY}{AY}.\frac{CY'}{AY'}$$

$$=\frac{Aa}{Bb}.\frac{Bb'}{Aa'}.\frac{Bb}{Cc}.\frac{Cc'}{Bb'}.\frac{Cc}{Aa}.\frac{Aa'}{Cc'}=1.$$

Theorem 6. Every projective transformation of the kind T in the plane determines a characteristic anharmonic ratio along each of the invariant lines and through each of the invariant points. When these three anharmonic ratios are reckoned in the same order around the triangle their product is unity.

Thus we see that of these three anharmonic ratios only two are independent. Every transformation of T depends therefore upon 8 parameters, viz: the six co-ordinates of the three invariant points (or lines) and these two independent anharmonic ratios. Since each of these parameters may assume ∞^1 different values, we see again that there are ∞^8 transformations of the kind T in the plane.

We are also enabled to distinguish two distinct varieties of variable parameters, viz: co-ordinates of invariant points (or lines) and characteristic anharmonic ratios. This is an important distinction which will be of considerable use later on.

Theorem 7. Of the eight parameters which determine a transformation of the kind T six are coordinates of invariant points (or lines) and two are characteristic anharmonic ratios.

We proceed now to consider the system of transformations leaving a triangle invariant. In this case the six co-ordinate parameters are constant and the two anharmonic ratio parameters are variable; thus we see that there are ∞^2 transformations leaving a given triangle invariant. From another point of view we arrive at the same result. The two conics K and K^1 by means of which we can construct the transformation T touch four fixed lines l, x, y, z. K and K^1 therefore belong to a range of ∞^1 conics touching the same four lines. Any pair of conics taken from this range determines a transformation leaving the triangle (ABC) invariant. ∞^2 pairs of conics may be formed from this range, thus showing that there are

∞^2 transformations which leave the triangle invariant. By Theorem 5 these ∞^2 transformations form a two-termed group G_2 or G (ABC).

Since there are ∞^6 different triangles in the plane, it follows that there are ∞^6 such two-termed groups. Hence the general projective group G_8 is composed of ∞^6 two-termed groups G_2; thus $G_8 = \infty^6 G_2$. No two of these two-termed groups can have a transformation of the kind T in common; for if two transformations T and T_1 are identical the eight parameters of the one must be equal to the eight parameters of the other. Now the two anharmonic ratio parameters of one of the transformations may readily be equal to those of the other; but if the transformations leave different triangles invariant, all of the six co-ordinate parameters of the one can not be equal to those of the other. Hence T and T_1 can not be identical. (Later it will be shown that for particular positions of the triangles two or more groups G_2 may have common many transformations of the type S.)

Theorem 8. The general projective group G_8 is composed of ∞^6 two-termed sub-groups G_2. Each of these two-termed sub-groups has an invariant triangle. No two of these two-termed groups can have a transformation of the kind T in common.

We now proceed to show that there are other sub-groups of the general projective group G_8 that can be constructed out of these two-termed sub-groups of the type G (ABC). Suppose the vertex A of the triangle ABC to move along the side AC. It may assume ∞^1 different positions on the line and thus form ∞^1 triangles of the type A_nBC. To each of these triangles belongs a two-termed group of transformations. Consider any two transformations taken from different groups of this series. These two transformations both leave invariant the points B and C, and the lines x and y; and they are together equivalent to a third transformation which leaves the same figure invariant and therefore belongs to some one of these ∞^1 groups G_2. Thus we see that the ∞^3 transformations leaving the lines x and y and the points B and C invariant form a three-termed group G_3, which is made up of ∞^1 two-termed groups. Thus $G_3 = \infty^1 G_2$. It is easily seen that the general projective group G_8 contains ∞^5 such three-termed sub-groups. Two three-termed groups, whose invariant figures contain no geometric element in common, contain no transformation in common. But it is possible to chose the invariant figures so that they shall contain a common triangle; the two three-termed groups then contain a common two-termed group.

Theorem 9. The general projective group G_8 may be decomposed into ∞^5 three-termed sub-groups each of which has for invariant figure two lines, their point of intersection, and another point on one of these lines.

In a similar manner three-termed groups of the type just discussed may be put together so as to form four-termed groups; and this may be done in three different ways. First, suppose that the line y is made to revolve about the point C; it thus assumes ∞^1 different positions. Belonging to each of these positions is a three-termed group, and by the principle of Theorem 5 these form a four-termed group $G_{4.a}$, whose invariant figure is composed of two invariant points and the line joining them.

In the second place, the point B may be supposed to assume all positions on the line x; corresponding to each position of the point B is a three-termed group, and the totality of all these three-termed groups is a four-termed group $G_{4.b}$, whose invariant figure is composed of two invariant lines and their point of intersection.

Again the point C may be made to move along the line y; to each position corresponds a three-termed group, and the totality of all these three-termed groups is a four-termed group $G_{4.c}$, whose invariant figure consists of the invariant line y and the invariant point B not on the line y. These three types of four-termed groups are the only possible ones that can be compounded out of three-termed groups of the kind G_3. We shall designate these by the symbols $G_{4.a}$, $G_{4.b}$, $G_{4.c}$.

Theorem 10. There are three types of four-termed groups which may be compounded out of three-termed gronps of the kind G_3 (and hence out of two-termed groups of the kind G (ABC)). Their invariant figures are respectively two points and their join; two lines and their intersection; a line and a point not on the line.

If ∞^1 four-termed groups of the kind $G_{4.a}$ be taken such that their invariant figures have common the line x and the point C on x, these form a system of ∞^5 transformations all of which leave invariant the linear element x, C. Hence these form a five-termed group. Again, if we take ∞^1 four-termed groups of the kind $G_{4.b}$ such that their invariant figures have common the line x and the point C, we have the same five-termed group as before. But if we take four-termed groups of the kind $G_{4.c}$ we can not put them together so as to form a five-termed group. This kind of a five-termed group with an invariant linear element is the only kind that can be built up out of two-termed groups G (ABC). Two such five-termed groups will generally have a two-termed group in com-

mon; for the common invariant figure is a triangle. If the two lines or the two points of the linear elements coincide, the two groups have in common a four-termed group.

Theorem 11. The ∞^5 *transformations which leave a linear element invariant form a five-termed group. The general projective group contains* ∞^3 *such five-termed groups.*

If the point C be made to move along the line x, to each position of the point belongs a five-termed group. The sum total of the transformations belonging to all these five-termed groups forms a six-termed group whose invariant figure is a straight line. It is clear that to every line in the plane belongs a six-termed group of this kind. The general projective group therefore contains ∞^2 such six-termed groups.

In like manner if the line x be made to revolve around the point C, to every position of the line x belongs a five-termed group. These ∞^1 five-termed groups make up a six-termed group whose invariant figure is a point. The general projective group contains ∞^2 of these six-termed groups.

Theorem 12. The ∞^6 *transformations which leave a line or a point invariant form a six-termed group. The general projective group contains* ∞^2 *sub-groups of each kind.*

This completes the enumeration of the sub-groups of the general projective group, which can be built up out of the two-termed sub-groups of the kind G (ABC). We have a list of nine kinds of groups, as follows:

$$G_8;\ G_{6.p},\ G_{6.l};\ G_5;\ G_{4.a},\ G_{4.b},\ G_{4.c};\ G_3;\ G_2.$$

So far we have shown how to build up these groups of higher orders out of groups of lower orders. The reverse process might have been followed. We might have started with the general projective group and decomposed it into groups of lower orders. This we proceed to do briefly.

A transformation of the kind T is determined by eight parameters, six co-ordinate parameters and two anharmonic ratio parameters. When all eight of these vary they generate the general projective group. When two or more of the co-ordinate parameters are fixed quantities and the rest of them variables the various sub-groups are generated. In order that a point of the plane shall remain invariant it is necessary and sufficient that two of the co-ordinate parameters shall be fixed; the variation of the other six parameters generates a six-termed sub-group $G_{6.p}$. In like manner two conditions or parameters determine a line; the variation of the other six parameters generates a six-termed group $G_{6.l}$. The gen-

eral projective group G_8 contains ∞^2 sub-groups $G_{6.p}$, and also ∞^2 sub-groups $G_{6.l}$. Three conditions determine a linear element; if the co-ordinates of a linear element are fixed, the variation of the other five parameters generates a five-termed group G_5. If two points of the plane are invariant, four parameters are fixed and the variation of the remaining four produces the four-termed group $G_{4.a}$. If two lines of the plane are invariant, four parameters are fixed and the remaining four produce the four-termed group $G_{4.b}$. If a point and a line are invariant, four parameters are fixed and the remaining four produce the four-termed group $G_{4.c}$. If two points, their join and a line through one of them; or two lines, their intersection and a point on one of them are invariant, five conditions are satisfied; the variation of the remaining three parameters generates a three-termed group G_3. If three non-collinear points or three non-concurrent lines are invariant, all six co-ordinate parameters are constant and the two anharmonic ratio parameters generate a two-termed group G_2. (If three collinear points are invariant, all the points of the line are invariant; but the transformations leaving all the points of a line invariant are of the kind S and S′; the same is true of three concurrent lines. Groups of this kind will be discussed later.)

§3 One-Termed Groups of Transformations of the Kind T.

We shall next show that a two-termed group G (ABC) can be decomposed into one-termed sub-groups. To do this we proceed as follows:

Let T_1 be any transformation of the group G_2, and let its characteristic anharmonic ratios along the invariant lines x, y, z be $\lambda_1\ \mu_1\ \nu_1$ respectively. Let T_2 be another transformation of the group G_2, and let its characteristic anharmonic ratios be $\lambda_2\ \mu_2\ \nu_2$ respectively. These two transformations are together equivalent to another transformation of the same group G_2, whose characteristic anharmoic ratios are respectively $\lambda_3\ \mu_3\ \nu_3$. But these two-dimensional transformations each determine along the invariant lines one-dimensional transformations. The three one-dimensional transformations, one along each of the invariant lines, since they leave two points of the line invariant, belong respectively to one-termed groups of transformations of the points on a line. Hence we have by theorem 5. part I, $\lambda_1\lambda_2=\lambda_3$: $\mu_1\mu_2=\mu_3$: $\nu_1\nu_2=\nu_3$. (1) But by means of the relations $\lambda_1\mu_1\nu_1=1$, $\lambda_2\mu_2\nu_2=1$, and $\lambda_3\mu_3\nu_3=1$, we have $\nu_1=(\lambda_1\mu_1)^{-1}$; $\nu_2=(\lambda_2\mu_2)^{-1}$; $\nu_3=(\lambda_3\mu_3)^{-1}$. Now let us put $\mu_1=\lambda_1^{-a}$ where a is some unknown constant; let us also put $\mu_2=\lambda_2^{-b}$, where b is another unknown constant. The three charac-

teristic anharmonic ratios of the transformation T_1 are now $\lambda_1, \lambda_1^{-a}, \lambda_1^{a-1}$; those of T_2 are $\lambda_2, \lambda_2^{-b}, \lambda_2^{b-1}$; those of T_3 can be expressed in terms of the others. The relations existing among these anharmonic ratios are given by the equations

$$\lambda_1\lambda_2 = \lambda_3; \quad \lambda_1^{-a}\lambda_2^{-b} = (\lambda_1\lambda_2)^{-a}\lambda_2^{a-b} = \lambda_3^{-a}\lambda_2^{a-b};$$

$$\lambda_1^{a-1}\lambda_2^{b-1} = (\lambda_1\lambda_2)^{a-1}\lambda_2^{b-a} = \lambda_3^{a-1}\lambda_2^{b-a}. \qquad (2)$$

If now $b = a$ these equations reduce to

$$\lambda_1\lambda_2 = \lambda_3; \quad \lambda_1^{-a}\lambda_2^{-a} = (\lambda_1\lambda_2)^{-a} = \lambda_3^{-a}; \quad \lambda_1^{a-1}\lambda_2^{a-1} = (\lambda_1\lambda_2)^{a-1} = \lambda_3^{a-1}. \qquad (3)$$

Hence we see that if the two transformations T_1 and T_2 are so related that their characteristic anharmonic ratios along one of the invariant lines, for example along y, are each equal to the same power of the corresponding characteristic anharmonic ratios along another invariant line as x, then the resulting transformation T_3 has the same property; i. e. its corresponding anharmonic ratios have exactly the same relation. Thus we see that the two transformations T_1 and T_2 of this particular kind are together equivalent to a third T_3 of the same kind; i. e. T_3 is expressed in terms of λ_3 and a exactly as T_1 and T_2 are expressed in terms of λ_1 λ_2 and a. This is the fundamental property of a group. Hence we conclude that all the transformations of the group G_2, which have the characteristic anharmonic ratio along one of the lines as y equal to a constant power of that along another of the lines as x, form a sub-group.

This is a one-termed sub-group of G (ABC), the variable parameter of the group being the characteristic anharmonic ratio along some one of the invariant lines. This one-termed group contains ∞^1 transformations corresponding to the ∞^1 values of the variable parameter. This constant power a is the same for all transformations of the group. If we give to a different values we obtain different one-termed groups, and as many as there are values of a, viz: ∞^1. Thus we see that our two-termed group G_2 falls apart into ∞^1 one-termed groups G_1.

Theorem 13. The two-termed group of transformations G_2, which leaves a triangle invariant, consists of ∞^1 one-termed groups G_1. All the transformations belonging to one of these one-termed groups have the common property that the characteristic anharmonic ratio of each transformation along one of the invariant lines is a constant power of its characteristic anharmonic ratio along another of the invariant lines; this constant power is the same for all transformations of a one-termed group, but is different for different one-termed groups.

We shall now proceed to study in detail the properties of one of these one-termed groups. Since the variable parameter of the one-termed group in the plane is the characteristic anharmonic ratio of a one-termed group of one-dimensional transformations along one of the invariant lines, we may expect that the properties of the group of the kind G_1 on a line, (See Part I.)

The characteristic anharmonic ratios of a transformation T along the invariant lines are λ, λ^{-n}, λ^{n-1}; hereafter we shall speak of λ as the characteristic anharmonic ratio of the transformation T. We have already shown in equation (1) that λ_3, the characteristic anharmonic ratio of the resultant transformation T_3, is equal to the product of λ_1 and λ_2, the characteristic anharmonic ratios of the component transformations T_1 and T_2. By combining T_3 with T_4 we obtain T_5; so that T_5 is equivalent to the combination of T_1, T_2, T_4; thus $T_1 T_2 T_4 = T_5$. Also $\lambda_3\lambda_4 = \lambda_5$ i. e. $\lambda_1\lambda_2\lambda_4 = \lambda_5$. The same reasoning may be extended to any number of transformations.

Property 1. Any two or more transformations of the group G_1 are equivalent to some single transformation of the same group; the characteristic anharmonic ratio of the resultant transformation is equal to the continued product of the characteristic anharmonic ratios of the component transformations.

If λ 1, then $\lambda^{-n} = 1$, and $\lambda^{n-1} = 1$; but for $\lambda = 1$ the transformation along an invariant line is an identical transformation. Hence every point on the invariant lines x, y, and z are invariant points; also all lines through A, B, and C are invariant lines; therefore every line of the plane and every point of the plane is invariant, The transformation of the group given by $\lambda = 1$ is therefore an identical transformation.

Prop. 2. The group G contains one identical transformation whose characteristic ratio is unity.

Two transformations of the group whose characteristic anharmonic ratios are reciprocals of one another are said to be inverse transtransformations. It is evident that all transformations of the group may be arranged in inverse pairs, and that the two transformations of a pair are together equivalent to the identical transformation of the group. Hence we see that if any transformation T moves P to P', the inverse transformation T' moves P' back to P.

Prop. 3. The transformations of a group G may be arranged in inverse pairs; the characteristic anharmonic ratios of the transformations forming an inverse pair are the reciprocals of one another. Any transformation of the group and its inverse are together equivalent to the identical transformation of the group.

We must examine the two transformations corresponding to $\lambda=0$ and $\lambda=\infty$. We learned in one-dimensional groups to call these psuedo-transformations. In order to understand the psuedo-transformations in the plane we must consider the value of the constant a. First let a be positive between 0 and 1; second let a be positive between 1 and ∞; third let a be negative. Let ABC be the invariant triangle; and let the characteristic anharmonic ratio along BC be λ, along CA be λ^{-a}; and along AB be λ^{a-1} all taken in the same order around the triangle. For $\lambda=0$ and a a positive fraction these ratios are respectively 0, ∞, ∞. Hence (Kansas University Quarterly Vol. IV, page 79) all points of the plane except the line AB are transformed into the point C; the line AB is indeterminately transformed. For $\lambda=\infty$ and a a positive fraction the values of these ratios are respectively ∞, 0, 0. Hence all points of the plane except the points on the line AC are transformed to B; the points on AC are indeterminately transformed. In the second place let a be positive between 1, and ∞; for $\lambda=0$ the three anharmonic ratios in the order mentioned above are 0, ∞, 0. Hence in this case all points of the plane except the line AB are transformed to the point C. For $\lambda=\infty$ and a between 1 and ∞ the values of the three ratios are respectively ∞, 0, ∞. Hence all points of the plane are transformed to A except those on BC. In the third place let a be negative; for $\lambda=0$ the three values are respectively 0, 0, ∞. Hence all points except those on BC are transformed to A. For $\lambda=\infty$ the values are respectively ∞, ∞, 0; hence all points of the plane are transformed to B except those on AC. In general it can be shown that when a is any complex quantity for $\lambda=0$ and for $\lambda=\infty$ all points of the plane are transformed into some one of the invariant points.

Prop. 4. The group G contains two pseudo-transformations whose characteristic anharmonic ratios are 0 and ∞ respectively. A pseudo-transformation transforms all points of the plane to one of the invariant points except the opposite side of the invariant triangle; this is indeterminately transformed.

The group G contains an identical transformation for which $\lambda=1$. Let $\lambda=(1+\delta)$ where δ is infinitesimally small. Since the identical transformation transforms each point of the plane into itself, it is evident that the infinitesimal transformation moves every point of the plane an infinitesimal amount. If this infinitesimal transformation be repeated n times it will give rise to a transformation of the group $\lambda=(1+\delta)^n$. When $n=\infty$ and δ is a complex infinitesimal, λ can be made to equal any finite real or complex quantity by a proper

choice of δ. Consequently any transformation of the group can be generated from the infinitesimal transformation of the group. (See Vol. IV, page 170, of this Quarterly).

Prop. 5. The group G contains one infinitesimal transformation whose characteristic anharmonic ratio is $(1+\delta)$. *Any transformation of the group or the whole group itself may be generated from the infinitesimal transformation.*

The foregoing properties of the most general form of a one-termed group in the plane are almost identical with the properties of the most general form of a one-termed group on a line. Both sets of properties depend upon the variation of an anharmonic ratio parameter.

We proceed now to examine certain properties of these one-termed groups of transformations and their relations to the conics K and K′ which are used to construct the transformation. Since four points and their four corresponding points completely determine a transformation, we should be able to construct the conics K and K′ when the invariant triangle ABC and one other pair of corresponding points are given. We first show how to do this.

The conics K and K′ belong to a range of ∞^1 conics touching the lines x, y, z, and l. If we take any conic K of this range S and consider the transformations formed by taking K with all conics of the range, we shall have a system of ∞^1 transformations which may be represented by T (KS). Each transformation of this system transforms any point P of the plane into points P′, P″, P‴, . . P^n. We wish to find the locus of these points P′, P″, P‴, . . P^n. The tangents from P to K intersect l in Q and R. The tangents from Q and R on the line l to the conics of the range S form two projective pencils of rays. The intersection of corresponding rays are the points P′, P″, P‴, . . P^n, which therefore lie on a conic through Q and R. This conic also passes through the points A, B, C; for the segments AA_1, BB_1, CC_1 are conics of the range S, and the tangents from Q and R to AA_1 intersect in A. Hence this conic which we shall call K passes through A and likewise through B and C.

Hence if we have given the invariant triangle and any pair of corresponding points P and P′, we can construct K and K′, the conics which determine the transformation, and therefore construct the whole transformation. The points A, B, C, P, and P′ determine a conic K which cuts l in two points Q and R; connect P with Q and R; these two lines and the lines x, y, z, and l all touch a conic K of the range S. The lines joining P′ to Q and R touch

another conic K' of the range S. Having found the conics K and K' all the rest of the transformation can be constructed. If the conic *K* cuts the line l in two real points Q and R, then P and P' are outside of K and K' respectively; but if Q and R are a pair of conjugate imaginary points, then P and P' are inside of K and K' respectively; if the conic *K* should touch l, then P and P' are on K and K' respectively.

The conics K and K' are each characterized by a certain numerical constant. Any tangent to the conic K cuts the four fixed tangents x, y, z, l in a constant anharmonic ratio which we shall designate by k. In like manner every tangent to K' cuts the same four tangents in a constant anharmonic ratio which we shall designate by k'. We shall call these two anharmonic ratios the *tangential anharmonic ratios* of the conics K and K'.

Let the conics K and K' touch l in the points L and L'; let them touch x in X and X', y in Y and Y', z in Z and Z'. The tangential anharmonic ratio k along the fixed tangent l is that of the four points A_1, B_1, C_1, and the point of contact L; thus $k=(A_1B_1C_1L)$; likewise $k'=(A_1B_1C_1L')$. Along the line x these same tangential anharmonic ratios are respectively $k=(XCBA_1)$ and $k'=(X'CBA_1)$. Along z they are $k=(BAZC_1)$ and $k'=(BAZ'C_1)$. Along y they are $k=(CYAB_1)$ and $k'=(CY'AB_1)$.

We shall now show that the three characteristic anharmonic ratios λ_x, λ_y, λ_z can be expressed in terms of the two tangential anharmonic ratios k and k'. X and X' are corresponding points on the invariant line x; hence $\lambda_x=(BCXX')$. In like manner $\lambda_y=(CAYY')$ and $\lambda_z=(ABZZ')$.

Taking the ranges of points along the line z, we have

$$\lambda_z=(ABZZ')=\frac{AZ}{ZB}:\frac{AZ'}{Z'B}.$$ Also $k=(BAZC_1)$; hence

$$\frac{1}{k}=(ABZC_1)=\frac{AZ}{ZB}:\frac{AC_1}{C_1B};$$ and likewise we have

$$k'=(BAZ'C_1)=(ABC_1Z')=\frac{AC_1}{C_1B}:\frac{AZ'}{Z'B}.$$

Therefore $$\frac{k'}{k}=\frac{AZ}{ZB}:\frac{AZ'}{Z'B}=(ABZZ')=\lambda_z.$$ Thus we have expressed λ_z in terms of k and k'.

Let us next take the ranges along the invariant line y. Here we have $\lambda_y=(CAYY')$, $k=(CYAB_1)$, and $k'=(CY'AB_1)$; whence we

infer that

$$1-k=(CAYB_1)=\frac{CY}{YA}:\frac{CB_1}{B_1A};$$

and $1-k'=(CAY'B_1)=\frac{CY}{YA}:\frac{CB_1}{B_1A}$. Dividing the first by the second we get

$$\frac{1-k}{1-k'}=\frac{CY}{YA}:\frac{CY}{Y'A}=(CAYY')=\lambda_y.$$

Again taking the ranges along the line x, we have $\lambda_x=(BCXX')$. $k=(XCBA_1)$, and $k'=(X'CBA_1)$; therefore

$$\frac{k}{k-1}=(BCXA_1)=\frac{BX}{XC}:\frac{BA_1}{A_1C},$$

and $\frac{k'-1}{k'}=(BCA_1X')=\frac{BA_1}{A_1C}:\frac{BX'}{X'C}$. Multiplying together these two results we have

$$\frac{k(k'-1)}{k'(k-1)}=\frac{BX}{XC}:\frac{BX'}{X'C}=\lambda_x.$$

By multiplying together these values of λ_x, λ_y, λ_z we can verify the former theorem that the product of the three characteristic anharmonic ratios along the three invariant lines is unity; thus

$$\lambda_x\lambda_y\lambda_z=\frac{k(k'-1)}{k'(k-1)}.\frac{k-1}{k'-1}.\frac{k'}{k}=1.$$

Theorem 14. For any projective transformation of the kind T the three characteristic anharmonic ratios along the three invariant lines x, y, z may be expressed in terms of the two tangential anharmonic ratios of the two conics K and K' which determine the transformation; thus

$$\lambda_z=\frac{k'}{k},\ \lambda_y=\frac{k-1}{k'-1},\ \lambda_x=\frac{k(k'-1)}{k'(k-1)}.$$

If we express these in terms of λ and a, these relations are found,

$$\lambda=\frac{k'}{k};\ \lambda^{a-1}=\frac{k-1}{k'-1};\ \lambda^{-a}=\frac{k(k'-1)}{k'(k-1)}.$$

We wish to find out how to select the pairs of conics which produce transformations belonging to a one-termed group. We must first express k and k', the tangential anharmonic ratios of the conics K to K', in terms of λ, the characteristic anharmonic ratio of the transformation. By theorem 14 we have

$$\lambda=\frac{k'}{k} \text{ and } \mu=\lambda^{-a}=\frac{(k'-1)}{(k-1)};\frac{k}{k'},$$

therefore $\frac{k-1}{k'-1}=\frac{k'^{a-1}}{k^{a-1}}$; by means of the relation $\lambda=\frac{k'}{k}$ we get after reduction

$$k=\frac{\lambda^{a-1}-1}{\lambda^{a}-1}, \text{ and } k'=\frac{\lambda^{a}-\lambda}{\lambda^{a}-1}. \tag{4}$$

When the fixed constant a is given, the conics K and K′ corresponding to a given value of λ are at once determined,

We can now determine the positions of the conics K and K′ for particular values of λ. When $\lambda=1$, the transformation is an identical one for the whole plane; substituting this value of λ in the last equations and evaluating the indeterminate expressions we find $k=k'=\frac{a-1}{a}$. Thus in the case of the identical transformation of the group the conics K and K′ are coincident, and touch the line l at the point L, such that $(A_1B_1C_1L)=\frac{a-1}{a}$. When the conics K and K′ are coincident, it is easy to see from the construction of the transformation that every point of the plane is unaltered in position; in other words the transformation in the whole plane is an identical one.

If we consider the construction of any transformation T (KK′) by means of the conics K and K′, we see that the transformation determined by the same two conics taken in the reverse order, T (K′K), is the inverse of the first; i. e. if T (KK′) transforms P to P′, then T (K′K) transforms P′ back to P; and so with every point of the plane. It is clear that every transformation of the group has an inverse belonging to the same group and that any transformation and its inverse are together equivalent to the identical transformation of the group.

In considering the positions of the conics which produce a pseudo-transformation of the group it is necessary to consider the value of the constant a. We shall consider the case where a is between 0 and 1, and a real quantity. The coincident conics producing the identical transformation of the group touch the line l between A_1 and B_1. Let λ gradually decrease in value; then the two conics separate, the point of contact of K approaching A_1 and the point of contact of K′ approaching B_1. When $\lambda=0$, $k=-\infty$ and $k'=0$; the conic K then becomes the degenerate conic AA_1, while K′ becomes BB_1, Thus the pseudo-transformation is pro-

duced by the two degenerate line conics AA_1 and BB_1. Let λ decrease still further and become negative; the point of contact of K approaches C_1 from one side and the point of contact of K′ approaches the same point from the other side. For some value of λ (usually an imaginary root of unity) the two conics coincide with the degenerate line conic CC_1. Let λ approach $-\infty$; the conic K then approaches its limiting form BB_1, while K′ approaches its limiting form AA_1. Thus we reach the second pseudo-transformation of the group which is produced by the same two degenerate conics BB_1 and AA_1; but now taken in the reverse order, showing that the two pseudo-transformations form an inverse pair. If a be taken not between 0 and 1, another combination of line conics will produce the pseudo-transformations.

The real group contains two real infinitesimal transformations which are inverse to one another. The conics K and K′ which determine these infinitesimal transformations differ by an infinitesimal amount from the coincident conics which produce the identical transformation of the group. (The case where the transformations of the group are not real will be discussed elsewhere.)

The analytical expressions for a one-termed group G_a can readily be written down from the properties pointed out above. Let the invariant triangle be ABC, and let the transformation T whose equations we wish to find be that one which transforms the point P (x, y, z) to the point P_1 (x_1, y_1, z_1). The anharmonic ratio of the pencil $C(ABPP_1)$ is λ; in terms of the co-ordinates of the points P and P_1 this is seen to be $\lambda = \frac{y_1}{x_1} : \frac{y}{x}$. Hence $\frac{y_1}{x_1} = \lambda \frac{y}{x}$. In like manner the ratio of the pencil $A(BCPP_1)$ is λ^{-a}; and its analytic expression in the co-ordinates of P and P_1 is $\lambda^{-a} = \frac{z_1}{y_1} : \frac{z}{y}$; hence we have $\frac{z_1}{y_1} = \lambda^{-a} \frac{z}{y}$. The anharmonic ratio of the pencil $B(CAPP_1)$ is similarly found to lead to $\frac{x_1}{z_1} = \lambda^{a-1} \frac{x}{z}$.

These three equations express the transformation T which transforms P to P_1; if we have a second transformation T_1 of the same group which transforms P_1 to P_2, its equations will be

$$\frac{y_2}{x_2} = \lambda' \frac{y_1}{x_1}; \quad \frac{z_2}{y_2} = \lambda'^{-a} \frac{z_1}{y_1}; \quad \frac{x_2}{z_2} = \lambda'^{a-1} \frac{x_1}{z_1}.$$

If we eliminate x_1, y_1, z_1 from these two sets of equations, we are able to express the co-ordinates of P_2 in terms of those of P. Setting $\lambda\lambda' = \lambda_1$, the elimination gives

$$\frac{y_2}{x_2} = \lambda_1 \frac{y}{x}; \quad \frac{z_2}{y_2} = \lambda_1^{-a} \frac{z}{y}; \quad \frac{x_2}{z_2} = \lambda_1^{a-1} \frac{x}{z}.$$

This shows that the two transformation T and T_1 are together equivalent to T_2; another transformation of the same group which transforms P directly to P_2.

This analytical expression for a one-termed group G_a is in fact identical with Lie's expression in homogeneous co-ordinates.

[*To be Continued.*]

Theory of Compound Curves in Railroad Engineering.

BY ARNOLD EMCH.

1. It is the purpose of this note to treat the problem of compound curves as it occurs in railroad engineering from a general geometrical stand point which enables us to discuss in an easy manner all the essential parts of the problem. It will be seen that the theory of compound curves is identical with the theory of two projective special pencils of circles. In Vol. III, No. 5, of the *American Mathematical Monthly*, the author has treated of projective pencils of circles in connection with a special complex of lines of the second degree*.

The theorem has been established:

The locus of the points of tangency of both tangent-circles of two pencils of circles is a bi-circular curve of the fourth order. The same curve is also produced by one of the pencils and the projective conjugate pencil of the other pencil.

This curve, of course, passes through the four fundamental points of the pencils of circles. Now we may take the special case where the two fundamental points of each pencil of circles coincide, or where all the circles of the pencil are tangent to a fixed line at a fixed point. This, however, represents precisely the case of compound curves in railroad engineering. Evidently the bicircular curve of the fourth order, having also two finite double points, must degenerate into *two circles*.

2. In order to apply the previous result we will verify it directly. First we will write the equations of the two special pencils of circles in the form

$$\begin{aligned} U-2\lambda V&=0, \\ U'-2\lambda' V'&=0, \end{aligned} \qquad (1)$$

and assume as the double points (coinciding fundamental points) of these pencils the points (0,0) and (a,b) fig. 1.

*A Special Complex of the Second Degree and its Relation with the Pencils of Circles.

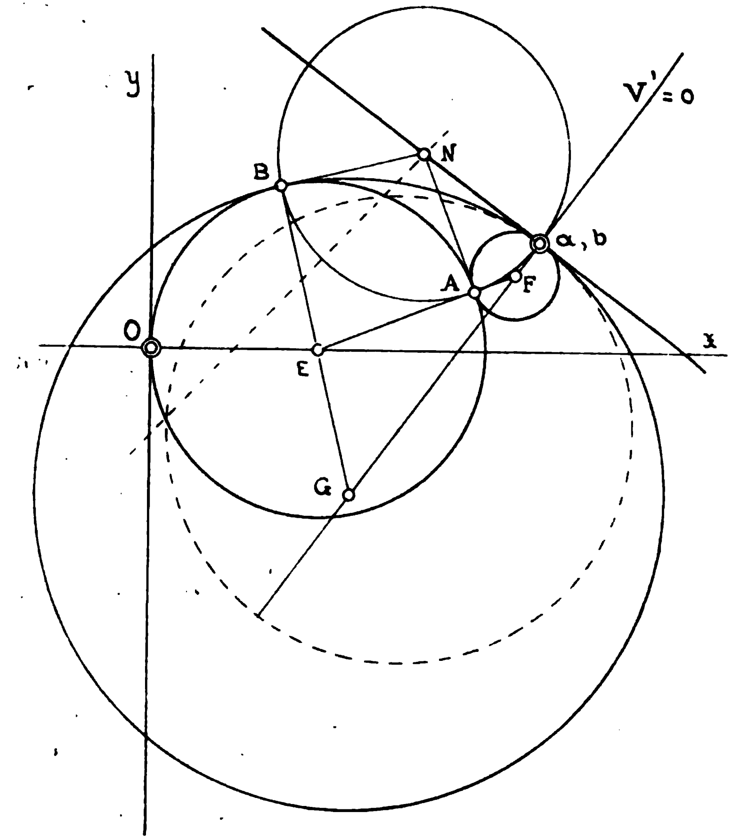


Fig. 1.

The circles of the first pencil we assume tangent to the y—axis ($x=0$), and those of the second tangent to the line

$$\frac{x-a}{y-b}=k,$$

$$\text{or}\quad x-yk-a+bk=0 \qquad (2)$$

at the point (a,b).

Now any two circles of a pencil of circles determine all the other circles of the pencil, as indicated in the formulæ (1). We may also choose special circles of these pencils, for instance, the tangents at their double points and the zero-circles in these points. Thus we have

$$U=x^2+y^2, \qquad V=x$$

$$U'=(x-a)^2+(y-b)^2, \quad V'=x-yk-a+bk.$$

The equations of our special pencils of circles are therefore

$$x^2+y^2-2\lambda x=0, \tag{3}$$

$$(x-a)^2+(y-b)^2-2\lambda'(x-yk-a+bk)=0. \tag{4}$$

It is required that any of the circles (3) is orthogonal to as many circles of (4) as is possible. Designating the co-ordinates of the centers of any two circles by (α,β) and (α',β') and their radii respectively by ρ and ρ', the condition for the orthogonality of the two circles is

$$(\alpha-\alpha')^2+(\beta-\beta')^2=\rho^2+\rho'^2. \tag{5}$$

Associating the values α, β, ρ for a special value of λ with the corresponding circle of the pencil (3) we have

$$\alpha=\lambda,\ \beta=0,\ \rho=\lambda.$$

In the same way we associate the values α', β', ρ' with the pencil (4) and have

$$\alpha'=a+\lambda',\ \beta'=b-\lambda'k,\ \rho'=\lambda'\sqrt{(1+k^2)}.$$

Substituting these values in equation (5) there is

$$(\lambda-a-\lambda')^2+(\lambda'k-b)^2=\lambda^2+\lambda'^2(1+k^2),$$

or after some reductions

$$2\lambda'(a-\lambda-bk)=2\lambda a-a^2-b^2,$$

whence

$$\lambda'=\frac{2\lambda a-a^2-b^2}{2a-2\lambda-2bk}. \tag{6}$$

According to this condition, to each value of λ belongs one and only one value of λ', i. e., taking any circle of the pencil (3), there is one and only one circle in the pencil (4) orthogonal to that circle. If we substitute in formula (6) for λ' and λ successively the values:

$$\lambda'=\alpha'-a,\ \lambda=\alpha,\ \text{and}\ \lambda'=\frac{b-\beta'}{k},\lambda=\alpha,$$

we obtain the two expressions

$$\alpha'-a=\frac{2a\alpha-a^2-b^2}{2a-2\alpha-2bk},$$

and

$$\beta'-b=\frac{k(2a\alpha-a^2-b^2)}{2\alpha-2a+2bk},$$

or

$$\alpha'=\frac{a^2-b^2-2abk}{2a-2\alpha-2bk},$$

$$\beta' = \frac{2a(b+ak)-2ab-a^2k}{2a-2a+2bk}.$$

From this is seen that the centers of corresponding orthogonal circles in the two pencils form projective point-ranges. The two pencils are, therefore, also projective and their product is a bi-circular curve of the fourth order which degenerates into two circles. To obtain the equation of these circles we have to eliminate λ and λ' from the following equations:

$$x^2+y^2-2\lambda x=0 \qquad \text{(I)}$$

$$(x-a)^2+(y-b)^2-2\lambda'(x-yk-a+bk)=0 \qquad \text{(II)}$$

$$\lambda' = \frac{2\lambda a-a^2-b^2}{2a-2\lambda-2bk}. \qquad \text{(III)}$$

From (I) follows

$$\lambda = \frac{x^2+y^2}{2x},$$

hence

$$\lambda' = \frac{a(x^2+y^2)-x(a^2+b^2)}{2x(a-bk)-(x^2+y^2)}.$$

Substituting this value in II, there is

$$\left[(x-a)^2+(y-b)^2\right]\left[2x(a-bk)-(x^2+y^2)\right]-$$
$$-2\left[x-yk+bk-a\right]\left[a(x^2+y^2)-x(a^2+b^2)\right]=0.$$

After some transformations and reductions this equation may be written in the conspicuous form

$$\left[x^2+y^2-x(a-bk-b\sqrt{1+k^2})-y(b+ak+a\sqrt{1+k^2})\right]\times$$
$$\times\left[x^2+y^2-x(a-bk+b\sqrt{1+k^2})-y(b+ak-a\sqrt{1+k^2})\right]=0. \qquad (7)$$

This is the equation of the product of the two projective special pencils of circles and, evidently, represents two circles

$$x^2+y^2-x(a-bk-b\sqrt{1+k^2})-y(b+ak+a\sqrt{1+k^2})=0, \qquad (8)$$

$$x^2+y^2-x(a-bk+b\sqrt{1+k^2})-y(b+ak-a\sqrt{1+k^2})=0, \qquad (9)$$

which both pass through the origin and through the point (a,b), the two finite double points.

The co-ordinates of the center of the first circle are

$$m=\frac{a-bk-b\sqrt{1+k^2}}{2},$$

$$n=\frac{b+ak+a\sqrt{1+k^2}}{2}, \qquad (10)$$

and of the second

$$m'=\frac{a-bk+b\sqrt{1+k^2}}{2},$$

$$n'=\frac{b+ak-a\sqrt{1+k^2}}{2}. \qquad (11)$$

The radii of these circles respectively are $\sqrt{m^2+n^2}$ and $\sqrt{m'^2+n'^2}$. It is easily verified that

$$(m-m')^2+(n-n')^2=m^2+n^2+m'^2+n'^2.$$

This, however, is the condition that two circles are normal to each other. Hence:

The two circles forming the locus intersect each other at right angles.

From this follows, that the points P, Q, O, M, T in fig. 2, all lie on the same circle with the line $\overline{PQ}$ as a diameter.

3. The normal pencil of circles of the pencil (4) is obtained by considering the normal to the straight line (2), $\frac{x-a}{y-b}=k$, which is

$$xk+y-ak-b=0,$$

and the zero-circle at the point (a,b) as two circles of the pencil. The required normal pencil is therefore given by the equation

$$(x-a)^2+(y-b)^2-2\lambda''(xk+y-ak-b)=0. \qquad (12)$$

For a fixed value of λ'' the co-ordinates of the center of the corresponding circle are

$$\alpha''=a+\lambda''k,$$

$$\beta''=b+\lambda'',$$

and

$$\rho''=\lambda''\sqrt{1+k^2}.$$

The condition for the tangency of the circle (12) and of the original circle (3) is

$$(\alpha-\alpha'')^2+(\beta-\beta'')^2=(\rho\pm\rho'')^2.$$

Substituting the values of α, β, ρ and α'', β'', ρ'' and developing we find the expression

$$\lambda''=\frac{a^2\times b^2-2a\lambda}{2\lambda(k\pm\sqrt{1+k^2})-2(ak+b)},$$

which shows that to each value of λ belong two values of λ'', or that each circle of the pencil

$$x^2+y^2-2\lambda x=0$$

is touched by two and only two circles out of the pencil

$$(x-a)^2+(y-b)^2-2\lambda''(xk+y-ak-b)=0.$$

These results are all well known from the theory of pencils of circles and it is for the present purpose not necessary to develope further details.

We will now show that any circle C' of the pencil

$$(x-a)^2+(y-b)^2-2\lambda'(x-yk-a+bk)=0$$

which is normal to a certain circle C of the pencil

$$x^2+y^2-2\lambda x=0$$

cuts the latter circle in two points, A and B, which are precisely the points of tangency of the two possible tangent circles C_1'' and C_2'' out of the normal pencil of circles

$$(x-a)^2+(y-b)^2-2\lambda''(xk+y-ak-b)=0.$$

In fig. 2, C_1'' and C_2'' are the two circles tangent to the circle C.

Now the tangent to C or C_2'' at B, intersects the tangent V_1 in the point $(\alpha', \beta',)$, or N, such that NB = NM = NA. Hence the normal circle of C', C_1'', and C_2'' pass through A and B, q. e. d.

The locus of the points of tangency of the circles of our special pencils of circles is, therefore, the same as the product of projec-

tivity of one of the pencils with the normal pencil of the other, i. e., consists of two circles which both pass through M and O.

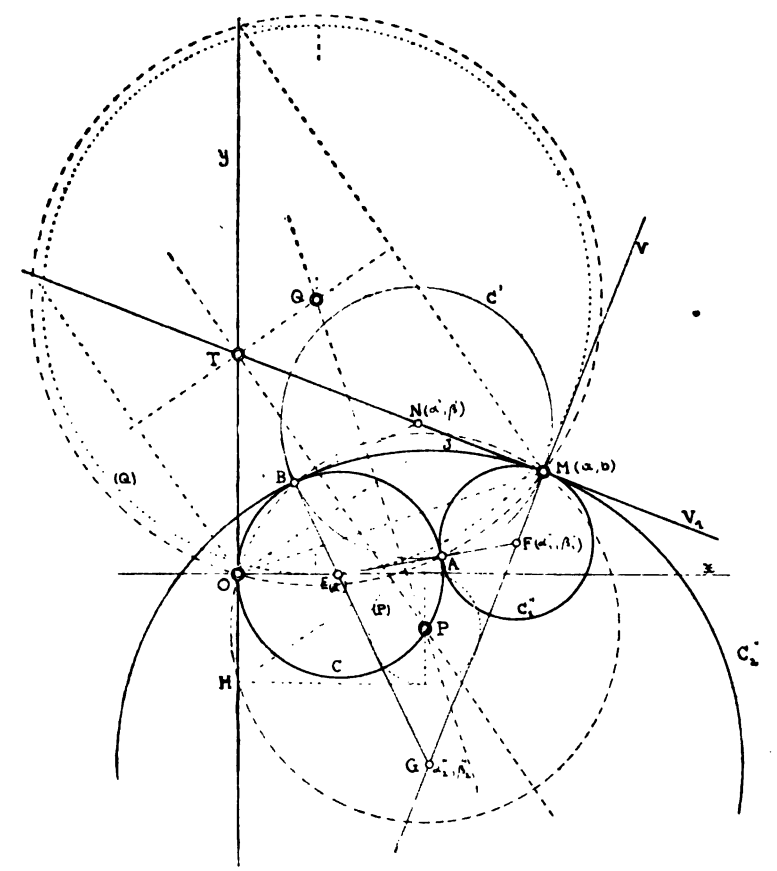


Fig. 2.

The equation of the line V' is

$$xk+y-ak-b=o,$$

or

$$\frac{xk}{\sqrt{1+k^2}}+\frac{y}{\sqrt{1+k^2}}-\frac{ak+b}{\sqrt{1+k^2}}=o,$$

and of the y—axis

$$x=o.$$

The equations of the inner and outer bi-sector are, therefore, given by the expressions

$$x(k-\sqrt{1+k^2})+y-ak-b=o, \tag{13}$$

and

$$x(k+\sqrt{1+k^2})+y-ak-b=0. \qquad (14)$$

As is easily verified, the co-ordinates (10) which represent the center of one of the circles of the locus, satisfy the first of these equations and those of (11) the second.

If we now use the technical terminology, i. e. designate the arcs MB, OB, MA, AO, etc., as arcs of compound curves and their points of tangency as points of compound curves we may state the theorem:

The locus of all points of compound curves between two tangents and points consists of two circles which pass through the two given points on two given tangents and whose centers lie on the bi-sectors of the two given tangents.

To construct these centers we may, therefore, connect A with B, erect a perpendicular to AB in the middle of AB, which will intersect the bi-sectors in the required points P and Q.

Considering any point of compound curve as B, then it lies on the same right line with the centers of the corresponding arcs of compound curves OB and MB. Since O, and B lie also on the circle of the locus of points of compound curves with the center P, the perpendicular to the chord OB through E passes through P. Hence

$$\angle PEX = \angle PDG.$$

This means that every ray connecting the centers of two compound curves whose point of tangency, or point of compound curve, lies constantly on one of the circles of the locus, is tangent to a fixed circle which is concentric with the circle of the locus. The same can be proved for the ray EF. The two concentric circles one with P, the other with Q as a center, in fig. 2, are designated by (P) and (Q).

The circle of the locus with P as a center intersects the y—axis and the line V' in two other points J and H, such that JM=OH, and TM—TO=OH. Evidently OH is equal to the diameter of the circle (P). In a similar manner it is proved that the diameter of the circle (Q) is equal to TM+TO. To sum up we may say:

The locus of points of compound curves of all compound curves between two tangents, TM and TO, and two tangent points, M and O, consists of two circles which pass through the points M and O and whose centers lie on the bi-sectors of the tangents TM and TO.

By this condition, the centers P and Q of these circles and, therefore, the circles themselves are perfectly determined. In all compound curves the radial lines through the points of compound curves belonging to this

system are all tangent to either one or the other of two fixed circles having as their centers the points P and Q and for their radii the values

$$\frac{TM-TO}{2} \text{ and } \frac{TM+TO}{2}.^*$$

The points P, Q, O, M, T, in fig. 2, all lie on the same circle, having the line PQ as a diameter.

4. Among the great number of special cases we will consider the problem where the two tangents are parallel. The general theorem and construction still hold, so that the solution is simply a matter of reduction for special values. To find the equations of the locus we have to put $k=\infty$ in formulæ (8) and (9). Observing that for an indefinitely large value of k

$$\lim\left(-bk+b\sqrt{1+k^2}\right)_{k=\infty}=0$$

$$\lim\left(ak-a\sqrt{1+k^2}\right)_{k=\infty}=0$$

these equations become respectively

$$bx-ay=0 \qquad (15)$$

and

$$x^2+y^2-ax-by=0, \qquad (16)$$

or

$$(x-\frac{a}{2})^2+(y-\frac{b}{2})^2=\frac{a^2+b^2}{4}. \qquad (17)$$

The meaning of the equations (15) and (16) is clear: The first represents a straight line through the point (a,b) and the origin; the second a circle with the point $(\frac{a}{2}, \frac{b}{2})$ as a center, and OM as a diameter, fig. 3.

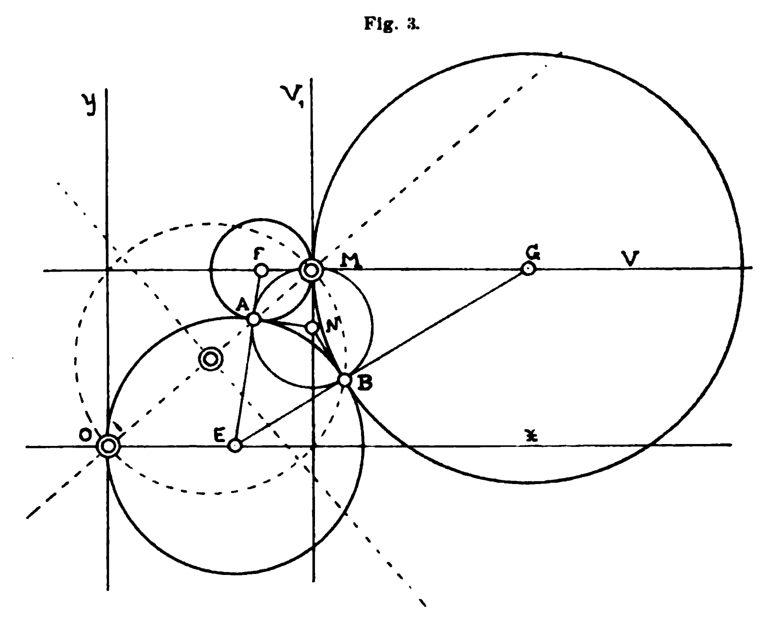
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*In practical treatises on this subject the conception of compound curves is not given under this general point of view Thus in Mr. W. H. Searles's treatise on Field Engineering the following restriction is made:

"A compound curve consists of two or more consecutive circular arcs of different radii, having their centers on the same side of the curve; but any two consecutive arcs must have a common tangent at their meeting point. or their radii at this point must coincide in position "

This result is also obtained from the expressions (10) and (11). For $k=\infty$ the first indicates that the center of the circle (8) is at

Fig. 3.



an infinite distance in a direction whose trigonometric tangent is

$$\lim \left\{ \frac{b+ak+a\sqrt{1+k^2}}{a-bk-b\sqrt{1+k^2}} \right\}_{k=\infty} = -\frac{a}{b}.$$

The second expression gives for the co-ordinates of the circle (9)

$$x=\frac{a}{2},\ y=\frac{b}{2}.$$

The Visual Perception of Distance.

BY JOHN E. ROUSE.

If we omit Descartes, the scientific study of the perception of distance began with Bishop Berkeley. Assuming that a difference in the distance of a point can make no difference in the nature of the retinal image, since "distance being a line directed endwise to the eye projects only one point upon the fund of the eye—which point remains invariably the same, whether the distance be greater or smaller," he concluded that distance could not be a visual sensation, but must be an intellectual "suggestion," due to some non-visual experience, and this experience he considered *tactile*.

According to his view, visual perception of distance is the acquired interpretation of light and color differences in terms of distance already gained by skin and muscle. To say that an object is a certain distance, is to assert that so much sensation of skin and muscle must be had before the object can be touched.

But the notion that distance is not a visual, but a tactile form of consciousness, suggested by visual signs, though endorsed by many later psychologists, is by no means *generally* accepted. Some argue that the estimation of distance by the eye, is, as Berkeley said, a result of suggestion and experience, but that visual experience alone is adequate, and this Berkeley denied. It is further maintained that depth feeling is just as optical in its nature as either hight or breadth, and that in the absence of motion of the body, or any part of it, toward or away from objects observed, the movement of the objects themselves may be substituted, with similar experience resulting.

Persons blind from birth and acquiring their sight in later years, have thus *at first* experienced distance by touch, and *afterward* both by touch and sight. As these persons (about twelve cases having been reported) *generally* maintain that all objects seemed to be, when first seen, in one plane near the globe of the eye, and that optical perception of their distance was learned by "associa-

tion" with the tactile sense, it seems that a strong argument for Berkeley's theory has been found. But there have been a few people of the above class to whom objects, when first seen, did not appear in the same plane, but nearer and farther, although, of course, experience enabled them to locate the objects more accurately.

The difficulty of finally settling the question of whether or not distance is a visual as well as a tactile form of consciousness, is greatly augmented by the fact that, although we find persons who at one time perceived distance tactually, and later both tactually and visually, as above, we do not have at hand to compare with them, persons who at first have no tactile sensations, but only visual, and then later both kinds of sensations.

Numerous experiments and observations have led psychologists to conclude that distance perception may be regarded as the product of three ever varying factors: *retinal, muscular*, and *intellectual*, as may be seen in the following so-called "clues"—accommodation, double and disparate images, difference in parallactic displacement of objects when the head is moved, faintness of tint, dimness of outline, and smallness of retinal images of objects named and known, together with various comparisons and allowances made, voluntarily and involuntarily. All of the above have *something to do* with our notions of "far" and "near;" but when we consider that these "aids" have a way of overcoming and overbalancing each other, especially when influenced by the presence of some other sensible quality in the object, and that definite tactile and retinal modifications do not accompany differences in distance, and further, that there are many other irregularities, it then becomes evident to us that the act of judging distance follows no simple law. But that there are certain tendencies shown in our acts of judgment, a number of psychological experiments have plainly indicated; and it was to continue the examination of various estimations of distance that the following investigation was made. The accompanying drawing is intended to represent the large room in which the tests of judging given distances were made, and to show the mechanism and arrangement of the apparatus used.

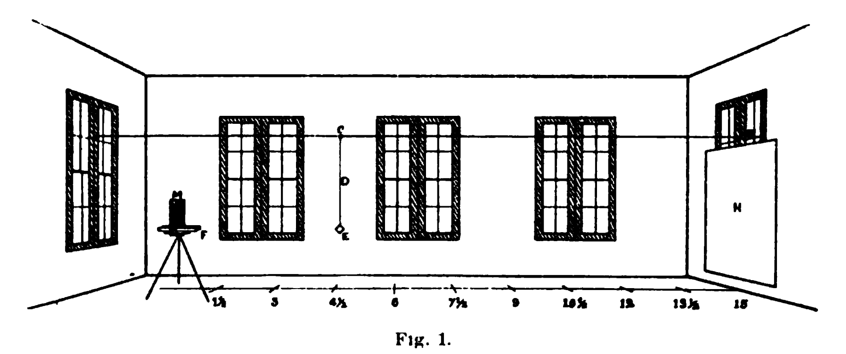


Fig. 1.

In the figure above, E represents a light-brown paste-board box, suspended with an invisible wire (D) to a larger wire (AB), by means of a smooth ring (C) capable of sliding back and forth. Four such cubical boxes were employed, their edges measuring, respectively, 2, 3, 4, and 5 centimeters. The larger wire (AB), supporting them, was tightly drawn in a horizontal position overhead and extended the entire length of the 52 foot room. Two tin tubes (F), well smoked inside, were so fixed at one end of the room that they were directly below the horizontal wire (AB), parallel with it, and in the same horizontal plane with the boxes when suspended successively. A curtain of plain dark material (H) was placed at the further end of the room, just below and at right angles to the wire drawn above.

Two persons were required (besides the subject) to perform the experiment, one to move the boxes back and forth (with a long stick or pointer) to correspond with divisions of a chalk line drawn upon the floor directly below the horizontal wire (AB), and another to give the subject views of the boxes when placed at proper positions, not permitting him to see them moved, or to know when one box was exchanged for another, and to keep account of estimations made, thus leaving the subject free to judge the distance of the objects at different positions. Views at the proper time were given by uncovering the farther ends of the tubes, a large piece of pasteboard (M), perforated with holes through which the nearer ends of the tubes passed, cutting off all view in front of the subject except through the tubes.

In the experiment ten young men from the higher classes of the university were used as subjects, each sitting at the opposite end of the room from the curtain, and judging the distance of each of the four boxes, when placed in a definite series of positions. When judging distance at the first of the experiment the subject looked through the tubes and saw nothing but the suspended boxes

and the screen (H), without knowing the size of the former or the distance of the latter. Then afterwards he was shown the boxes, and allowed to handle them and to learn their respective dimensions, informed of the distance of the curtain, and permitted to judge the distance of the objects, when again suspended, but without looking through the tubes. In this way the judgment was assisted in every way possible, except in seeing the objects moved, which was in no case permitted.

In the former case, when the tubes were used, the same definite series of positions was estimated in three ways: with *right* eye, with *left* eye, and with *both* eyes, one tube being closed for monocular vision. In seeing directly (without the tubes) only binocular vision was used.

In addition to the above series of tests a shorter one was given, using the tubes and binocular vision (with 2 in. box), to illustrate Wundt's Law regarding judgment of the distance of objects when moving closer and closer, or farther and farther.

Each of the ten subjects made observations requiring a sitting of an hour or more. Care was taken to have the room lighted evenly in different parts, and the same set of tests was given to each subject in as nearly the same manner as possible.

Below is shown a tabulated report of 160 average judgments, made from 1,600 tests upon ten subjects. Arabic numerals at the top of each of the four columns are used to indicate different sized boxes used, the boxes being named in the order of their sizes, beginning with the smallest box.

TABLE I.

	Using the tubes, and seeing only objects and curtain beyond, size of former and distance of latter not being known.												Without tubes, size of objects and distance of curtain known.			
Real Dist.	(I.) Right Eye.				(II.) Left Eye				(III.) Both Eyes.				(IV.) Both Eyes (fr e)			
Meters	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1½	1.95	1.[illegible]0	1.05	1.10	2.05	1.55	1.55	1.30	1.75	1.65	1.40	1.25	1.50	1.40	1.45	1.35
3	3.75	2.60	2.45	2.35	3.65	3.05	2.65	2.45	3.45	2.80	2.65	2.55	2.80	3.05	3.00	2.85
4½	5.60	3.75	3.65	3.15	5.35	4.05	3.65	3.40	4.25	3.70	3.95	3.25	4.05	4.40	4.30	4.25
6	7.55	5.20	5.05	4.55	7.00	5.25	4.70	4.65	5.45	4.65	4.90	4.45	5.55	5.70	5.80	5.50
7½	8.75	6.75	6.15	5.60	8.75	6.45	6.00	5.70	6.55	5.95	6.45	5.50	6.35	6.90	7.10	6.85
9	10.50	8.00	7.45	6.55	10.15	7.90	7.20	6.60	7.85	7.25	7.45	6.75	7.65	8.35	8.40	8.00
10½	11.55	9.40	8.55	7.65	11.35	8.85	8.45	7.60	9.15	8.35	8.50	8.20	9.55	9.55	9.10	9.35
12	12.45	10.55	9.70	8.75	12.45	10.15	9.50	8.50	10.45	9.45	9.65	9.30	10.05	11.10	11.55	11.40
13½	13.25	11.30	10.80	9.55	13.15	11.00	10.65	9.50	11.55	10.75	10.75	10.50	12.80	12.90	13.15	13.00
15	14.85	13.00	12.30	10.85	13.60	12.70	11.90	10.90	12.90	12.40	12.00	11.20	14.[illegible]0	14.80	14.90	14.80
82½	90.20	72.05	67.15	60.00	87.50	70.95	68.25	60.60	73.85	66.95	67.70	62.95	76.10	78.15	78.75	77.35

It will be seen that the average estimations generally vary inversely as the size of the object observed, i. e., as the box used is larger, the distance judged is shorter, and vice versa. E. g., for 3 meters in (I), the smallest box (1) was thought to be 3.75 meters distant, and the next larger ones, (2), (3) and (4), 2.60, 2.45 and 2.35 meters, respectively. This is a common illusion, and it is natural that it should be shown here. To this tendency there is but one exception in (I) and one exception in (II), while in (III) there are a half dozen exceptions, and in (IV) the illusion almost wholly disappears, showing that in unassisted binocular vision there is a *slight* tendency to overcome the mistake of judging a larger object to be nearer, and a smaller object to be farther, while in binocular vision assisted in different ways the illusion, in a great measure, disappears. A comparison of the sums of the averages for each box in different columns will show the same relation more plainly. Observing the same order of boxes, the smallest first, we find the sums as follows:

TABLE II.

	1	2	3	4
(I)	90.20	72.05	67.15	60.00
(II)	87.50	70.95	66.25	60.60
(III)	73.35	66.95	67.70	62.95
(IV)	76.10	78.15	78.75	77.35

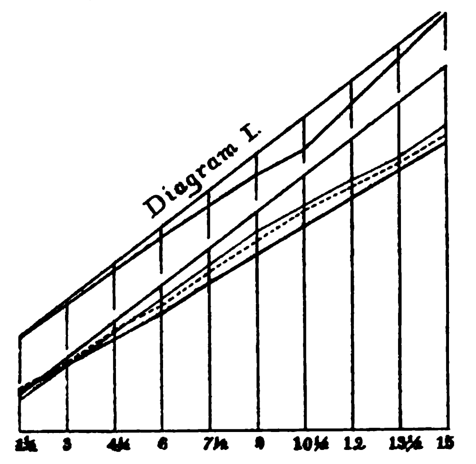
By averaging (1), (2), (3) and (4) of each of the four columns of table No. 1, the final average estimations (with different kinds of vision) appear as follows:

TABLE III.

Real Dist.	(I.) Right.	(II.) Left.	(III.) Both.	(IV.) Both (free)
1½	1.3750	1.6125	1.5125	1.4250
3	2.7875	2.8500	2.8625	2.9250
4½	4.0375	4.1125	3.7875	4.2500
6	5.5875	5.4000	4.8625	5.6375
7½	6.8125	6.7250	6.1125	6.8000
9	8.1250	7.9625	7.3250	8.1000
10½	9.2875	9.0525	8.5500	9.3875
12	10.3625	10.1500	9.7125	11.2750
13½	11.2250	11.0750	10.8875	12.9625
15	12.7500	12.2750	12.1250	14.8250
82½	72.3500	71.3250	67.7375	77.5875

Table III seem to indicate that, within a scope of 15 meters, distance is nearly always *underestimated*, and appears *less* to the *left* eye than to the *right* eye, and less to *both* eyes (unassisted) than to the *left* eye, as is more plainly shown by the following diagram. The perpendicular lines represent on a small scale the actual distances indicated at their lower extremities; and the *length* of these perpendiculars *from the horizontal base line to where they are cut* by the different curves, shows the *respective estimates* of these "actual distances."

A



Comparing the sum of the actual distances (82½ m.), shown in table III, with the sums of the estimates in columns (I), (II) and (III), the order of accuracy is shown as follows:

Real Distance.	Right Eye.	Left Eye.	Both, unassisted.
82½	72.35	71.325	67.7375

A previous experiment was made in nearly the same manner as this one, except that no distance greater than 10 meters was shown (instead of 15, as in this), and that a greater number of subjects were used with fewer tests each, and finally, that instead of giving the tests in the order of right eye, left eye and both eyes, it was given in this order: both eyes, right eye and left eye; so the order of giving the tests could not have influenced the results of the two experiments to be similar to each other.

The averages of the results obtained from the preceding experiment were as follows:

Real Distance.	Right Eye.	Left Eye.	Both, unassisted.
92½	87.96¼	86.03¾	79.72½

Simplifying these two sets of results by reducing the real distances (82½ and 92½) to unity, we have the following comparisons:

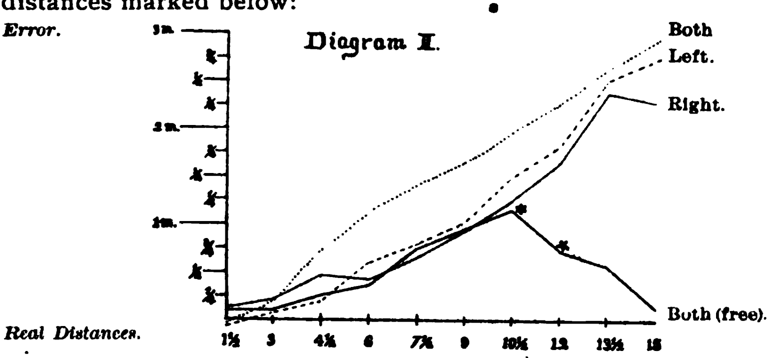
	Real Distance.	Estimations. Right.	Left.	Both.
For 15 meters,	1 meter.	.87	.86	.82
For 10 meters,	1 meter.	.95	.93	.86

These figures show the accuracy of judgment to be greater within a scope of 10 meters than 15, which might have been expected, since in the curve in diagram I the oblique lines representing the relative judgments, diverge more and more from the true line as the distance increases. This is better shown below:

TABLE IV.

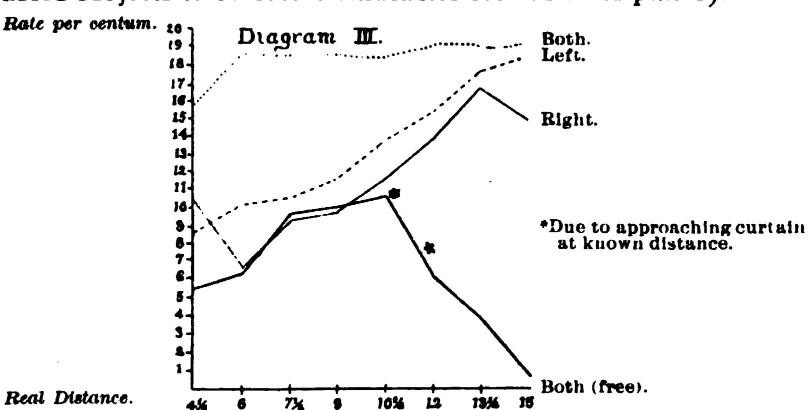
Dist.	Actual Errors.				Per centum of Errors.			
	Right.	Left.	Both.	Both(free)	Right.	Left.	Both.	Both(free)
1½	.12½	—.11¼	—.01¼	.07½	8.33	—7.50	—.83	5.00
3	.21¼	.05	.13¾	07½	7.08	1.66	4.58	2.50
4½	.46¼	.38¾	.71¼	25	10.27	8.61	15.88	5.55
6	.41¼	.60	1.13¼	.36¼	6 87	10.00	18 78	6.04
7½	.68¾	.77½	1.38¾	.70	9.16	10.83	18.50	9 83
9	.87½	1.03¾	1.67½	.90	9.72	11.52	18.61	10 00
10½	1.21¼	1.43¼	1.95	1.11¼	11.54	13.69	18.57	10.59
12	1.63¾	1.85	2.28¼	.72½	13 64	15.41	19.06	6.04
13½	2.27½	2 42½	2.61¼	.53¾	16.85	17.96	19.35	8.98
15	2.25	2.72½	2.87½	.11½	15.60	18.16	19.16	.76
82½	10.15	11.17½	14.96¼	4.91¼				

The curves in the following diagram show the *actual error* for distances marked below:



*Due to approaching curtain at known distance.

Below is a plot showing *per centum* of errors (omitting first two positions, where close view and some imperfections of method enabled subjects to overcome tendencies seen in other places).



The above shows that the error has a strong tendency to increase as the distance increases, with few exceptions.

The following is a report (illustrated by curve) of a series of 190 tests to explain Wundt's Law:

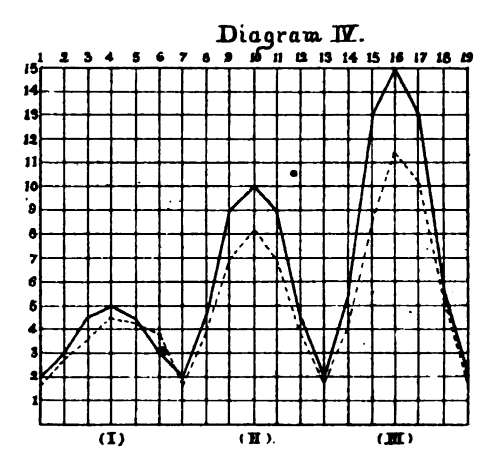


TABLE V.

Dist.,	2 00	3 00	4.50	5 00	4.50	3 00	2.00	4 50	9 00	10.00	9 00	4 50	2.00	3.50	13.00	15.00	13.00	5.50	2.00
Est.,	1.60	2.75	3.65	4.45	4.40	3.75	1 65	3.95	6 90	8.15	6.90	3.90	1.55	4.15	8.60	11 35	10.10	5.30	1.55

In the table a series of positions is shown, such that the object observed is part of the time approaching the observer and part of the time receding, at equal distances each. In the plot the heavy line represents the true distances and the dotted line the judgments, the left hand side of the figures showing the forward movement, the right hand side the backward.

Remembering that the relative accuracy is shown in the above diagram by the tendency of the two lines to come together, we conclude that the accuracy in (I) and (III) is greater when the object seen approaches than when it recedes, although in (II) there seems to be no marked tendency either way. A numerical statement of all the different judgments is shown in the following table.

In the following table Roman numerals correspond to those in the preceding diagram.

TABLE VI.

Parts.	Real Distance.	Estimations. Approaching.	Estimations. Receding.
(I.)	14 50	14 25	12.45
(II.)	25 50	20 50	20 65
(III.)	35 50	28.30	25 65
Total.	75 50	60.05	58.75

From the above numerical statement it would appear that, within

a range of 15 meters, the accuracy of judging the distance of an approaching object is greater than in determining that of a receding one by the ratio of 63.05 to 58.75.

In making estimates it was noticed that subjects quite generally moved the head in different ways while looking at the objects, apparently to give motion to the eye. Many were unable to tell with which eye they saw the object, often mistaking monocular for binocular vision, and one kind of monocular vision for another. Some subjects made their estimates quickly, others slowly. Some quickly at one time, but slowly at another. A comparison of the results of these different subjects did not indicate that the time element entered into the problem at all. Each subject required at least an hour for the whole series of tests.

GENERAL CONCLUSIONS.

1. There is a strong tendency to *underestimate* visible distances.

2. The illusion of judging a large object to be nearer and a small one farther, is *less common* in *binocular* than in *monocular* vision, although the value of this advantage must not be considered too high, as it only shows a less variable relation between different estimates of binocular vision, all of which may be farther wrong than the aggregate of monocular estimates, though severally more variable.

3. The greatest accuracy of judgment is that attained in *binocular vision, assisted* by various clues (as size of objects, comparison of other distances).

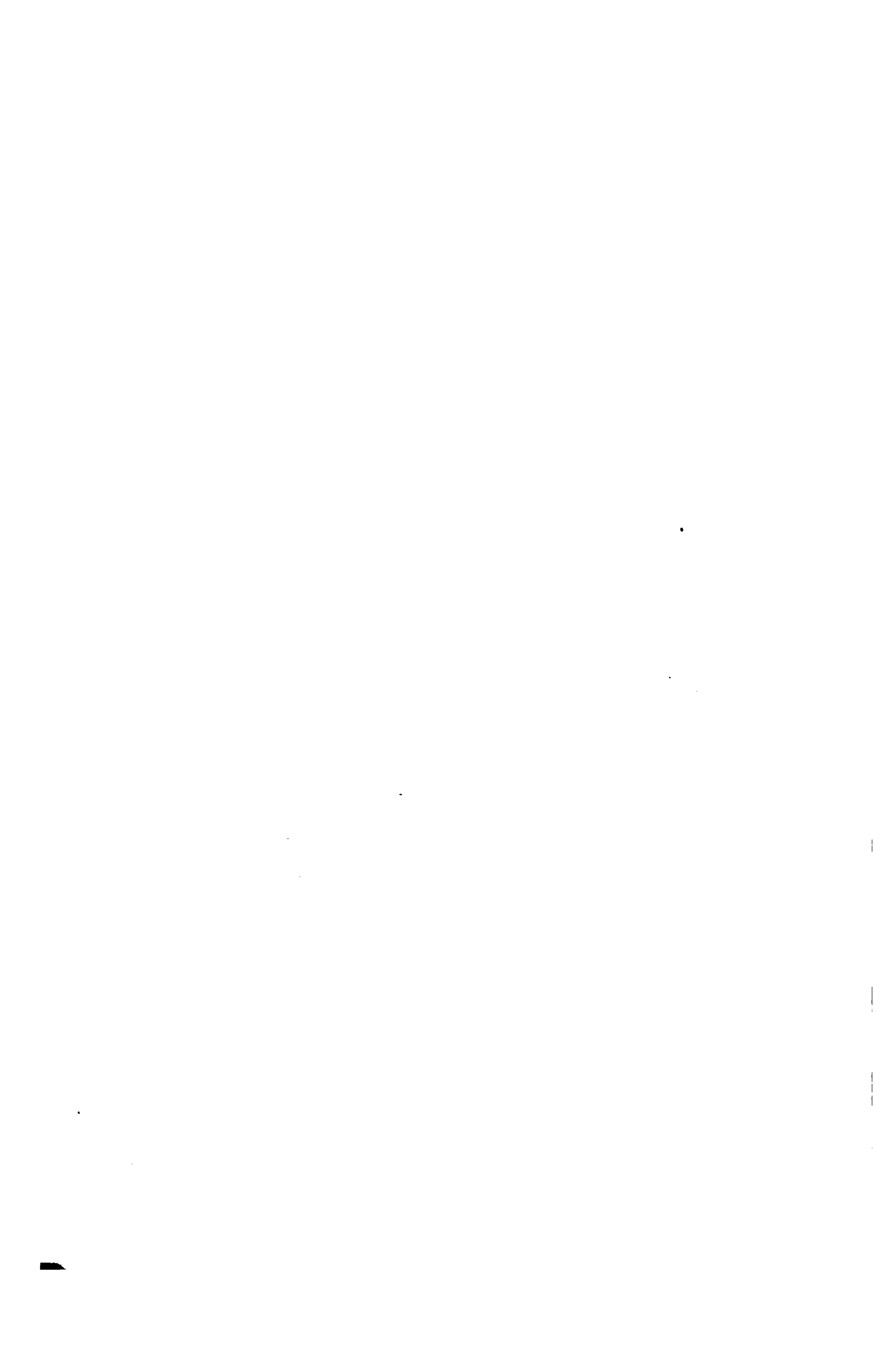
4. When vision is in no way assisted, the order of accuracy is: *right eye, left eye,* and *both eyes,* in the ratio of .87: .86: .82 (true distance being *unity*).

5. The distance of *approaching* objects is more truly judged than that of *receding* ones, in the ratio of 63.05 to 58.75 (true distance being 75.50).

6. As the true distance increases the *error steadily increases* also.

7. Distance perception has *little dependence* upon the *time* consumed in the process.

8. *Movement of head* to give motion to eye appears to be a factor in distance perception.



The Limitations of the Composition of Verbs with Prepositions in Thucydides.

DAVID H. HOLMES.

In Greek the subject of composition in general has received but little attention. So far as I know, the particular chapter which I have chosen has not been treated at all. But no attempt has been made in this paper to discuss such problems as the change of meaning caused by composition, or the case-constructions of compounds, or the influence of the preposition on the voice of the verb. As these subjects have been uniformly passed over by grammarians, we cannot reproach ourselves for treating them with the same respect.

The object of the present investigation is rather to see from an examination of the material offered by Thucydides what the indications are from this source regarding the principles underlying the composition of verbs with prepositions, and the limitations affecting the operation of these principles.

If any justification is needed for the undertaking of such a task, it is found in the interest and instruction which attach to the answers to such questions as: the range of prepositions of the different verbs; the relative affinity of verbs for prepositions; the lines of favoritism between verbs and prepositions; causes and results of loss of color in the preposition. The same inquiries will be extended to diprothetics and triprothetics.

Such is the modest aim of this paper. But whatever the results may be, they can not of course be considered binding, except so far as the language of Thucydides is concerned, until, at least, other authors are investigated in the same manner.

With this aim in view, then, I shall present the following material: First, A consideration of the individual prepositions; Second, Statistical tables for monoprothetics, diphrothetics and triprothetics; Third, An examination of the statistics.

I. A Consideration of the Individual Prepositions.

The test of a proper preposition is its ability to combine with verbs. It is only necessary to strike ἀμφὶ out of the list of proper

prepositions to get the range of combinable prepositions in Thucydides. They all occur in their simple form (ἀνὰ and ἀμφὶ twice each). The compounds of ἀμφὶ like the preposition, are mostly confined to poetry.

ἀνά. The case of ἀνὰ is different. While the simple preposition is confined mostly to phrases and poetry, it survives in composition, having a range of 77 verbs in Thucydides. Its favorite verb is χωρέω with which it occurs 144 times. It is the favorite preposition of 5 verbs, not counting its exclusives. It combines exclusively with 17 verbs, of which 9 are ἅπαξ εἰρημένα. In one of these, ἀνοίγνυμι, the place of the simple has been usurped by the compound in prose. The simple οἴγνυμι belongs to poetry. In ἀναλίσκω and ἀναλόω, we have probable usurpations of old simples which had passed out of the language in pre-historic times. Ἀνὰ does not occur as first element in diprothetics or triprothetics. The range of the simple ἀνά, like ἀμφί, is largely poetic.

ἀντί. The simple preposition ἀντὶ occurs 52 times in Thucydides. It is found in composition with 80 verbs, of which 48 are monoprothetic, 27 diprothetic and 5 triprothetic. No other preposition occurs more than once in triprothetics Its favorite verb is ἔχω with which it combines 41 times. Other favorites are ἵστημι and εἶπον. It combines exclusively with 10 verbs, of which 7 are ἅπαξ εἰρημένα.

ἀπό. The simple ἀπὸ occurs 634 times. It has a combinable range of 114 verbs, of which 112 are monoprothetic, and 2 diprothetic. The favorite verb is ἱκνέομαι, in composition with which it occurs 192 times. It is the favorite preposition of 22 verbs, not counting its exclusives. It is the exclusive preposition of 23 verbs, of which 15 are ἅπαξ εἰρημένα. In ἀπαντάω, we have a usurpation of the simple ἀντάω, which is limited to poetry. The compounds ἀποκτείνω, its passive ἀποθνήσκω, and ἀφικνέομαι are equivalents of their respective simples, except in the perfect and pluperfect of θνήσκω, which are rarely compounded in Attic Greek, never in Thucydides. In ἀπόλλυμι, we have a complete usurpation, the form ὄλλυμι being restricted to poetry. Homer has ἀπο....ὄλλυμι in so-called tmesis, where the prepositional element was strongly felt. To say, however, with Liddell and Scott, that ἀπόλλυμι is a stronger form of ὄλλυμι, presupposes a weaker ὄλλυμι for Attic prose, which does not exist. ἀπόλλυμι is stronger than ἀποκτείνω, just as ὄλλυμι is stronger than κτείνω. Ἀπεχθάνομαι is a usurpation of the poetic ἔχθω.

διά. In the simple form διὰ occurs 534 times. It has a range of 101 verbs, 98 of which are monoprothetic and 3 diprothetic. Φθείρω is its favorite verb with which it combines 151 times. It is the favorite of 14 verbs, though the favoritism is not so sharply defined as

in the prepositions treated above. It has an exclusive range of 18 verbs, of which 10 are ἅπαξ εἰρημένα. In διαφθείρω, we have an effort to usurp φθείρω, the proportion standing 3.75:1. The place of νοέω, largely confined to poetry, is taken in prose by its compounds; διὰ being its favorite preposition by more than 4 to 1.

ἐξ. The preposition ἐξ occurs in simple form 897 times. It has a range of 89 verbs of which 85 are monoprothetic, 4 diprothetic. Ἔρχομαι (ἐλθεῖν) is its favorite verb, with which it combines 47 times. Πέμπω is also a marked favorite. Ἐξ is favorite preposition of eight verbs, not counting its exclusives. The favoritism of ἐξ for verbs or of verbs for ἐξ is not strongly marked. Its exclusive range consists of 17 verbs of which 9 are ἅπαξ εἰρημένα. The simple ἀρτύω is superseded by the compounds in κατὰ and ἐξ, ἐξ alone occurring in Thucydides.

ἐν. The preposition ἐν occurs 1794 times, in which respect it stands first in the list. This fact is rather remarkable considering that it governs but one case. It has a combinable range of 67 verbs, 55 being monoprothetic, and 12 diprothetic. Its favorite verb is δίδωμι with which it combines 38 times. It is the favorite preposition of three verbs, the preference being marked with πίμπρημι which is superseded in prose by ἐμπίμπρημι. Its exclusive range consists of 13 verbs of which 11 are ἅπαξ εἰρημένα. In ἐναντιόομαι and ἐμπίμπρημι we have usurpations, the simple of the former being restricted to Ionic Greek, of the latter to poetry.

ἐπί. The simple ἐπὶ occurs 1216 times. It has a range of 156 verbs in which respect it heads the list of prepositions. 117 are monoprothetic, 39 diprothetic. Its favorite verb is εἰμι; γίγνομαι and ἐλθεῖν are also favorites, all three having ἐπὶ for their favorite preposition. It is the favorite prepositional element of 23 verbs. Here as in all cases exclusives are not counted. It has an exclusive range of 20 verbs, of which 5 are ἅπαξ εἰρημένα. There is no case of complete usurpation with ἐπὶ in Thucydides. Though the simple of ἐπιμελέομαι or ἐπιμέλομαι does not occur, yet its meaning is sharply differentiated from that of the simple. The spheres are different.

ἐς. Ἐς occurs 1692 times in Thucydides, ranking next to ἐν, and like ἐν, governing but one case. Its range of verbs is limited to 23, all of which are monoprothetic. Its combinable range is less than that of any other preposition in proportion to the number of its occurrences as a simple preposition. Its favorite verb is βάλλω, with which it is found 65 times, and of which it is also a favorite preposition, ranking next to πρός. It is the favorite preposition of only one verb, ἀκοντίζω, and has no exclusives and no usurpations.

κατά. The preposition κατὰ occurs in simple form 861 times. It

has a range of 105 verbs, 104 monoprothetic and 1 diprothetic. Ἵστημι is its favorite verb with which it occurs 260 times, and of which it is also the favorite preposition. It is the favorite preposition of 16 verbs and has an exclusive range of 25 verbs, of which 12 are ἅπαξ εἰρημένα. In κατάγνυμι we have a usurpation in the active voice. Καθέζομαι, κάθημαι and καθίζω are usurpations. The simples are poetic, ἕζομαι and ἵζω are late Greek.

μετά. Μετὰ occurs 619 times. It is restricted in the range of its verbs to 24, of which 22 are monoprothetic and 2 diprothetic. Its favorite verb is ἵστημι. Leaving out its only exclusive, μεταμέλει, it can not be said to be the favorite preposition of any verb. Μετὰ is not a general favorite in composition.

ξύν. Ξὺν occurs 35 times. It is not, strictly speaking, an Attic preposition, surviving chiefly in legal and religious phrases. It has a range of 153 verbs, of which 102 are monoprothetic, 50 diprothetic and 1 triprothetic. In respect of range of combinable verbs, it stands second in the list of prepositions, being next to ἐπί. Its favorite verb is βαίνω, with which it combines 130 times and of which it is the favorite preposition. It is the favorite preposition of 10 verbs. It has an exclusive range of 19 verbs, of which 13 are ἅπαξ εἰρημένα. There are no usurpations with ξύν.

παρά. The preposition παρὰ occurs in simple form 282 times. It combines with 54 verbs of which 48 are monoprothetic and 6 diprothetic. Εἶμι is its favorite verb, with which it occurs 173 times and of which it is the favorite preposition. It is the favorite preposition of 7 verbs and has a range of 8 exclusives, 4 being ἅπαξ εἰρημένα. Παρὰ has no usurpations. While αἰνέω is found in Attic prose only in composition (except twice in Plato), and in Thucydides only with παρὰ and ἐπί (κατά, once), yet the spheres of each are sharply defined.

περί. Περὶ occurs 478 times. It has a range of 43 verbs, all of which are monoprothetic. Its favorite verb is γίγνομαι, with which it combines 48 times. It is the favorite preposition of 3 verbs and is the exclusive of 2, both of which are ἅπαξ εἰρημένα. Περὶ has no usurpations.

πρό. Πρὸ occurs 80 times. It has a combinable range of 105 verbs, 69 being monoprothetic, 35 diprothetic and 1 triprothetic. Χωρέω is its favorite verb with which it combines 37 times. It is the favorite preposition of 7 verbs and is the exclusive preposition of 6, one of which is ἅπαξ εἰρημένον. Πρὸ has no usurpations.

πρός. The preposition πρὸς occurs in simple form 861 times. It has a combinable range of 74 verbs of which 56 are monoprothetic, 17 diprothetic and 1 triprothetic. Its favorite verb is βάλλω, with which it occurs 67 times and of which it is also the favorite prepo-

sition, being a little in advance of ἐς. It is claimed by 11 verbs as a favorite and by 2 as an exclusive. No ἅπαξ εἰρημένα and no usurpations occur with πρός.

ὑπέρ. Ὑπὲρ occurs 64 times and has a range of 11 verbs, all of which are monoprothetic. Its favorite verb is βαίνω, with which it occurs 9 times. It is not a favorite of any verb and has but 1 exclusive which is ἅπαξ εἰρημένον. No usurpations.

ὑπό. The simple ὑπὸ occurs 422 times. Its range of combinable verbs consists of 58, of which 45 are monoprothetic, 12 diprothetic and 1 triprothetic. Its favorite verb is ἄρχω, with which it combines 94 times and of which it is the favorite preposition. 3 verbs claim it as their favorite preposition and 5 as an exclusive, of which 1 is ἅπαξ εἰρημένον. In ὑποπτεύω and ὑποτοπέω we have usurpations of ὀπτεύω found only in Aristophanes, and τοπέω used once by Eustathius, the Homeric commentator.

II. Statistical Tables.

This portion of the work consist of four tables. The first shows all the simple verbs in Thucydides which combine with prepositions to form other verbs. It indicates the prepositions so used and the number of occurrences of both compounds and simples. It gives the complete statistics for monoprothetics based on simple verbs. I have taken no account of compounds whose verbal elements are not referable to simple verbs. Accordingly I have omitted verbs like ἐπικουρέω, προθυμέομαι, ἐγχειρέω referable to ἐπίκουρος, θυμὸς and χεὶρ respectively. On the other hand such verbs as ἐνδίδωμι, ξυμπροθυμέομαι, are included, being referable to the simples δίδωμι and προθυμέομαι. A compound like κατηγορέω, although the verbal element *ηγορέω does not exist, is included, since *ηγορέω is referable to ἀγορεύω. Another example is ἐκδιαιτάομαι (διαιτάω). Such verbs are starred. The second table shows the same facts for the diprothetics and triprothetics as the first table for the monoprothetics. The third table shows the different combinations of prepositions as seen in diprothetics and triprothetics. The fourth shows the relative range of the prepositions, their favorite verbs and statistics. It also combines for the sake of convenience some of the more salient points of the other tables.

It is impossible that the statistics shown by the appended tables should be absolutely without error. Infallibility belongs only to the enthusiasm of youth. But it is believed that no false impressions can be gotten from the figures indicated.

III. An Examination of the Statistics.

INTRODUCTORY.

The preposition is a local adverb.

The prevalent definition of the verb is predication.

There is no kind of predication that does not imply motion, actual or potential. At any rate in the consideration of the preposition or its relation to the verb, we are justified in making that element predominant which is necessarily the most fundamental. Motion in a verb, then, is that quality in a verb which is capable of direction.

The fundamental notion of the preposition is one of place. The deviations from this notion, the transfers from place to time, or the paling out of the original color, all have their basis in the primal notion of place.

It is unnecessary to demonstrate the interdependence and kinship of the notions of motion and place. Place involves motion just as the preposition involves the verb. It also lies implicitly in the nature of the subject that certain forms of motion will have a natural affinity for certain relations of place, while some forms of both motion and place will absolutely refuse to coalesce. This is due to the different modifications of motion assumed by the verb. By modification of motion, we mean: *the alteration of its color, the definition of its kind, or the indication of its direction.* Absolutely pure motion is free from such modification. If there were a verb which designated motion without reference to color, direction, or kind, it could be said to express pure motion. But pure motion does not exist in language. Language begins with concrete notions, however general the application which the expression of that notion may have had, after the notion had once taken form. Thus there are verbs which express motion in a more general way than others. E. g., εἶμι, however concrete the notion for which it originally stood, is used for so many different kinds of motion, that, for purposes of this paper, it can be said to express relatively pure motion.

The motion in a verb may be modified either internally or externally.

Internal Modification.

For purposes of the present paper, verbs may be divided into two classes: those expressing actual motion, and those expressing potential motion. Verbs of actual motion include those verbs which express motion with its kind, direction or color more or less distinctly marked. Verbs of potential motion include verbs of existence, speech, thought, perception.

Verbs expressing relatively pure motion are rare, but language does not require many. The verbs **εἶμι, ἔρχομαι (ἐλθεῖν)** and more

remotely, *βαίνω*, furnish the best examples of relatively pure motion in the language.

That *εἶμι* is well selected is attested by the following considerations: I. It is used for various kinds of motion without distinction. Thus, for *walking:* Il. 7, 213: *ποσσὶν ἤϊε μακρὰ βιβάς*; for *hastening:* Od. 15, 213: *ἀλλ' αὐτὸς καλέων δοῦρ' εἴσεται*; for *flight of birds:* Il. 17, 756; for the *motion of things*: Il. 3, 611: *πέλεκυς εἶσιν διὰ δουρός*; &c., II. It is shown by the almost equal balance of the "whither" and "whence" relations as seen in the composition of the verb with the prepositions *ἀπὸ* and *πρός*, *ἄπειμι* occurring 33 times and *πρόσειμι* 29 times. This consideration is not set aside by the fact that *ἐπὶ* occurs 83 times in composition with this verb, because *ἐπὶ* is hostile, the sphere of *ἐπιέναι* in Thucydides being military—a fact constantly to be borne in mind. Hence the preponderance of *ἐπὶ* is of no account in this connection.

Ἔρχομαι (*ἐλθεῖν*) is a good example also, as shown by the following facts: I. It is frequently used with a supplementary participle showing the manner or the kind of the motion. Thus, Il. 11, 715: *ἦλθε θέουσα*; id. 10, 510: *πεφοβημένος ἔλθῃς*; Od. 6, 40: *πόδεσσιν ἔρχεσθαι*; Il. 5, 204: *πεζὸς εἰλήλουθα*; of *flying:* Od. 14, 334. In fact the use of this verb of the motion of spears, javelins, or of natural phenomena such as rivers, wind and storm, clouds and stars, time and sound, is too frequent to need confirmatory references and quite sufficient to denote the relative purity of the idea of motion contained in it. II. Another evidence is furnished by the fact that *ἔρχομαι* plays the part of present to both *ἥκω* and *οἴχομαι*, two verbs of motion with exactly opposite points of view. III. Here again we find that same prepositional balance as in the case of *εἶμι*, except that in this case the prepositions are *ἀπὸ* and *ἐπί*, *ἀπέρχομαι* (*ἀπελθεῖν*) and *ἐπέρχομαι* (*ἐπελθεῖν*) each occurring 76 times.

Next to *εἶμι* and *ἔρχομαι* (*ἐλθεῖν*), though by a considerable interval, ranks *βαίνω*. In *βαίνω* at least the color becomes visible. Yet no little freedom is also here manifest, as a participle often accompanies the verb to show the kind of motion. Thus, Il. 2, 167: *βῆ ἀΐξασα*; and id. 2, 665: *βῆ φεύγων*. Another evidence is that certain tenses of *βαίνω* are represented by *εἶμι* and *ἔρχομαι* (*ἐλθεῖν*).

These three verbs, *εἶμι*, *ἔρχομαι* (*ἐλθεῖν*) and *βαίνω*, sustain very much the same relation to what are ordinarily classed in the grammars as verbs of motion, as *ποιέω* does to what are more broadly termed verbs of action.

The moment color is given to the motion of a verb, that moment internal modification sets in and the sphere of the verb is narrowed.

The first curtailment is given to the idea of motion in the expression of its character or kind. Thus, βάλλω, πέμπω, πίπτω, φέρω, ἵστημι, τίθημι, ἔχω; and πλέω, θέω, τρέφω, &c. Still further curtailment, and more important in this connection, is seen in verbs which express with greater or less definiteness, the direction of their motion. Thus, ἥκω, οἴχομαι, διώκω, ἀκολουθέω, &c. Verbs in which the idea of motion is obscured or even lost in the color of the action, form another group, by far the largest, owing to the almost endless varieties of activity. As soon as a new activity is introduced into life, a new verb is created in language. Thus the history of the verb becomes the history of civilization. It is evident that verbs like τειχίζω, βοηθέω, μάχομαι, &c., have more color or are more picturesque than εἶμι, πέμπω or ἥκω; while verbs like ἄρχω, κλέπτω, ὄλλυμι, καίω, &c., possess still less motion if not indeed also still more color. Thus, the idea of motion may be almost wholly supplanted as in verbs like εὕδω and θνήσκω. Thus we see that the idea of motion in a verb is modified internally in color, kind or direction.

External Modification.

In external modification the problem is simpler. It is not germane to our subject to discuss here the external limitations of motion effected by adverbial or adnominal means. Such influences do not effect any change in the character of the motion expressed by the verb. I have already defined what I mean by the term modification. External modification is limited to direction and hence to the prepositions. We have to do here with prepositions in composition only. Our subject might be stated thus: The limits set to external modification by internal modification. It is evident that certain kinds of motion are inconsistent with certain varieties of direction. Such limitations are natural. Again certain other kinds of motion may be so characteristic of certain departments of literature as to be confined more or less strictly to these departments. On the other hand, the department may be of such a nature as to exclude certain varieties of direction or of modification. Again, the affiliation of a certain kind of motion for a certain direction may be so strong as by that very fact to refuse affiliation with other directions in no way hostile in themselves, thus bringing about *usurpation* from the point of view of the direction, and *exclusion* from the point of view of the motion. Such limitations are empirical and artificial.

Having thus seen that the principal elements at the basis of verb and preposition are motion, place, direction, let us see how these

elements affect the composition of verbs with prepositions, so far as indicated by the language of Thucydides; and what light they throw on the questions of range, affinity, favoritism, loss of color, &c., announced at the beginning of our discussion.

Perhaps the most practical way of getting at a result is to collect all the verbs having the greatest combinable range of prepositions together, and place side by side with them those verbs having the next highest range, and so on to a point where a clear observation can be made of the change which takes place in the kind, direction or character of the motion expressed by them, as their prepositional ranges become narrower. See page 16 of the accompanying statistical tables for a list arranged for this purpose.

As I have already shown, relatively pure motion is best seen in εἶμι, ἔρχομαι (ἐλθεῖν) and βαίνω. This motion is stamped with a certain character in the verbs βάλλω, ἄγω, ἔχω, φέρω, &c., is given manner in πλέω, πίπτω, ἵστημι, θέω, &c., direction in ἥκω, λείπω, ἕπομαι, διώκω, &c., while in verbs like μάχομαι, ἀναγκάζω, &c., the color of the action is more prominent than the notion of motion, which continues to grow less in ἄρχω, δέω, γελάω, and is scarcely felt at all in ἀδικέω, εὕδω, θνήσκω.

The same variation in color is also seen in verbs expressing potential motion. Thus, in verbs of existence, εἰμὶ and γίγνομαι may be taken as being most nearly colorless. The metaphysical idea of motion in such verbs often becomes physical when given direction. But the idea of motion fades out as the idea of existence gives place to condition. Cf. ζάω, εὐδαιμονέω.

In like manner, in the case of verbs of speech, ἀγορεύω, εἶπον and λέγω (φημὶ not occurring in composition) may be said to be most nearly colorless. But the idea of speech assumes character in καλέω and γράφω,* still more so in βοάω, δείκνυμι, still more so in ψηθίζω-ομαι, ὄμνυμι, μαρτυρέω-ομαι, and becomes faint in διδάσκω, ὁμολογέω.

Again in verbs of thought and perception. This variety of potential motion finds its purest expression in the verbs νοέω-ομαι, γιγνώσκω (οἴομαι not being used in composition), becoming colored in κρίνω-ομαι on the one hand, and in εἶδον, ὁράω and ἀκούω on the other; while in μιμνήσκω, φοβέω and ἐλπίζω the mobility of the thought is replaced by color, and in αἰσθάνομαι and μανθάνω the notions of thought and perception are mixed.

It appears therefore from this general survey of the combinable verbs, with the aid of the statistical tables given below, that the

*The constructions of γράφω justify this classification.

range of prepositions is largest in the case of those verbs which express motion most nearly in its purity, actual or potential, physical or in the form of existence, speech, thought, or perception; and as those notions give place to definition of color, kind or direction, the range of prepositions grows less. That is to say:

In general, the range of combinable prepositions of a verb is in direct ratio to the nearness with which the verb expresses pure motion.

Until other authors are examined in the same way, however, we cannot safely go further than to say that the indications for Thucydides point in this direction, and even here there are a few possible objections. These are not many and not difficult to answer.

I. It may be urged that βάλλω, although not expressing pure motion as we have defined it, inasmuch as the character of the motion is designated, nevertheless has a larger range of prepositions than any other verb including any of those instanced as verbs of relatively pure motion. That is to say, βάλλω heads the list with a range of 16 prepositions, no other *single* verb in Thucydides having more than 14. Ὑπὸ is the only preposition out of the 17 proper prose prepositions with which it does not combine. But both ἀμφὶ and ὑπὸ are in its Homeric range. On the other hand, εἶμι has a range of only 12, ἔρχομαι (ἐλθεῖν) 13, and βαίνω 13. In reply, there are three considerations that must not be overlooked: (1) Not one of the verbs εἶμι, ἔρχομαι (ἐλθεῖν) and βαίνω has in its simple form a complete tense-system, and hence they supply each other's deficiencies. Take the three verbs as one, however, and the range of prepositions increases to 15. (2) The absence of ἀντὶ from the range of εἶμι, ἔρχομαι (ἐλθεῖν) and βαίνω is significant. It is due to the intense feeling of ἀντί. This consciousness of ἀντὶ shows itself in other ways to be noticed later on. Its sensitiveness is so marked as to attract a verb of more feeling or color than mere motion, and hence it is found with verbs like ἀγωνίζομαι, εἶπον, ἵστημι, τάσσω, &c. This community of feeling between verb and preposition we shall have occasion to notice again in still other manifestations. That the feeling of ἀντὶ in composition is stronger than that of any other preposition, appears in diprothetics and triprothetics. Its range of diprothetics relative to its whole combinable range is greater than that of any other preposition and it is first element in 5 out of 9 triprothetics. (3) βάλλω is a military term. Thucydides' is a military history. Every possible turn to perhaps the most comprehensive military term in the whole range of the language, would most naturally be necessary, owing to the military character of the department. This would account for the large prepositional

range of βάλλω in Thucydides. In Homer on the other hand, where the department is the same, its range is limited to 14 including the poetic ἀφμί, while βαίνω alone has a range of 15, which increases to 17, counting ἀμφί, in connection with ἔρχομαι (ἐλθεῖν) and εἶμι, ἀντὶ still being the missing link. This influence of department manifests itself again in a negative way in Demosthenes, where βάλλω stands 15 (εἰσβάλλω, the most military of all military terms, naturally being missing), while the εἶμι-ἔρχομαι- and βαίνω-combination stands 16.

II. Another objection of very much the same sort might be raised from the fact that γράφω in the verbs of speech, has a range of 9 prepositions, which is larger than that of any of the verbs cited as expressing relatively pure speech, ἀγορεύω, εἶπον or λέγω. True, ἀγορεύω has a range of but 4, εἶπον 7, and λέγω 5. But here again, as in the case of verbs of relatively pure physical motion, no one of the verbs makes a complete system of Attic tenses. Taken collectively they have a range of 10 prepositions. Γράφω had the advantage in that it started life as a verb of actual motion. Its later legal sphere was again in its favor. That γράφω should get the better of the verbs most nearly colorless in the orators, is what would be expected from the legal technique employed in that department. Accordingly, in Demosthenes, the proportion is 13 for γράφω, as against 8 for the group ἀγορεύω, εἶπον and λέγω.

III. A third objection may be found in the narrow range of prepositions of the verbs ἱκνέομαι and στέλλω, in which the notion of motion clearly predominates. Here again the community of feeling between verb and preposition comes into play, especially in the case of ἱκνέομαι. In ἱκνέομαι, "arrive", the point of view of the motion is "whence". The notion is not so much "come to", as "come from —— to". Hence ἀπὸ is the preposition for which ἱκνέομαι has the strongest affinity. But the addition of ἀπὸ did not create any modification in the idea of the verb. The notion was still "arrive", the point of view of the motion being simply reinforced. Now began a race between ἱκνέομαι and ἀφικνέομαι in which ἀφικνέομαι was the winner, debarring its rival entirely from the track of prose. The problem which the language then had to solve had changed from defining the direction of ἱκνέομαι to defining the direction of ἀφικνέομαι, that is, from defining the direction of a simple, to defining the direction of a compound. But the language does not take so kindly to diprothetic composition as to monoprothetic, and although attempt was made even here toward diprothetic composition, of which occasional evidence remains (εἰσαφικνέομαι Od. 12, 40; 8 times in Homer; προ- and προσαφικνέομαι, see Table II.), yet

it preferred in this case to show the direction by prepositions in the simple form. The combination ἀφικνέομαι made the loss of color of ἀπὸ merely a matter of time. Thus in ἀφικνέομαι the compound has usurped the place of the simple, the preposition ἀπὸ having come in to the exclusion (or nearly so) of other prepositions, though a few cases exist of ἱκνέομαι in composition with διά, ἐκ (one each in Thuc.), with διά, ἐκ and κατὰ (Hom.) and ἐπί (Dem.).

The case of στέλλω is of the same kind with the additional circumstance that the official character of στέλλω gives it a much narrower range of prepositions (see Table I.). When στέλλω fails, πέμπω supplies the deficiencies.

Additional evidence for the truth of our main thesis is derived from a consideration of the diprothetics and triprothetics. Here as in monoprothetic composition, where there exists most mobility, there exists also most modification. The more nearly the idea of the simple verb approaches pure motion, the wider its range of diprothetic combinations. Pursuing the same method as in monoprothetics, we find that, with reference to range of diprothetic combinations, the verbs run as follows:

ἵστημι	13	πέμπω	5
εἶμι	11	χωρέω	5
ἔρχομαι (ἐλθεῖν)	10	λείπω	4
ἄγω	9	στέλλω	4
βαίνω	9	*ἁλίσκω	3
βάλλω	8	βιβάζω	3
αἱρέω	6	γιγνώσκω	3
ἔχω	6	δίδωμι	3
ἧμαι	6	ἕζομαι	3
πλέω	6	ὄλλυμι	3
λαμβάνω	5	τίθημι	3
		φέρω	3

For further particulars see Table II.

The εἶμι—ἔρχομαι (ἐλθεῖν)—βαίνω—combination gives us here a remarkable range of 22 prepositional doublets. The prominence of ἵστημι in this connection is interesting. The large number of combinations possible with ἵστημι is due to the predominance of κατὰ and ἀνὰ as second elements in its diprothetics. The modification produced in the motion of ἵστημι by κατὰ and ἀνὰ in composition with it, is not so much a change in its direction as a reinforcement and an extension of it, from opposite points of view. "Up" and "down", like "high" and "deep", are the same idea logically, but from exactly opposite points of view. So the diprothetic compounds of ἵστημι which have κατὰ or ἀνὰ as second element, give, in feeling, practically a monoprothetic resultant. In this same

way κάθημαι astonishes us with a range of 6 combinations, although both a usurpation and an exclusive in its monoprothetic form. The explanation, however, is easy, as the diprothetics of ἧμαι are practically monoprothetic in feeling, owing to loss of color in κατά, the second element in composition.

In the case of triprothetics, 7 in all, the range of verbs is too narrow for valuable results from comparison, but so far as they go, they fall into line with the views advanced in this paper. Four of them, ἄγω, εἶμι, ἔρχομαι (ἐλθεῖν), ἵστημι, all in the foreground as verbs of motion, will also be remembered as the most prominent diprothetics and among the most prominent monoprothetics. It is a curious fact that the remaining three, ἐλαύνω, σείω and εὑρίσκω, in which the idea of motion is by no means secondary, are not found among the diprothetics of Thucydides. The discussion of these verbs in their triprothetic relation belongs to the ἅπαξ εἰρημένα of Thucydides, a treatment of which is not germane to this investigation.

Suggested Corollaries.

Growing out of the above discussion are several special phenomena, from a consideration of which can be deduced corollaries to the main theorem. Within the limits of the present study we cannot hope to be exhaustive or more than suggestive, as many of the points alluded to could, of themselves, be carried out to the point of special monographs.

Favoritism of Verbs for Certain Prepositions.

One of the first things to strike the eye in an examination of the foregoing statistical tables, is the great preponderance of some prepositions over others with certain verbs. Let us see if there is any principle underlying such favoritism, and what light it throws on the general subject of the composition of verbs with prepositions. It is not our purpose to discuss each individual case, but merely to point out general tendencies. A few examples from Table I. bearing on each point will suffice.

I. Extension and Reinforcement.

'Αλλάσσω combines with ἀπὸ 27 times to 24 times in all with 6 other prepositions. The idea of "change", "alter", naturally carries with it a very strong feeling for the relation "from", and hence the marked preference for ἀπό.

Βοηθέω, as would naturally be expected from its meaning, is found with ἐπὶ 27 times, with πρὸς 25 times, twice as frequently as with any other preposition. In like manner δέχομαι favors πρός, the

ratio being πρός : 5 others : : 55 : 36; thus διώκω favors κατὰ and ἐπί: ἥκω favors πρός; θνήσκω favors ἀπό; ἵστημι favors κατά; πέμπω favors ἀπὸ and ἐκ; στέλλω favors ἀπό; ἐλαύνω favors ἐκ; ἕπομαι favors ἐπί; &c. Thus we see that the first movement between the verb and the preposition is in the line of the least resistance—*extension* and *reinforcement*. The nature of a verb can be best appreciated by a study of its favorite prepositions, the nature of a preposition, from its favorite verbs.

II. Exclusion.

This preference of a verb for a preposition may be so strong as to drive out all other prepositions, as in the case of ἀγείρω with σὺν, after Homer. In like manner, καίω with κατά; κτείνω with ἀπό; φθείρω with διά (with ἀπὸ once in Thuc.). This gives rise to what we may term *exclusion*. Verbs which combine with only one preposition may be called exclusives. Exclusives, however, are to be sharply distinguished from ἅπαξ εἰρημένα, since a single occurrence would not generate sufficient force to produce exclusion.

III. Usurpation.

Again the preference of the verb for the preposition may be so marked as to bring about *usurpation*, or a complete effacement of the simple by the compound. Such usurpations are most notable among exclusives, though cases are not infrequent where the different compounds have acted conjointly in the displacement of the simple. Thus of the first sort are ἀνοίγνυμι, ἀναλόω, ἐναντιόομαι, καθέζομαι, κάθημαι, &c. Examples of the latter are: the compounds of αἰνέω, νοέω, &c.

IV. Phraseological Expressions.

This preference for a certain preposition is often due merely to a transferred signification imported by the prepositional element which gives a phraseological resultant. Thus, ξυμβαίνω, ὑπάρχω, πάρειμι, παρέχω, &c.

V. Loss of Color of Prepositions.

Another natural concomitant of this principle of favoritism is the loss of color of the preposition. This has already been incidentally alluded to. This loss of color is most prominent in compounds which are mere reinforcements of the meanings of the simples. Where least needed, the feeling is least. We look for loss of color, therefore, first in extensions, exclusions, and usurpations. In extensions, the similarity in meaning, which was the basis of the attraction, became the cause of the fading out of the color. What became the life of the compound became the death

of the preposition in the compound. In exclusions and usurpations the loss of color became easier by reason of the absence of contrast with other prepositions which would have operated to some extent in keeping up the difference in feeling. The function of the simple becomes the function of the compound, the simple often being relegated to poetry, while the compound does duty in prose. The simple often reappears in late Greek, a striking parallel to which is found in the Silver Latinity. Thus, καθέζομαι, ἕζομαι being poetic and late Greek. Cf. also ἀφικνέομαι, ἀνοίγνυμι and ἀπόλλυμι. The preposition is sometimes ignored in augment. Thus, ἠνέῳγμαι, N. T. Rev. 10, 8; Heliodor. 9, 9; ἠνεῴχθην, Dio Cass. 44, 17; ἐκαθεζόμην Xen. An. 1, 5, 9; and frequently in Attic. The emergence in late Greek of strengthened compounds often follows loss of color in the preposition. Thus, the strengthened combinations προσεπι-, ἐπιπροσ-; ἐξαπο-, ἀπεξ-; συμμετα-, μετασυν-; προσεισ-; καταντι- and ἀντικατα-, are not uncommon in late Greek, but rare in classical Greek. Cf. Table III.

VI. Relative Consciousness of Prepositions.

The loss of color in the preposition naturally suggests the relative consciousness of the prepositions. Here again we cannot hope to be more than suggestive. Valuable service is rendered in this connection by the diprothetics. A careful examination of Tables II. and III. will show the operation of two principles in diprothetic composition. First—a desire for reinforcement—the extension side. Second—a desire for modification—the plastic side. Now reinforcement implies weakness. Language is continually building itself up where long use or abuse has broken it down. In the case of monoprothetics it is evident that most weakness is found in extensions and usurpations. A monoprothetic whose prepositional element has faded out is felt as a simple. This leads either to a discarding of the preposition altogether and a restoration of the simple, as actually occurs in late Greek, or to reinforcement. Reinforcement of such monoprothetics gives a diprothetic form but a monoprothetic feeling. The language of Thucydides presents us with a range of 387 separate monoprothetics, but only 86 diprothetics. It is fair then to conclude that the language may consent to a single union but resist a double one, and since the growth of language is along the line of the least resistance, we find that diprothetics having reinforcement as their cause, greatly preponderate over the plastic use. Now when language reinforces it brings to bear the most powerful means at its command. This is seen in the predominant prepositions in

diprothetic composition. Those prepositions hold their color longest which play the most prominent role in diprothetics and triprothetics. Thus, ἀντὶ appears as first element in 27, ἐπὶ in 39, ξὺν in 50, πρὸ in 35, and πρὸς in 17 diprothetics, while of triprothetics, ἀντὶ has 5, and ξύν, πρό, πρὸς and ὑπό, have each one. The absence of ἐπὶ in triprothetics would seem to militate against this view, but coincident with this absence, it occurs as a second element in 8 out of the 9 triprothetics in Thucydides, reinforced by ἀντὶ in 5 out of the 8 cases and by πρὸς in one, thus indicating the fading out of the color of ἐπὶ in diprothetics.

This tendency to make that combination in which there will be the most strength, shows itself also in another way. In the formation of a diprothetic, when there exists a choice between monoprothetics in ἐκ or ἀπό, or between εἰς and πρός, or between κατὰ and ἀντί, the forms in ἐκ, εἰς and κατὰ are chosen. The exceptions can usually be explained. Thus, ἄγω (see Table II.) has ἐκ instead of ἀπὸ as second element in diprothetics; εἶμι, ἐκ 3 times, ἀπὸ once; ἔρχομαι, ἐκ instead of ἀπό: ἵστημι does not count, as other considerations are involved, such as loss of color of κατὰ and the military character of ἀφίστημι, accounting for the preponderance of these elements here. In this phenomenon we are limited to the class of diprothetics which represent the plastic side. Naturally enough, those simples predominate here in which the motion is least obscured. Where modification is necessary, room and mobility are needed. It follows that the second elements of diprothetics represent two opposite conditions of things: 1st, loss of color of the preposition; 2nd, vividness of preposition. In the first case, reinforcement was aimed at; in the second, modification of the idea of the verb. Hence there is greater diprothetic feeling in the latter class than in the former, and from this follows the comparative ease with which diprothetics of the former class were formed and their consequent preponderance over the latter class.

In triprothetics, the principle of reinforcement again is chiefly operative, and here naturally enough, the second element is the least conscious. It is noticeable that ἐπὶ is second element in 8 of the 9 triprothetics in Thucydides.

Summary.

In the foregoing discussion I have endeavored to prove the theorem for Thucydides:

In general the range of combinable prepositions of a verb is in direct ratio to the nearness with which the verb expresses pure motion.

From the demonstration of this theorem can be deduced the following corollaries:

1. A verb unites most readily and first with that preposition which is in a sense an extension of its own meaning.

2. The converse is also true, that that preposition has the greatest affinity for those verbs which are in line with its own direction.

3. The character of a verb is best shown by its favorite prepositions, or more narrowly, the best index of a verb is its favorite preposition.

4. The converse is also true, that the character of a preposition is best shown by its favorite verbs.

5. Favoritism is extension, extension leads to exclusion, exclusion leads to usurpation. All contribute toward the loss of color of the preposition.

6. Loss of color in the preposition is attended with a decline of the simple, a narrow range of combinable prepositions, followed, perhaps, by emergence in late Greek of the simple or of a strengthened compound.

7. Those monoprothetics which are extensions of their simples or which reinforce the point of view of the simple, enter most into diprothetic composition.

8. Those prepositions which preponderate in monoprothetics, preponderate also as second elements in diprothetics.

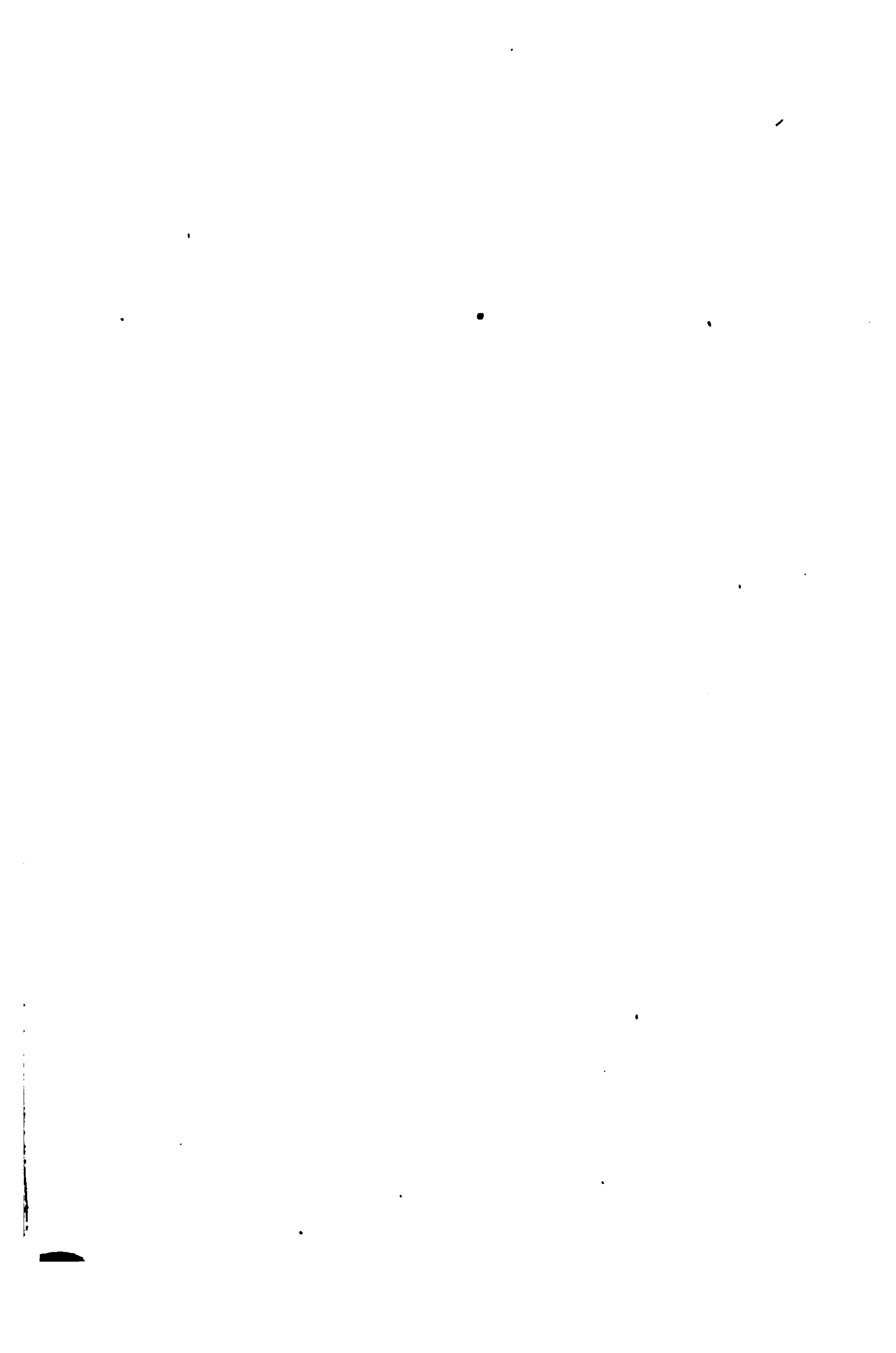
9. Those prepositions have lost most color which appear most as second elements in diprothetics.

10. Those prepositions are most conscious which appear most as first elements in diprothetics.

11. In general, in the formation of diprothetics from a given simple, the formation is made on the basis of the monoprothetics in ἐκ, εἰς and κατά, instead of in ἀπό, πρὸς and ἀντί, where choice is possible.

12. In triprothetics, the first element is the most conscious, the second the least, while the third is variable.

It is the operation of the above principles that defines the Limitations of the Composition of Verbs with Prepositions in Thucydides.



Editorial Notes.

ANNOUNCEMENT—Beginning with January 1897, the Kansas University Quarterly will be published in two series: Series A. for Science and Mathematics; Series B. for Literature and History. The management is assured that this arrangement will be much more satisfactory to those who consult the Quarterly and exchange with it.

Volume V will close with the present issue, having, therefore, but two numbers. A complete file of the Quarterly for the scientist will therefore be vols. I—V, and from Vol. VI on in series A. For students in history the file will run from Vol. I to Vol V, and then via Series B. Subscribers will for the present receive both series. Exchanges will receive the series suited to their character. Libraries and institutions will receive both.

During the summer of 1896, Prof. Haworth, of the Department of Physical Geology and Mineralogy of the University of Kansas, was engaged in making investigations principally in the western part of Kansas. He had a total of eleven helpers during the greater part of the summer. Early in the season Mr. W. R. Crane spent about four weeks in a further study of the coal beds, and now has the field work principally done for a Report on the Coal Deposits of Kansas. Later in the season he did work in the extreme northwestern part of the state in connection with the investigations of the water supply of the state conducted by Prof. Haworth for the State Board of Irrigation. Mr. W. N. Logan took the field in May to continue investigations begun last season on the general stratigraphy of the Benton and Niobrara. After devoting about six weeks to this work he spent from three to four more weeks in the northern tier of counties investigating the underground water. Dr. George I. Adams, who had spent the greater portion of two previous summer vacations in connection with the field work of the University Geological Survey, devoted ten weeks to field work in west central Kansas. He had with him one assistant all the time and two part of the time.

Dr. G. P. Grimsley, of Washburn College, has undertaken the task of making a careful study of the Gypsum deposits of Kansas and, in connection with Prof. Bailey of the Dept. of Chemistry, is preparing a Report on the Gypsum of Kansas which will treat the subject from the standpoints of Geology, Chemistry and economic value.

Prof. C. S. Prosser, of Union College, Schenectady, New York, devoted eight weeks to a study of the Lower Cretaceous in Kansas, and to the "Red Beds," which are of somewhat doubtful age. It is hoped that he will be able to decide many mooted questions now connected with this interesting area.

Prof. Haworth himself took up field work in April by continuing his investigations of the lead and zinc deposits in the southern part of the state, and later made a detailed study of an area covered by four U. S. Topographic sheets lying principally south of the Arkansas river and including Dodge City and Garden City. This latter work was done under the auspices of the United States Geological Survey, the report on which will be published by that bureau.

The draughting and literary work is now being done for Vol II of the University Geological Survey of Kansas, a volume to be devoted to the general stratigraphy

of the Cretaceous and the Tertiary of the state with such other matters as are closely allied. Also laboratory and literary work is being carried rapidly forward on the several chapters on coal, gypsum, etc. to constitute Vol. III of the Survey, a volume to be devoted to economic geology entirely.

It is hoped that the Legislature at its next session will make provision for the proper illustration and publication of each of these volumes.

The Elements of Physics, by E. L. Nichols and W. S. Franklin, Vol. II, Electricity and Magnetism. This book is of a decidedly higher grade than the majority of text-books upon the subject, and really forms a connecting link between them and the more elaborate treatises upon special departments of physics. Starting with a chapter upon the properties and analysis of distributed scalars and vectors, the work discusses clearly and fully the topics usually treated under the subjects of electricity and magnetism.

The proofs of the various propositions are in general clear and as simple as the difficulties of the subject permit. A good knowledge of calculus is required, and the student should be especially familiar with the idea of infinitesimals

The book is thoroughly up to date, even such a recent subject as Roentgen's discovery being treated. It may be highly recommended to those students possessing the requisite mathematical knowledge, as a thoroughly scientific and accurate presentation of the subject. In order, however, to derive the greatest benefit from its study, the student should do a large amount of outside reading, describing, in detail, the phenomena treated - A. St. C. D.

With this number Mr. George Wagner is added to the Editorial Board of the QUARTERLY. He has already been helpful in rectifying the exchange list, and will in future be in charge of the circulation.

The Editor of the UNIVERSITY QUARTERLY has studied the type-writer question with some care. He has come to the conclusion that every professional scholar should have and use a typewriter, for the sake of accuracy, neatness and economy of time. What machine is the best for a professional stenographer he does not pretend to know, but he has concluded that the Hammond is the best for the professional man who is to operate his own machine. And this from the following grounds: Having watched the work of four or five standard machines in the hands of his colleagues he finds that the Hammond work is vastly superior in alignment and uniformity of impression; it is lighter and less bulky than other standard machines; the single keyboard with shift-keys is much sooner learned, and as rapidly worked by any but professionals; it makes less noise than some; the quick and inexpensive change of font makes it convenient for all, and especially desirable for language men. For these reasons the Editor uses and recommends the Universal Hammond.

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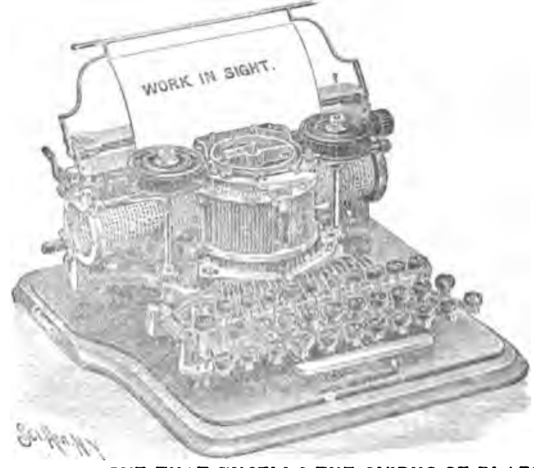
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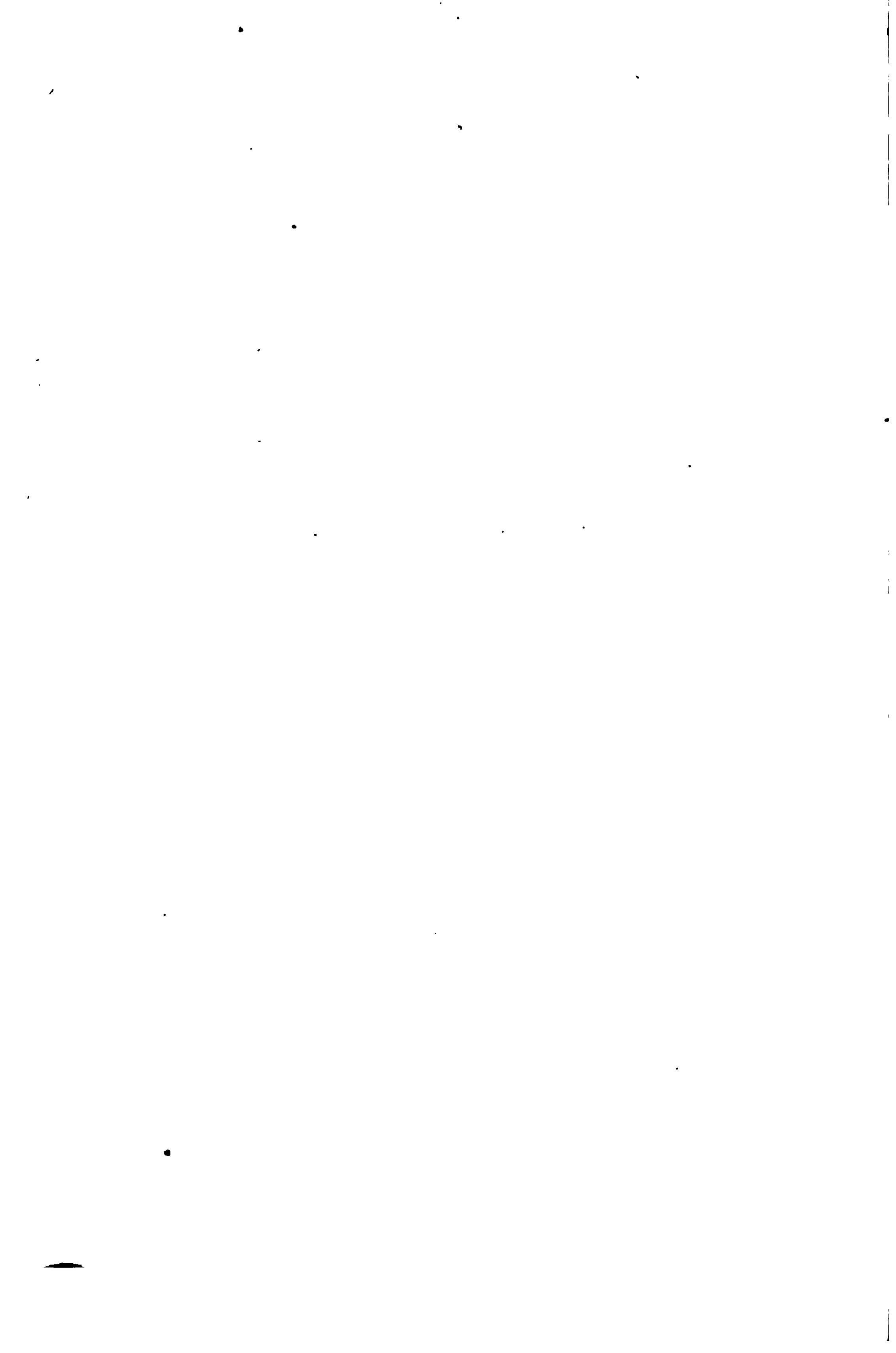
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Tables to "Composition of Verbs in Thucydides."

STATISTICS

for the Composition of Verbs with Prepositions in Thukydides.

Prepared by D. H. HOLMES, Ph. D.

Tafel I.

Statistik für monoprothetische Verben.

Sie giebt an:

a) alle kombinierbaren Verben bei Thukydides;
b) die Reihe der Präpositionen eines jeden Verbums und umgekehrt die Verbenreihe einer jeden Präposition;
c) die erforderlichen Zahlenangaben.

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
ἀγγέλλω*)	1		20	1	1		15	8				13	10	1				9	ἀπὸ	43
ἀγείρω											6							1	ξὺν	6
ἄγνυμι									2									1	κατὰ	
ἀγορεύω									2		3			5	2			4	πρὸ	1
ἄγχω			2															1	ἀπὸ	
ἄγω	36		33	7	18	9	43	13	17		14	8	1	8	45		7	14	πρὸς, ἐπὶ	110
ἀγωνίζομαι		2		3		1					4			1				5	ξὺν	22
ἀδικέω											2							1	ξὺν	119
ἀθρέω	1																	1	ἀνὰ	1
ἀθροίζω											2							1	ξὺν	8
αἰνέω							14		1			40						3	παρὰ	
*αἱρέομαι		2					2								3			3		
αἱρέω	32		28	17	11				42		7	1	5	1			3	9	κατὰ	154
αἴρω		2	10				22		7		4							5	ἐπὶ	58
αἰσθάνομαι														10				1	πρὸ	120
αἰσχύνω									1									1	κατὰ	8
*αἰτέομαι												1						1	παρὰ	
αἰτέω		1	6															2	ἀπὸ	21
αἰτιάομαι							6		2									2	ἐπὶ	15
ἀκολουθέω							7				3							2	ἐπὶ	13
ἀκοντίζω								8	2									2	ἐς	4
ἀκούω							2	13	2								27	4	ὑπὸ	74
ἀλγέω			1										2					2		2
ἀλείφω					2													1	ἐκ	3
ἀληθεύω							2											1	ἐπὶ	
*ἁλίσκω	2																	1	ἀνὰ	
*ἀλλαξείω			1															1	ἀπὸ	
ἀλλάσσω		2	26	9	1	1			3	1	7							8	ἀπὸ	1
*ἀλόω	16																	1	ἀνὰ	
ἁμαρτάνω				3	2													2	διὰ	41

*) In der alphabetischen Anordnung sind dieselben Prinzipien wie in den n Indices und Wörterbüchern befolgt worden.

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
ἀμάω				1														1	διά	
ἀμελέω												1						1	παρά	5
*ἀμύνομαι		2												2				2		
ἀμύνω							6											1	ἐπί	117
ἀμφοτερίζω							1											1		
ἀναγκάζω							1		5						8			3	πρός	67
ἀντάω			18															1	ἀπό	
ἀντιόομαι						25												1	ἐν	
ἀξιόω		1	1															2		98
ἀπατάω					4													1	ἐκ	8
*ἅπτομαι		2																1	ἀντί	
ἅπτω									2		2						1	3		19
ἀράσσω			1						1									2		
*ἀργυρίζω					1													1	ἐκ	
ἀρέσκω			1															1	ἀπό	21
ἀρκέω		1																1	ἀντί	12
ἁρμόζω											1							1	ξύν	
ἀρνέομαι			1															1	ἀπό	2
ἁρπάζω				4														1	διά	3
ἀρτάω			1		1						1							3		2
ἀρτύω					16													1	ἐκ	
ἄρχω											9						94	2	ὑπό	277
αὐλίζω						3	2											2	ἐν	17
αὐξάνω							2											1	ἐπί	10
αὐτομολέω			1															1	ἀπό	7
αὐχέω																1		1	ὑπέρ	1
βαδίζω				1														1	διά	
βαίνω	26		48	34	7	1	11	20	25		130	19		2	4	9		13	ξύν	1
βάλλω	5	1	3	22	8	12	8	65	9	10	9	9	4	6	67	6		16	πρός, ἐς	24
*βασείω											1							1	ξύν	
*βιάζομαι															1	1		2		
βιάζω									1									1	κατά	54
βιβάζω	2		3	3	5	2	2	2	2		1							9	ἐκ	
βιόω							2											1	ἐπί	
βλέπω			1															1	ἀπό	2
βοάω	1			2		2	9		3									5	ἐπί	5
βοηθέω		2			1		27				11	13			25			6	ἐπί, πρός	136
*βουλεύομαι				3														1	διά	

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
βουλεύω							34				2			2				3	ἐπὶ	109
βυρσόω									1									1	κατά	
γελάω									1									1	κατά	
γηράσκω						1												1	ἐν	
γίγνομαι			5	1	1	9	74				5	44	48	7	32			10	ἐπὶ	834
γιγνώσκω	3			10			4		8	7	3			3				7	διά	130
γράφω	2	1				3	3	1		2	16			1			1	9	ξὺν	19
δαμάζω									1									1	κατὰ	
δαρθάνω									1									1	κατὰ	
δατέομαι	1																	1	ἀνά	
δείδω									3									1	κατὰ	140
δείκνυμι			14			2	4							1			2	5	ἀπὸ	19
δέχομαι	1		9			13		2							55		11	6	πρός	110
δέω	9								3								1	3	ἀνά	10
δέω			2												11			2	πρός	181
δηλόω														2				1	πρὸ	59
*διαιτάομαι					1	1					1							3		6
διδάσκω				1										1				2		26
διδράσκω	3		5	1	2													4	ἀπὸ	
δίδωμι	2	5	81	2	4	38	6			3		82		31				10	παρά ἀπὸ	113
δικάζω									2									1	κατὰ	7
δικαιόω				1														1	διά	11
διώκω			2		1		11		14		2			1				6	κατά	24
δοιάζω						3												1	ἐν	
δοκέω											3			2				2	ξὺν	309
δουλόω									2									1	κατά	29
δράω											1							1	ξὺν	84
δυναστεύω												1						1	παρά	2
δύω			1	1		1			11									4	κατά	
δωσείω												1						1	παρά	
ἐγγυάω				1														1	διά	
ἕζομαι									17									1	κατὰ	
ἐθίζω											1							1	ξὺν	1
εἶδον			2			3	4		6				10	10		6		7	περὶ πρὸ	64
εἴκω												2					2	2		
εἴλλω						1												1	ἐν	
εἰμὶ			10		35	28	1			3	3	173	25		8		2	10	παρὰ	2389
εἶμι			33	6	25		83	11	7	3	11	13	1	15	29			12	ἐπὶ	116

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
εἶπον	3	15	5		1		1							20			3	7	πρὸ	144
εἴργω od. εἵργω			4	3	4				6				2					5	κατὰ	33
ἐλαύνω		2		17									1		4			4	διὰ	12
ἐλέγχω					1													1	ἐκ	3
ἐλευθερόω											4							1	ξὺν	31
ἕλκω	12	1	2				2		7								1	6	ἀνὰ	4
ἐλπίζω		1					2											2		53
ἐπείγω									1									1	κατὰ	26
ἔοικα			1															1		
ἑορτάζω				1														1	διὰ	1
ἐπιχειρέω														1				1	πρὸ	
ἕπομαι							11				8							2	ἐπὶ	15
ἐργάζομαι					9				8		1			3	1			5	ἐκ κατὰ	17
ἐρεθίζω	1																	1	ἀνὰ	
ἔρομαι							3											1	ἐπὶ	3
ἔρχομαι (ἐλθεῖν)	1		76	20	47		76	34	12	3	18	41	3	26	18			13	ἀπὸ ἐπὶ	172
ἐρωτάω							4											1	ἐπὶ	13
ἐτάζω					3													1	ἐκ	
εὐδαιμονέω						1												1	ἐν	1
εὕδω									1									1	κατὰ	
εὑρίσκω	1				2													2		42
ἐρῶ	1	5	3											8				4	πρὸ	5
*ἐχθάνομαι			4															1	ἀπὸ	
ἔχω	16	41	49	6			29		48	24	3	126	9	31	12	4	4	14	παρὰ	767
ζάω			1	1														2		20
ζεύγνυμι	1																	1	ἀνὰ	1
ζητέω	2																	1		7
ἡγέομαι				1	11			4	1								1	5	ἐκ	127
*ἠγορέω									10						2			2	κατὰ	
ἥκω	1			1					4		1	1		1	34			7	πρὸς	86
ἧμαι									13									1	κατὰ	
ἡσσάομαι		1																1	ἀντὶ	34
θάπτω											1							1	ξὺν	14
θαρσέω	6																	1	ἀνὰ	39
θαρσύνω												5						1	παρὰ	7
θειάζω							2											1	ἐπὶ	1
θεραπεύω							2											1	ἐπὶ	22
θέω				1					2						1			3		5

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
θνήσκω			65											1				2	ἀπὸ	32
θορυβέω				1														1	διά	18
θροέω				2														1	διά	
ἵζω									12									1	κατά	
ἵημι	21		47		2	2	15		6		1	5		11	2			10	ἀπὸ	1
ἱκνέομαι			192	1	1													3	ἀπὸ	2
ἱππεύω												2			1			2		
*ἰσόομαι		1																1	ἀντὶ	
ἰσόω					3													1	ἐκ	
ἵστημι	43	24	149	9	3	2	5		260	25	31	14	23	11			12	14	κατά	78
*ἰσχνέομαι																	26	1	ὑπὸ	
ἰσχυρίζω		1	1															2		7
ἴσχω							3		2				1	5	2			5	πρὸ	5
καίω									12									1	κατά	5
καλέω	5					4	30		1	1	12	23		22	3			9	ἐπὶ	78
κάμνω					1													1	ἐκ	5
καρπόω					1													1	ἐκ	2
καρτερέω						1												1	ἐν	4
κεῖμαι	2			5		12	30	1			14		1	4	18		1	10	ἐπὶ	38
*κελεύομαι				1								24						2	παρά	
κελεύω		2					2				1							3		148
κεράννυμι											1							1	ξὺν	2
*κηρυκεύομαι				1			17								1			3	ἐπὶ	
κηρύσσω	1																	1		9
κινδυνεύω			3	12							3	2		2				5	διά	55
κινέω				1														1	διά	
κλάω	2						3											2		
κλέπτω				1														1	διά	
κληρόω			2	1														2		1
κλῄω			11						4		6		3					4	ἀπὸ	5
κλίνω					1		1											2		
κλύζω							1		1				1					3		
*κοινόομαι											1							1	ξὺν	
κολάπτω					1													1	ἐκ	
κολπίζω									1									1	κατά	
κολυμβάω			1						1									2		
κομίζω	4		9	14	3			13	2		2	6	2		5			10	διά / ἐς	90
κομπέω							2											1	ἐπὶ	1
κόπτω				2	1				4				1	2				5	κατά	8

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
κοσμέω				5														1	διά	7
κράζω						1												1	ἐν	
κρατέω							17		1									2	ἐπὶ	72
κρεμάννυμι					1		4											2	ἐπὶ	
*κρίνομαι			36	10													1	3	ἀπὸ	
κρίνω	1				2									1				3		38
κρούω	3	1	5		4						2							5	ἀπὸ	7
κρύπτω			3				1											2	ἀπὸ	6
κτάομαι	1						3				2				5			4	πρὸς	46
κτείνω			68															1	ἀπὸ	20
κτίζω											1							1	ξύν	14
κύπτω						1												1	ἐν	
κυρόω							2											1	ἐπὶ	2
κωλύω			3	13			1											3	διά	66
λαμβάνω	27	14	27	1			5		67	5	27	36	1	2	24		11	13	κατὰ	207
λαμπρύνομαι						1												1	ἐν	1
λανθάνω				2														1	διά	60
*λαύω			3															1	ἀπὸ	
λέγω		18	3	3			2							1				5	ξὺν	261
λέγω					4				3		37							3		
λείπω			23	4	27	7	5		34			4		3			23	9	κατὰ	21
λεύω									1									1	κατὰ	1
λιμνάζω													1					1	περὶ	
λιμπάνω									1									1	κατὰ	
λογίζομαι	2				4													2	ἐκ	14
λογίζω														3				1	πρὸ	2
λυπέω												2						1	παρὰ	10
λύω			14	35					44			3					1	5	κατὰ / διά	46
μανθάνω							1							1				2		15
μαρτυρέω											1							1	ξὺν	
μαρτύρομαι							1											1	ἐπὶ	2
*μαχέω							1				15							2	ξὺν	
μάχομαι	1		1	5							2			1				5	διά	51
*μελέομαι							7											1	ἐπὶ	
μέλομαι							2											1	ἐπὶ	
μέλω										7								1	μετὰ	1
μέλλω		1		10														2	διά	181
μέμφομαι				1					3									2	κατὰ	13
μένω	6					17	4				4	11	15		1		29	8	ὑπὸ	57

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
μετρέω											2							1	ξύν	
μηχανάομαι		1																1	ἀντί	7
μίγνυμι							3				11				25		1	4	πρός	
μιμνήσκω	4		1				2										6	4	ὑπό	21
μίσγω							3				3				4			3		
μισθόω			1												2			2		4
μνημονεύω				1														1	διά	2
μονόω			1															1	ἀπό	7
μυθέομαι												5						1	παρά	
ναυμαχέω				4							1							2	διά	58
ναυπηγέω		2																1	ἀντί	5
νείφω																	2	1	ὑπό	
νέμω				2			1		1									3		39
*νεόομαι	9																	1	ἀνά	
νεύω					1										1			2		
νέω								1			1							2		2
νικάω					2									2				2		109
*νοέομαι			1	63													8	3	διά	
νοέω							15		6					7				3	ἐπί	
νοστέω			1														1	2		
ξηραίνω			1															1	ἀπό	
οἴγνυμι	15																	1	ἀνά	
οἶδα											11			8				2	ξύν	123
οἰκέω			1	1	1	8	2		8		3	3			3			9	κατά	85
*οἰκίζομαι	4					1												2	ἀνά	
οἰκίζω			1		2				21		7							4	κατά	26
οἰκοδομέω	2		2	2		6	1	1			1	4	1		2			10	ἐν	22
οἰκτίζω		1																1	ἀντί	1
οἰμώζω	1																	1	ἀνά	
ὀκέλλω							2											1	ἐπί	3
ὀκνέω			9						2									2	ἀπό	4
ὀλισθάνω			1															1	ἀπό	
ὄλλυμι			41	3														2	ἀπό	1
ὀλοφύρομαι	1		1												1			3		4
ὁμιλέω															2			1	πρός	5
ὄμνυμι			1				1				12							3	ξύν	20
ὁμολογέω											1							1	ξύν	18
ὀνομάζω		1					4											2	ἐπί	17
ὀξύνω												4						1	παρά	
ὀπτεύω																	16	1	ὑπό	

Verben	ἀνὰ	ἀντὶ	ἀπὸ	διὰ	ἐκ	ἐν	ἐπὶ	ἐς	κατὰ	μετὰ	ξὺν	παρὰ	περὶ	πρὸ	πρὸς	ὑπὲρ	ὑπὸ	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
ὁράω						1	3		6				28	6		3	1	7	περὶ	155
ὀρθόω	1								21									2	κατὰ	10
ὁρκόω					4													1	ἐκ	2
ὁρμάω			6		3													2	ἀπὸ	28
ὁρμέω		5					22						2					3	ἐπὶ	19
ὁρμίζω							1		7	1			1	1			1	6	κατὰ	15
ὀρύσσω				1					1			1						3		1
ὀτρύνω					2		2											2		
ὀφείλω		1					1							1	2			4		
παλάσσω						1												1	ἐν	
πάσχω		2												3				2		91
παταγέω		1																1	ἀντὶ	
πατέω									2									1	κατὰ	
παύω	4				1				4									3	ἀνὰ / κατὰ	41
πείθω	15										1							2	ἀνὰ	213
πειράω	3		9															2	ἀπὸ	86
πέμπω	1	4	45	9	42		2	9		21	10	5	6	14	6		1	14	ἀπὸ / ἐκ	203
περαιόω				3														1	διὰ	27
πήγνυμι									2									1	κατὰ	4
πηλακίζω														2				1	πρὸ	
πίμπλημι	1		1	1	1	1												5		1
πίμπρημι						15							1					2	ἐν	1
πίπτω	1		1		22	11	18	16	2	1	9	1	8		53		1	13	πρὸς	9
πλέκω											1							1	ξὺν	
πλέω	1	2	59	11	37	1	42	24	29		15	61	35	1	22			14	παρὰ / ἀπὸ	203
*πλήγνυμι					1													1	ἐκ	
πληρόω	1	3					1				2				3			5		51
πλήσσω					18				13									2	ἐκ	5
πνέω					2	2												2		
ποιέω		2				4				2		1	8	1	17			7	πρὸς	433
πολεμέω		2		4	1				5		16				3			6	ξὺν	109
πολεμόω					2										1			2		4
πολιορκέω		1		1	11						3							4	ἐκ	59
πολιτεύω						2					3							2		14
πονέω					1													1	ἐκ	9
πορεύω				2		1					1							3		47
πορθέω				3	2													2		14

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
πορίζω					4						3							2		20
πράσσω	2			6	1						13							4	ξύν	191
πρεσβεύω		1									2							2		20
προθυμέομαι											3							1	ξύν	15
πυνθάνομαι														2				1	πρό	66
ῥάσσω											1							1	ξύν	
ῥέω									1				2					2		5
ῥήγνυμι	3		2						1		1	3						5		
ῥίπτω	4																	1	ἀνά	3
ῥώννυμι	1						8											2	ἐπί	5
σαλεύω			3															1	ἀπό	
σείω	1								1					1				3		2
σημαίνω			1				1										2	3		17
σιμόω			1															1	ἀπό	
σιτίζω							3											1	ἐπί	
σκάπτω									4									1	κατά	1
σκεδάννυμι			1	2														2		7
σκέπω									3									1	κατά	
σκευάζω	2			1			8		10		1	164						6	παρά	3
σκήπτω							2		1									2		1
σκοπέω	2			5									2	5				4		37
σκοτάζω											2							1	ξύν	
σπάω	3		2	6			5		1									5	διά	
σπείρω				2														1	διά	
σπένδω							1											1	ἐπί	41
σπέργω							1		1									2		
σπεύδω			1															1	ἀπό	10
σταυρόω	1		3	1									1	1	1			6	ἀπό	2
στέλλω	3		64				12				1			1				5	ἀπό	4
στερέω			6															1	ἀπό	16
στρατεύω					8		8				28							3	ξύν	111
στρατοπεδεύομαι		2			1	1												3		27
στρέφω	5		6				3		19		9						2	6	κατά	
σύρω			1															1	ἀπό	
σφάζω			2															1	ἀπό	2
σῴζω			_	23							1							2	διά	54
ταλαιπωρέω							1								1			2		16
ταράσσω											1							1	ξύν	13
τάσσω		10		6			11			1	14	19		3	22			8	πρός / παρά	65

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d. einfach. V.
ταχύνω							1											1	ἐπὶ	
τείνω												3				1	2	3		
τειχίζω			19		4	1	2				2		11		1		1	8	ἀπὸ	61
τεκμαίρομαι											1							1	ξὺν	3
τελέω			3	3			10				2				1		1	6	ἐπὶ	4
τελευτάω						1												1	ἐν	89
τέμνω			1			1					3							3	ξὺν	35
τεχνάομαι					1													1	ἐκ	3
τηρέω							3											1	ἐπὶ	16
τίθημι	13	3	1	3			39	1	19	2	18	1	3	19	36		4	14	ἐπὶ / πρός	39
τίκτω						1												1	ἐν	
τιμάω							3							6				2	πρὸ	24
τιμωρέω		1												2				2		25
τίνω					2													1	ἐκ	
τολμάω		2	1															2		33
τοξεύω									1									1	κατὰ	4
*τοπεύω																	1	1	ὑπὸ	
τοπέω																	8	1	ὑπὸ	
τραυματίζω									5									1	κατὰ	4
τρέπω			22		1		24					1		2				5	ἐπὶ / ἀπὸ	105
τρέφω				1														1	διὰ	11
τρέχω	1			2	1		3	2	4		1							7	κατὰ	
τρίβω				7					1		1							3	διὰ	7
τρυχόω					2													1	ἐκ	3
τυγχάνω		1				11	7				2	10	12	1				7	περὶ	135
ὑβρίζω					2		1											2		7
ὑστερίζω							1											1	ἐπὶ	1
φαίνω	1		19	4	1		1		1							2	1	8	ἀπὸ	101
φέρω	5		7	24	3		40	10	9	1	42	3	1	12	20	5		14	ξὺν / ἐπὶ	78
φεύγω			6	44	1				34								4	5	διὰ	58
φθάνω														3				1	πρὸ	50
φθείρω			1	151														2	διὰ	40
φθλέγω							1		1									2		
φοβέω					8				2	1								3	ἐκ	108
φοιτάω							2											1	ἐπὶ	7
φορέω	1			2				1			1							4		4
φράγνυμι			1															1	ἀπὸ	

Verben	ἀνά	ἀντί	ἀπό	διά	ἐκ	ἐν	ἐπί	ἐς	κατά	μετά	ξύν	παρά	περί	πρό	πρός	ὑπέρ	ὑπό	Zahl der Präpos.	Bevorzugte Pr.	Zahl des Vorkomm. d einfach. V.
φράσσω			1			2												2		4
*φρέω				1														1	διά	
φρονέω									13				1			3		3	κατά	12
φροντίζω					1													1	ἐκ	
φρουρέω						2							1					2		22
φυγγάνω				1														1	διά	
φυλάσσω				1										3				2	πρό	
φωράω									2									1	κατά	1
χειμάζω				4			1											2	διά	8
χειρίζω						2				5								2	μετά	
χειροτονέω		2																1	ἀντί	
χέω				2														1	διά	
χόω															2			1	πρός	4
χράω			3	3			1											3		143
χρηματίζομαι					1													1	ἐκ	
χρονίζω						1												1	ἐν	2
χωλόω			1															1	ἀπό	
χωρέω	144		52				1			2	25			37	45		30	8	ἀνά	103
ψηφίζομαι				1					1									2		48
ψηφίζω	1						4											2	ἐπί	
ψύχω			1	1														2		
ὠθέω	1		17	2	9								2					5	ἀπό	6
(397)																				

Tafel II.

Statistik für diprothetische Verben.

Verben	Zahl der Kombin.	Präpositionale Kombinationen	Zahl d. Vork.	Verben	Zahl der Kombin.	Präpositionale Kombinationen.	Zahl d. Vork.
ἀγορεύω	1	προσκατα	1			προανα	1
ἄγω	10	ἀνθυπο	1			προεκ	2
		ἀντανα	12	αἰνέω	2	ξυνεπι	1
		ἀντεπι	2			προεπι	1
		ἐξανα	2	αἱρέω	6	ἐπικατα	1
		ἐπανα	2			ξυνανα	1
		ἐπεκ	3			ξυναπο	1
		ἐπικατα	2			ξυγκατα	5
		ξυνεπι	4			προσανα	1

Verben	Zahl d. Kombin.	Präpositionale Kombinationen	Zahl d. Vork.	Verben	Zahl d. Kombin.	Präpositionale Kombinationen	Zahl d. Vork.
		ὑπεκ	1			προδια	2
αἴρω	1	ἐπανα	1			προκατα	1
αἰτέω	1	ἀνταπο	1	δαρθάνω	1	ἐπικατα	1
αἰτιάομαι	1	ξυνεπι	1	δίδωμι	3	ἀνταπο	7
*ἁλίσκω	3	ἀπανα	4			καταπρο	7
		προανα	2			ὑπεν	1
		ὑπανα	2	διώκω	1	ξυγκατα	1
ἀλλάσσω	1	ἐξαπο	1	δουλόω	1	ξυγκατα	2
ἁμαρτάνω	1	ξυνεκ	1	ἕζομαι	3	ἀντικατα	2
ἀμύνω	1	ξυνεπι	1			ἐγκατα	2
ἀντάω	1	προαπο	3			προσκατα	6
*ἄρχομαι	1	προκατα	1	εἰμί	2	ἐπιπαρα	1
ἄρχω	1	προϋπο	5			ξυμπαρα	1
ᾄσσω	1	προεκ	1	εἶμι	11	ἀντανα	1
βαίνω	9	ἐπανα	1			ἀντεπι	5
		ἐπεκ	2			ἀντιπρο	1
		ἐπεσ	2			ἐπεκ	20
		ἐπιδια	1			ἐπικατα	1
		ἐπικατα	5			ἐπιπαρα	5
		ξυνδια	1			ξυνεκ	1
		ξυγκατα	1			ξυνεπι	1
		προανα	1			προεκ	1
		ὑποκατα	1			προσανα	1
βάλλω	8	ἐπεσ	1			ὑπαπο	2
		ξυνδια	2	εἶπον	1	ἐπανα	1
		ξυνεσ	2	ἐργάζομαι	1	ξυγκατα	1
		προδια	1	ἔρχομαι	10	διεκ	3
		προεν	1			ἐπανα	3
		προπαρα	1			ἐπεκ	23
		προσξυν	1			ἐπεσ	2
		προσπερι	2			ξυνεκ	1
βιβάζω	3	ἀντεν	1			ξυνεσ	2
		ἐπανα	1			παρεκ	1
		μετεν	1			προαπο	2
βοηθέω	1	ἐπεκ	1			προεκ	1
βουλεύω	2	ἀντεπι	3			ὑπεκ	5
		προεπι	1	εὔχομαι	1	ξυνεπι	1
γίγνομαι	2	ξυμπαρα	2	ἔχω	6	ἀντιπαρα	1
		προξυν	2			ἐμπαρα	2
γιγνώσκω	3	ξυνδια	2			παρακατα	1

Verben	Zahl d. Kombin.	Präpositionale Kombinationen	Zahl d. Vork.	Verben	Zahl d. Kombin.	Präpositionale Kombinationen	Zahl d. Vork.
		προανα	1			ξυνεπι	5
		προκατα	1			ξυγκατα	2
		προσπαρα	1			προκατα	16
ἧμαι	6	ἀντικατα	1	λέγω	1	ἐγκατα	1
		ἐπικατα	1	λείπω	4	ἐγκατα	11
		ξυγκατα	2			ξυγκατα	1
		παρακατα	1			παρακατα	1
		προκατα	1			προσκατα	2
		προσκατα	7	λυπέω	1	ἀντιπαρα	1
θέω	2	ἐπεκ	2	μελέομαι	1	ξυνεπι	1
		προεκ	1	μένω	2	ἀντανα	1
θνῄσκω	1	ἐναπο	2			ξυμπαρα	1
ἵημι	2	διεκ	1	νέμω	1	ξυγκατα	1
		ἐπανα	1	νεύω	1	ξυναπο	1
ἱκνέομαι	2	προαπο	2	νέω	1	ἐπιπαρα	1
		προσαπο	1	οἰκέω	1	προσξυν	3
ἰσοω	1	ἐπανα	1	οἰκίζω	1	ξυγκατα	4
ἵστημι	13	ἀντικατα	7	οἰκοδομέω	1	ἐγκατα	1
		ἀπανα	7	ὄλλυμι	3	ξυναπο	2
		ἐγκατα	3			προαπο	2
		ἐξανα	4			προσαπο	2
		ἐπανα	5	ὀπτεύω	1	ἀνθυπο	1
		ἐπικατα	1	ὀρθόω	1	ἐπανα	1
		μετανα	3	ὁρμίζω	1	ἐγκατα	1
		ξυναπο	8	πείθω	1	ξυνανα	1
		ξυνεπι	1	πέμπω	5	ἐπιμετα	2
		ξυγκατα	3			ξυμπρο	1
		προαπο	1			προαπο	1
		προκατα	1			προσμετα	2
		προσαπο	1			ὑπεκ	1
ἴσχω	1	παρανα	1	πήγνυμι	1	παρακατα	1
καλέω	2	ἀντιπαρα	1	πίπτω	1	ξυνεσ	2
		προσπαρα	3	πλέω	6	ἀντεκ	1
καλεύομαι	1	ἀντιπαρα	1			ἀντιπαρα	1
κεῖμαι	1	ὑπεκ	2			διεκ	2
κλάω	1	ἐναπο	1			ἐπεκ	1
κομίζω	2	ξυμπαρα	2			ἐπεσ	2
		ὑπεκ	1			ξυνεκ	1
λαμβάνω	5	ἐγκατα	8	πολεμέω	1	ξυνδια	1
		ἐπικατα	2	σκευάζομαι	1	ἀντιπαρα	3

Verben	Zahl d. Kombin.	Präpositionale Kombinationen	Zahl d. Vork.	Verben	Zahl d. Kombin.	Präpositionale Kombinationen	Zahl d. Vork.
σκευάζω	2	ξυγκατα	1			ξυνεκ	1
		προπαρα	3	φεύγω	2	πρ κατα	3
σκήπτω	1	ἐγκατα	1			ὑπεκ	2
στέλλω	4	ξυναπο	1	φθείρω	1	προδια	2
		προαπο	3	χωρέω	5	ἐξανα	11
		προσαπο	1			ἐπανα	15
		προσεπι	2			προανα	1
στρατεύω	1	ξυνεπι	1			προαπο	1
στρέφω	2	ἐπανα	2			ὑπανα	1
		ξυγκατα	1	ψεύδομαι (86)	1	ἐπικατα	1
σῴζω	1	ξυνδια	3	**Triprothetische Verben.**			
τάσσω	2	ἀντεπι	1				
		ἀντιπαρα	6	ἄγω	3	ἀντεπανα	1
τειχίζω	1	ἀντεπι	1			ἀντεπεκ	1
τίθημι	3	ἀντεπι	1			ὑπεξανα	1
		ξυνεπι	5	εἶμι	1	ἀντεπεκ	2
		ὑπεκ	1	ἐλαύνω	1	ἀντεπεκ	1
τρίβω	1	ἐνδια	5	ἔρχομαι	1	ἀντεπεκ	1
φαίνω	1	ἀνταπο	2	εὑρίσκω	1	προσεπεκ	1
φέρω	3	ἐπεσ	1	ἵστημι	1	ξυνεπανα	1
		ἐπιδια	1	σείω (7)	1	προεπανα	1

Tafel III
Präpositionale Kombinationen.

a) Diprothetische Verben.

Kombin.	Zahl d. Verben	Kombin.	Zahl d. Verben	Kombin.	Zahl d. Verben	Kombin.	Zahl d. Verben
		ἀντιπρο	1	ἐπιδια	2	ξυναπο	5
		ἀπανα	2	ἐπεκ	7	ξυνδια	5
ἀνθυπο	2	διεκ	3	ἐπες	5	ξυνεκ	5
ἀντανα	4	ἐξανα	3	ἐπικατα	9	ξυνες	3
ἀνταπο	3	ἐξαπο	1	ἐπιμετα	1	ξυνεπι	11
ἀντεπ	1	ἐναπο	2	ἐπιπαρα	3	ξυγκατα	13
ἀντεν	1	ἐνδια	1	καταπρο	1	ξυμπαρα	4
ἀντεπι	6	ἐγκατα	8	μετανα	1	ξυμπρο	1
ἀντικατα	3	ἐμπαρα	1	μετεν	1	παρανα	1
ἀντιπαρα	7	ἐπανα	12	ξυνανα	2	παρεκ	1

Kombin.	Zahl d. Verben
παραχατα	4
προανα	5
προαπο	8
προδια	3
προεχ	5
προεν	1
προεπι	2
προχατα	7

Kombin.	Zahl d. Verben
προξυν	1
προπαρα	2
προϋπο	1
προσανα	2
προσαπο	4
προσεπι	1
προσχατα	4
προσμετα	1

Kombin.	Zahl d. Verben
προσξυν	2
προσπαρα	2
προσπερι	1
ὑπανα	2
ὑπαπο	1
ὑπεχ	7
ὑπεν	1
ὑποχατα	1
(66)	

b) Triprothetische Verben.

Kombin.	Zahl d. Verben
ἀντεπανα	1
ἀντεπεχ	4
ξυνεπανα	1
προεπανα	1
προσεπεχ	1
ὑπεξανα	1
(6)	

Tafel IV.

Statistik für die Präpositionen.

Prä-posi-tionen	Zahl d. Vork. d. einfach. Pr.	Zahl d. Verb. in Komposit.	Zahl d. mono-proth. Verb.	Zahl d. di-proth. Verb.	Zahl d. tri-proth. Verb.	Zahl der Exklusiva	Zahl der ἅπαξ εἰρ.	Zahl d. Vork. m. bevorzug-tem Verb	Zahl d. Vork. m. bevorzugt. Präposition	Stelle in der Rangordn. d. verbund. Pr.	Stelle in der Rangordn. d. unverb. Pr.	Bevorzugte Verben
ἀμφὶ	2											
ἀνὰ	2	77	77			17	9	144	5	9	16	χωρέω
ἀντὶ	52	80	48	27	5	10	7	41		8	14	ἔχω, ἵστημι, εἶπον
ἀπὸ	634	114	112	2		23	15	192	22	3	6	ἱχνέομαι
διὰ	534	101	98	3		18	10	151	14	6	8	φθείρω
ἐχ	897	89	85	4		17	9	47	8	7	4	ἔρχομαι, πέμπω
ἐν	1794	67	55	12		13	11	38	3	11	1	δίδωμι
ἐπὶ	1216	156	117	39		20	5	83	23	1	3	εἶμι, γίγνομνι, ἔρχομαι
ἐς	1692	23	23					65	1	16	2	βάλλω
χατά	861	105	104	1		25	12	260	16	5	5	ἵστημι
μετά	619	24	22	2		1		25		15	7	ἵστημι
ξὺν	35	154	103	50	1	19	13	130	10	2	15	βαίνω
παρὰ	282	54	48	6		8	4	173	7	13	11	εἰμί
περὶ	478	43	43			2	2	48	3	14	9	γίγνομαι
πρὸ	80	105	69	35	1	6	1	37	7	5	12	χωρέω
πρὸς	861	74	56	17	1	2		67	11	10	5	βάλλω
ὑπὲρ	64	11	11			1	1	9		17	13	βαίνω
ὑπὸ	422	58	45	12	1	5	1	94	3	12	10	ἄρχω

Tafel V.

16 Präpositionen

βάλλω

(1)

14 Präpositionen

ἄγω
ἔχω
ἵστημι
πέμπω
πλέω
φέρω

(6)

13 Präpositionen

βαίνω
ἔρχομαι (ἐλθεῖν)
λαμβάνω
πίπτω
τίθημι

(5)

12 Präpositionen

αἱρέω (9) -ομαι (3)
εἰμί

(2)

10 Präpositionen

γίγνομαι
δίδωμι
εἶμι
ἵημι
κεῖμαι
κομίζω
οἰκοδομέω

(7)

9 Präpositionen

ἀγγέλλω
βιβάζω
γράφω
καλέω
λείπω
οἰκέω

(6)

8 Präpositionen

μένω
τάσσω
τειχίζω
φαίνω
χωρέω

(5)

7 Präpositionen

ἀλλάσσω
γιγνώσκω
εἶδον
εἶπον
ποιέω
τρέχω
τυγχάνω

(7)

6 Präpositionen

βοηθέω
δέχομαι
διώκω
ἕλκω
ἥκω
κρίνω (3) -ομαι (3)
νοέω (3) -ομαι (3)
οἰκίζω (4) -ομαι (2)
ὁράω
ὁρμίζω
πολεμέω
σκευάζω
σταυρόω
στρέφω
τελέω

(15)

5 Präpositionen

ἀγωνίζομαι
αἴρω
βοάω
δείκνυμι
εἴργω
ἐργάζομαι
ἡγέομαι
ἴσχω
κελεύω (3) -ομαι (3)
κινδυνεύω
κόπτω
κρούω
λέγω
λύω
μάχομαι
πίμπλημι
πληρόω
ῥήγνυμι
σπάω
στέλλω
τρέπω
φεύγω
ὠθέω

(23)

4 Präpositionen

ἀγορέω
ἀκούω
ἅπτω (3) -ομαι (1)
βουλεύω (3) -ομαι (1)
διδράσκω
δύω
ἐλαύνω
ἐρῶ
ἰσόω (3) -ομαι (1)
κλῄω
κτάομαι
μίγνυμι
μιμνῄσκω
ὀφείλω
πολιορκέω
πράσσω
σκοπέω
φορέω

(18)

3 Präpositionen

βιάζω (1) -ομαι (2)
αἰνέω
αἰτέω (2) -ομαι (1)
ἀμύνω (1) -ομαι (2)
ἀναγκάζω
ἀρτάω
δέω
*διαιτάομαι
θέω
ἱκνέομαι
κηρυκεύομαι
κλύζω
κωλύω
λέγω
μίσγω
νέμω
ὀλοφύρομαι
ὄμνυμι
ὁρμέω
ὀρύσσω
παύω
πορεύω
σείω
σημαίνω
στρατεύω
στρατοπεδεύομαι
τείνω
τέμνω
τρίβω
φοβέω
φρονέω
χράω

(32)

2 Präpositionen

(97)

s. Tafel I.

1 Präposition

(181)

s. Tafel I.

Summa: 387, wobei zehn Media nicht gezählt sind.

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